ROHINI COLLEGE OF ENGINEERING & TECHNOLOGY

DEPARTMENT OF MATHEMATICS

BA4201 / QUANTITATIVE TECHNIQUES FOR DECISION MAKING

UNIT V : SIMULATION

INTRODUCTION:

Simulation technique is considered as a valuable tool because of its wide area of application. It can be used to solve and analyze large and complex real world problems. Simulation provides solutions to various problems in functional areas like production, marketing, finance, human resource, etc., and is useful in policy decisions through corporate planning models.

Advantages

- Simulation is best suited to analyze complex and large practical problems when it isnot possible to solve them through a mathematical method.
- Simulation is flexible, hence changes in the system variables can be made to selectthe best solution among the various alternatives.
- In simulation, the experiments are carried out with the model without disturbing the system.
- Policy decisions can be made much faster by knowing the options well in advanceand by reducing the risk of experimenting in the real system.

Disadvantages

- Simulation does not generate optimal solutions.
- It may take a long time to develop a good simulation model.
- In certain cases simulation models can be very expensive.
- The decision-maker must provide all information (depending on the model) aboutthe constraints and conditions for examination, as simulation does not give the answers by itself.

MONTE CARLO SIMULATION

In simulation, we have deterministic models and probabilistic models. Deterministic simulation models have the alternatives clearly known in advance and the choice is made by considering the various well-defined alternatives. Probabilistic simulation model is stochastic in nature and all decisions are made under uncertainty. One of the probabilistic simulation models is the Monte Carlo method. In this method, the decision variables are represented by a probabilistic distribution and random samples are drawn from probability distribution using random numbers. The simulation experiment is conducted until the required number of simulations are generated. Finally, the best course of action is selected for implementation. The significance of Monte Carlo Simulation is that decision variables may not explicitly follow any standard probability distribution such as Normal, Poisson, Exponential, etc. The distribution can be obtained by direct observation or from past records.

Procedure for Monte Carlo Simulation:

Step 1: Establish a probability distribution for the variables to be analyzed.

Step 2: Find the cumulative probability distribution for each variable.

Step 3: Set Random Number intervals for variables and generate random numbers.

Step 4: Simulate the experiment by selecting random numbers from random numberstables until the required number of simulations are generated.

Step 5**:** Examine the results and validate the model.

Problem 1: An ice-cream parlor's record of previous month’s sale of a particular varietyof ice cream as follows

Demand (No. of Ice-creams)	No. of days
4	5
5	10
6	6
7	8
8	1

Simulate the demand for first 10 days of the month

Solution: Find the probability distribution of demand by expressing the frequencies in terms of proportion. Divide each value by 30. The demand per day has the following distribution

Probability Distribution of Demand

Demand	Probability
4	0.17
5	0.33
6	0.20
7	0.27
8	0.03

Find the cumulative probability and assign a set of random number intervals to variousdemand levels. The probability figures are in two digits, hence we use two digit randomnumbers taken from a random number table. The random numbers are selected from the table from any row or column, but in a consecutive manner and random intervals areset using the cumulative probability distribution as shown in the following table.

Cumulative Probability Distribution

Demand	Probability	Cumulative Probability	Random Number Interval
4	0.17	0.17	00-16
5	0.33	0.50	17-49
6	0.20	0.70	50-69
7	0.27	0.97	70-96
8	0.03	1.00	97-99

To simulate the demand for ten days, select ten random numbers from random numbertables.

The random numbers selected are, 17, 46, 85, 09, 50, 58, 04, 77, 69 and 74

The first random number selected, 7 lies between the random number interval 17-49 corresponding to a demand of 5 ice-creams per day. Hence, the demand for day oneis 5. Similarly, the demand for the remaining days is simulated as shown in the following table

Demand Simulation

Day	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Random Number	17	46	85	09	50	58	04	77	69	74
Demand	5	5	7	4	6	6	4	7	6	7

Problem 2: A dealer sells a particular model of washing machine for which the probabilitydistribution of daily demand is the following table

Probability Distribution of Daily Demand

Demand/day	**-**	**0**	**1**	**2**	**3**	**4**	**5**
Demand	-	0.05	0.25	0.20	0.25	0.10	0.15

Find the average demand of washing machines per day.

Solution: Assign sets of two digit random numbers to demand levels are

Random Numbers Assigned to Demand

Demand	**Probability**	**Cumulative Probability**	**Random Number Intervals**
0	0.05	0.05	00-04
1	0.25	0.30	05-29
2	0.20	0.50	30-49
3	0.25	0.75	50-74
4	0.10	0.85	75-84
5	0.15	1.00	85-99

Ten random numbers that have been selected from random number tables are

68, 47, 92,76, 86, 46, 16, 28, 35, 54.

To find the demand for ten days

Ten Random Numbers Selected

Trial No	**Random Number**	**Demand / day**
1	68	3
2	47	2
3	92	5
4	76	4
5	86	5
6	46	2
7	16	1
8	28	1
9	35	2
10	54	3
Total Demand		**28**

Average demand =28/10 =2.8 washing machines per day.The expected demand /day can be computed as, Simulation Expected demand per day $\sum_{i=1}^{n} p_i x_i$

where, p_i = probability and x_i = demand

$= (0.05 \times 0) + (0.25 \times 1) + (0.20 \times 2) + (0.25 \times 3) + (0.1 \times 4) + (0.15 \times 5)$

= 2.55 washing machines.

The average demand of 2.8 washing machines using ten-day simulation differs significantly when compared to the expected daily demand. If the simulation is repeated number of times, the answer would get closer to the expected daily demand.

SIMULATION OF QUEUING PROBLEMS

Problem 1: Mr. Srinivasan, owner of Citizens restaurant is thinking of introducing separate coffee shop facility in his restaurant. The manager plans for one service counter for the coffee shop customers. A market study has projected the inter-arrival times at the restaurant as given in the following table. The counter can service the customers at the following rate:

Simulation of Queuing Problem

Inter–arrival times		**Service times**	
Time between two consecutive arrivals (minutes)	**Probability**	**Service time (minutes)**	**Probability**
2	0.15	2	0.10
3	0.25	3	0.25
4	0.20	4	0.30
5	0.25	5	0.2
6	0.15	6	0.15

Mr. Srinivasan will implement the plan if the average waiting time of a customers in the system is less than 5 minutes.

Before implementing the plan, Mr. Srinivasan would like to know the following:

i. Mean waiting time of customers, before service.

ii. Average service time.

iii. Average idle time of service.

iv. The time spent by the customer in the system.

Simulate the operation of the facility for customer arriving sample of 20 cars when the restaurant starts at 7.00 pm every day and find whether Mr. Srinivasan will go for the plan.

Allot the random numbers to various inter-arrival service times

Random Numbers Allocated to Various Inter-Arrival Service Times

Sl. No.	Random Number (Arrival)	Inter Arrival Time (Min)	Arrival Time at	Service Starts at	Random Number (service)	Service Time (Min)	Service Ends at	Waiting Time	
								Customer	Service (Min)
1	87	6	7.06	7.06	36	4	7.10	-	6
2	37	3	7.09	7.10	16	3	7.13	1	-
3	92	6	7.15	7.15	81	5	7.20	-	2
4	52	4	7.19	7.20	08	2	7.22	1	-
5	41	4	7.23	7.23	51	4	7.27	-	1
6	05	2	7.25	7.27	34	3	7.30	2	-
7	56	4	7.29	7.30	88	6	7.36	1	-
8	70	5	7.34	7.36	88	6	7.42	2	-
9	70	5	7.39	7.42	15	3	7.45	3	-
10	07	2	7.41	7.45	53	4	7.49	4	-
11	86	6	7.47	7.49	01	2	7.51	2	-
12	74	5	7.52	7.52	54	4	7.56	-	1
13	31	3	7.55	7.56	03	2	7.58	1	-
14	71	5	8.00	8.00	54	4	8.04	1	2
15	57	4	8.04	8.04	56	4	8.08	-	-
16	85	6	8.10	8.10	05	2	8.12	-	2
17	39	3	8.13	8.13	01	2	8.15	-	1
18	41	4	8.17	8.17	45	4	8.21	-	2
19	18	3	8.20	8.21	11	3	8.24	1	-
20	38	3	8.23	8.24	76	5	8.29	1	-
	Total	**83**				**72**		**20**	**17**

i. Mean waiting time of customer before service = 20/20 = 1 minute

ii. Average service idle time = 17/20 = 0.85 minutes

iii. Time spent by the customer in the system = 3.6 + 1 = 4.6 minutes.

Problem 2: Dr. Strong, a dentist schedules all his patients for 30 minute appointments. Some of the patients take more or less than 30 minutes depending on the type of dental work to be done. The following Table shows the summary of the various categories of work, their probabilities and the time actually needed to complete the work.

Simulation Problem

Category	**Time required (minutes)**	**Probability of category**
Filling	45	0.40
Crown	60	0.15
Cleaning	15	0.15
Extraction	45	0.10
Check-up	15	0.20

Simulate the dentist's clinic for four hours and determine the average waiting time for the patients as well as the idleness of the doctor. Assume that all the patients show up at the clinic exactly at their scheduled arrival time, starting at 8.00 am. Use the following random numbers for handling the above problem: 40,82,11,34,25,66,17,79.

Solution: Assign the random number intervals to the various categories of work as shown below

Category of work	**Probability**	**Cumulative probability**	**Random Number Interval**
Filling	0.40	0.40	00-39
Crown	0.15	0.55	40-54
Cleaning	0.15	0.70	55-69
Extraction	0.10	0.80	70-79
Check-up	0.20	1.00	80-99

Assuming the dentist clinic starts at 8.00 am, the arrival pattern and the service category are below

Arrival Pattern of the Patients

Patient Number	Scheduled Arrival	Random Number	Service category	Service Time
1	8.00	40	Crown	60
2	8.30	82	Check-up	15
3	9.00	11	Filling	45
4	9.30	34	Filling	45
5	10.00	25	Filling	45
6	10.30	66	Cleaning	15
7	11.00	17	Filling	45
8	11.30	79	Extraction	45

The arrival, departure patterns and patients' waiting time are tabulated.

Time	Event (Patient Number)	Patient Number (Time to go)	Waiting (Patient Number)
8.00	1 arrives	1 (60)	-
8.30	2 arrives	1 (30)	2
9.00	1 departure, 3 arrives	2 (15)	3
9.15	2 depart	3 (45)	-
9.30	4 arrive	3 (30)	4
10.00	3 depart, 5 arrive	4 (45)	5
10.30	6 arrive	4 (15)	5,6
10.45	4 depart	5 (45)	6
11.00	7 arrive	5 (30)	6,7
11.30	5 depart, 8 arrive	6 (15)	7,8
11.45	6 depart	7 (45)	8
12.00	End	7 (30)	8

The dentist was not idle during the simulation period. The waiting times for the patientsare as given in below.

Patient's Waiting Time

Patient	Arrival Time	Service Starts	Waiting time (minutes)
1	8.00	8.00	0
2	8.30	9.00	30
3	9.00	9.15	15
4	9.30	10.00	30
5	10.00	10.45	45
6	10.30	11.30	60
7	11.00	11.45	45
8	11.30	12.30	60
		Total	**285**

The average waiting time of patients = 285/8 = 35.625 minutes.