ROHINI COLLEGE OF ENGINEERING AND TECHNOLOGY
AUTONOMOUS INSTITUTION
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DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

VII Semester

AU3008 Sensors and Actuators

UNIT – I - INTRODUCTION TO MEASUREMENTS AND SENSORS

1.8 Mathematical model of transducers
- Zero, First and Second order transducers

Mathematical Model of Transducers:

- ❒ The mathematical models are the differential equations that describe the dynamics of transducers.
  - ❖ These models can be derived from the knowledge of the components, their interconnection and the physical laws governing their functioning.
  - ❖ A number of assumptions are needed to derive the equations representing the model.
  - ❖ But practically, the components used, their values, their behaviour, their interconnections and the physical laws followed by them may not be precisely known.
  - ❖ Therefore, using conventional method, the model cannot be obtained.
  - ❖ In such situations, the transducer can be assumed to be a black box, whose inputs and outputs are accessible for measurements.
  - ❖ Number of methods are available to identify the transducer model by measuring the inputs and outputs of the transducer.
  - ❖ If the order of the model is known already, then the method of identification becomes simple.

(1) Identification of transducer mathematical models:

Identification from Impulse response

- Let the transfer function of the transducer be of the form $\frac{K}{(1+\tau s)}$ where K and τ are the only unknown parameters.
- When this transducer is excited with an input impulse, the output transform

$$Y(s) = \frac{K}{(1+\tau s)}$$

as $$R(s) = 1$$

- Therefore $y(t) = L^{-1}\,Y(s)$

$$= Ke^{-t/\tau} \quad \text{... (2.45)}$$

- The output of the transducer is shown in fig. (2.20)

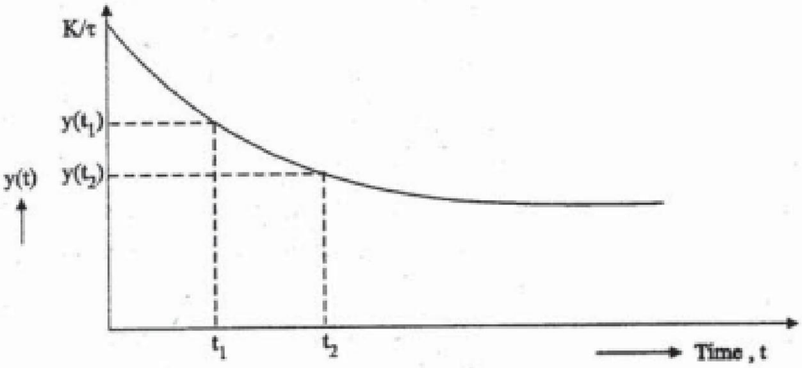

Fig. 2.20 First-order transducer response for impulse signal.

- From the experimentally obtained outputs of the transducer $y(t_1)$ and $y(t_2)$ at two different times t_1 and t_2, the two unknown parameters K and τ of the transducer can be estimated.

$$y(t_1) = \frac{K}{\tau} e^{-t_1/\tau} \quad \text{... (2.46)}$$

$$y(t_2) = \frac{K}{\tau} e^{-t_2/\tau} \quad \text{... (2.47)}$$

$$\frac{y(t_1)}{y(t_2)} = e^{(t_2 - t_1)/\tau} \quad \text{... (2.48)}$$

$$T = \frac{(t_2 - t_1)}{\ln \frac{y(t_1)}{y(t_2)}}$$

- K can be calculated by substituting τ in one of the above equations.
- If the transfer function is of the form

$$\frac{K}{\frac{s^2}{\omega_n^2} + \frac{2\xi}{\omega_n} s + 1}$$

- where ξ, ω_n and K are the unknown parameters.
- When such a system is subjected to an unit impulse, the response for the underemployed case will be as shown in fig. (2.21)

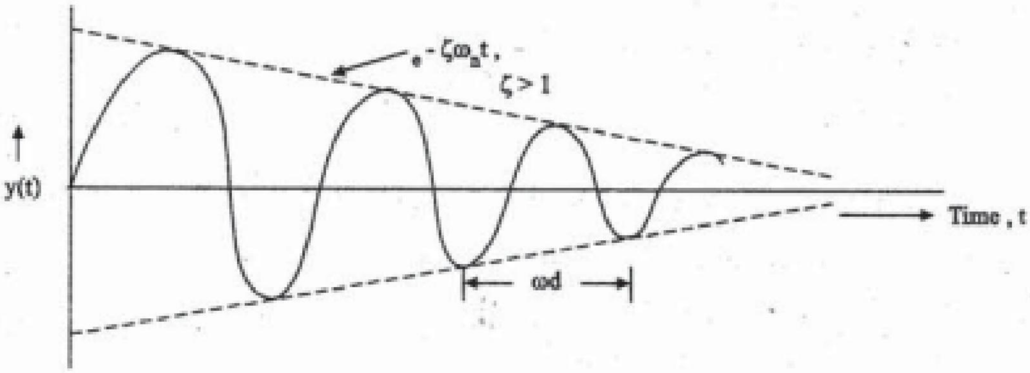

Fig. 2.21. Response of II - order transducer for impulse input signal.

$$y(t) = \frac{K\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin \omega_n [\sqrt{1-\xi^2}\, t] \qquad \text{... (2.49)}$$

- From the experimental output curve, ξ, ω_n is calculated taking the envelope (dotted line) only.
- As the envelope is a decaying exponential curve, $\frac{1}{\xi\omega_n}$ is the time constant of the exponential curve.
- The time between two successive peaks T_d is determined which is equal to

$$\omega_d = \frac{2\pi}{T_d} = \omega_n \sqrt{1-\xi^2} \qquad \text{... (2.50)}$$

- Now the values of ξ, and ω_n are determined from the values of $\frac{1}{\xi\omega_n}$ and the value of the envelope at $t = 0$ is

$$\frac{K\omega_n}{\sqrt{1-\xi^2}} \quad \text{... (2.51)}$$

which can be determined from the experimental response.

- As ξ and ω_n have already been evaluated, K can be calculated.

(2) Identification from step response

- When the transfer function of the transducer is of the form $\frac{K}{(1+\tau s)}$, the parameters K and τ have to be determined from the step response.
- The static sensitivity K is calculated as

$$K = \frac{\text{Steady state output charge}}{\text{Input change}} \quad \text{... (2.52)}$$

- For a second-order transducer, the parameters, K, ξ, and ω_n can be determined from the step response.
- The response of an under damped transducer for an unit step input is shown in fig. (2.22).
- The expression for the output $y(t)$ of the transducer is given by

$$y(t) = K\left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\right]\sin(\omega_n\sqrt{1-\xi^2}\,t + \phi) \quad \text{... (2.53)}$$

- The time instances at which the maximum and minimum values of the response curve occur can be found out by differentiating $y(t)$ with respect to time and equating to zero as shown below.

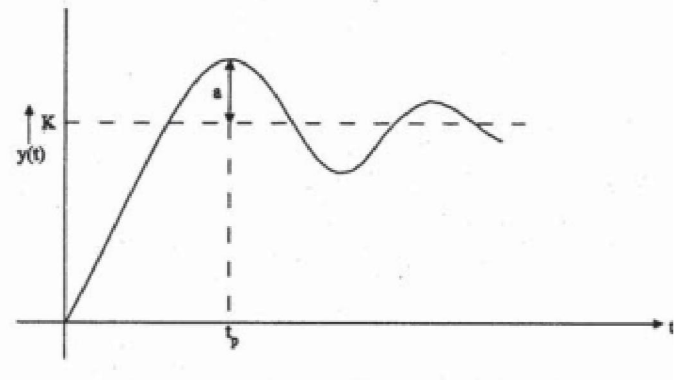

Fig. (2.22) second - order transducer response for step input.

$$\frac{dy(t)}{dt} = K \left| \xi\omega_n \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_n \sqrt{1-\xi^2}\, t + \phi) \right.$$

$$\left. - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot \cos(\omega_n \sqrt{1-\xi^2}\, t + \phi) \cdot \omega_n \sqrt{1-\xi^2} \right| \quad \text{... (2.54)}$$

When this expression is equated to zero, one gets,

$$\tan(\omega_n \sqrt{1-\xi^2}\, t + \phi) = \frac{\sqrt{1-\xi^2}}{\xi}$$

- $\phi = \tan^{-1} \frac{\sqrt{1-\xi^2}}{\xi^2}$ (by definition)
- Therefore, $\tan(\omega_n \sqrt{1-\xi^2}\, t + \phi) = \tan\phi$ ... (2.55)
- This equation is true for all values of

$$t = 0, \frac{\pi}{\omega_n \sqrt{1-\xi^2}}, \frac{2\pi}{\omega_n \sqrt{1-\xi^2}}, \frac{3\pi}{\omega_n \sqrt{1-\xi^2}} \text{ etc.} \quad \text{... (2.56)}$$

- when $t = 0$, $y(t)$ is 0, minimum value

$$t = t_p = \frac{\pi}{\omega_n \sqrt{1-\xi^2}} \quad \text{... (2.57)}$$

- $y(t)$ is the first maximum and t_p is the peak time.

$$t = t_v = \frac{2\pi}{\omega_n \sqrt{1-\xi^2}} \quad \text{... (2.58)}$$

- $y(t)$ - second minimum and t_v - valley time.
- As the oscillation is a damped one, the time at which the first maximum occurs will be the maximum overshoot.
- Therefore this overshoot shown as 'a' in fig. (2.22) can be obtained as

$$a = y(t)|_{max} - y(t)|_{steady\ state}$$

- $y(t)_{max}$ is obtained by substituting

$t_p = \dfrac{\pi}{\omega_n \sqrt{1-\xi^2}}$ in equation (2.59)

i.e.,

$$y(t)\big|_{max} = K\left\{\frac{1 - e^{-\xi\omega_n \frac{\pi}{\omega_n}\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}}\sin\left(\omega_n\sqrt{1-\xi^2}\cdot\frac{\pi}{\omega_n\sqrt{1-\xi^2}}+\phi\right)\right\} \quad \text{... (2.60)}$$

$$= K\left\{1 + \frac{e^{-\pi\xi/\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}}\sin\phi\right\} \quad \text{... (2.61)}$$

$$= K\left[1 + e^{-\pi\xi/\sqrt{1-\xi^2}}\right] \text{ as } \sin\phi = \sqrt{1-\xi^2} \quad \text{... (2.62)}$$

$$y(t)\big|_{steady\ state} = \underset{t\to\infty}{Lt}\ y(t)$$

$$= K(1-0)$$

$$= K$$

$$\therefore \text{ Overshoot, } a = K e^{-\pi\xi/\sqrt{1-\xi^2}} \quad \text{... (2.63)}$$

- From the step response plotted from experimental results, t_p, a and K can be obtained from equation (2.52). ξ can be calculated from equation (2.63) as a and K are already known.
- Substituting this value of ξ in equ. (2.58), ω_n can be determined.

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