MESH ANALYSIS:

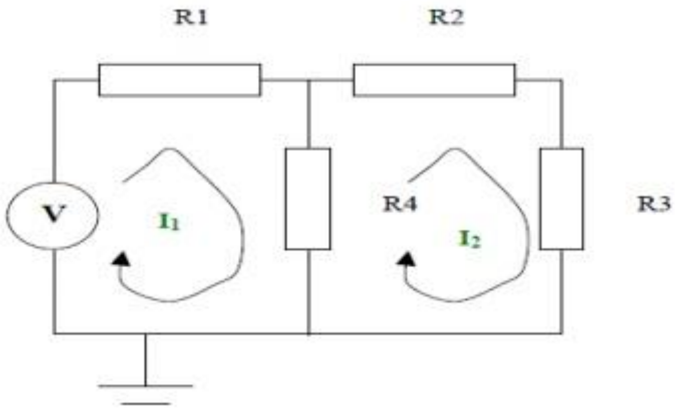
This is an alternative structured approach to solving the circuit and is based on calculating mesh currents. A similar approach to the node situation is used. A set of equations (based on KVL for each mesh) is formed and the equations are solved for unknown values. As many equations are needed as unknown mesh currents exist.

Step 1: Identify the mesh currents
Step 2: Determine which mesh currents are known

Step 2: Write equation for each mesh using KVL and that includes the mesh currents Step 3: Solve the equations

Step 1:
The mesh currents are as shown in the diagram on the next page

Step 2:
Neither of the mesh currents is known



Step 3:
KVL can be applied to the left hand side loop. This states the voltages around the loop sum to zero.

When writing down the voltages across each resistor equations are the mesh currents.

I1R1 + (I1 - I2) R4 - V = 0
KVL can be applied to the right hand side loop. This states the voltages around the loop sum to

zero. When writing down the voltages across ea the equations are the mesh currents.

I2R2 + I2R3 + (I2 - I1) R4 = 0
Step 4:
Solving the equations we get

$$I_1 = V\frac{R_2 + R_3 + R_4}{R_1R_2 + R_1R_3 + R_1R_4 + R_2R_4 + R_3R_4}$$

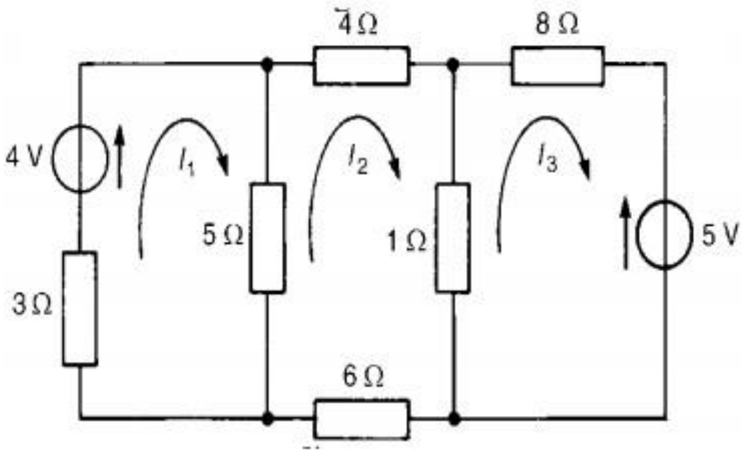
$$I_2 = V\frac{R_4}{R_1R_2 + R_1R_3 + R_1R_4 + R_2R_4 + R_3R_4}$$

The individual branch currents can be obtained from the these mesh currents and the node voltages can also be calculated using this information. For example:

$$V_C = I_2 R_3 = V \frac{R_3 R_4}{R_1 R_2 + R_1 R_3 + R_1 R_4 + R_2 R_4 + R_3 R_4}$$

Problem 1:

Use mesh-current analysis to determine the current flowing in (a the 1Ω resistance of the d.c. circuit shown in



The mesh currents I1, I2 and I3 are shown in Figure
Using Kirchhoff's voltage law:
For loop 1, (3 + 5) I1 −I2 = 4 ……………………………………………………………(1)
For loop 2, (4 + 1 + 6 + 5) I2 −(5) I1 −(1) I3 = 0……………………………………….(2)
For loop 3, (1 + 8) I3 −(1) I2 = 5− ……………………………………………………(3)
Thus

8I1 −5I2 −4 =0
−5I1 + 16I2 −I3 =0
−I2 + 9I3 + 5 =0

$$\frac{I_1}{\begin{vmatrix} -5 & 0 & -4 \\ 16 & -1 & 0 \\ -1 & 9 & 5 \end{vmatrix}} = \frac{-I_2}{\begin{vmatrix} 8 & 0 & -4 \\ -5 & -1 & 0 \\ 0 & 9 & 5 \end{vmatrix}} = \frac{I_3}{\begin{vmatrix} 8 & -5 & -4 \\ -5 & 16 & 0 \\ 0 & -1 & 5 \end{vmatrix}}$$

$$= \frac{-1}{\begin{vmatrix} 8 & -5 & 0 \\ -5 & 16 & -1 \\ 0 & -1 & 9 \end{vmatrix}}$$

Using determinants,

$$\frac{I_1}{-5\begin{vmatrix} -1 & 0 \\ 9 & 5 \end{vmatrix} - 4\begin{vmatrix} 16 & -1 \\ -1 & 9 \end{vmatrix}} = \frac{-I_2}{8\begin{vmatrix} -1 & 0 \\ 9 & 5 \end{vmatrix} - 4\begin{vmatrix} -5 & -1 \\ 0 & 9 \end{vmatrix}}$$

$$= \frac{I_3}{-4\begin{vmatrix} -5 & 16 \\ 0 & -1 \end{vmatrix} + 5\begin{vmatrix} 8 & -5 \\ -5 & 16 \end{vmatrix}}$$

$$= \frac{-1}{8\begin{vmatrix} 16 & -1 \\ -1 & 9 \end{vmatrix} + 5\begin{vmatrix} -5 & -1 \\ 0 & 9 \end{vmatrix}}$$

$$\frac{I_1}{-5(-5) - 4(143)} = \frac{-I_2}{8(-5) - 4(-45)}$$

$$= \frac{I_3}{-4(5) + 5(103)}$$

$$= \frac{I_3}{-4(5) + 5(103)}$$

$$= \frac{-1}{8(143) + 5(-45)}$$

$$\frac{I_1}{-547} = \frac{-I_2}{140} = \frac{I_3}{495} = \frac{-1}{919}$$

$$\text{Hence } I_1 = \frac{547}{919} = 0.595\text{ A},$$

$$I_2 = \frac{140}{919} = 0.152\text{ A, and}$$

$$I_3 = \frac{-495}{919} = -0.539\text{ A}$$

(a) Current in the 5 Ωresistance = I1 −I2
= 0.595 −0.152
= 0.44A

(b) Current in the 1 Ωresistance = I2 −I3
= 0.152 −(−0.539)

$= 0.69A$