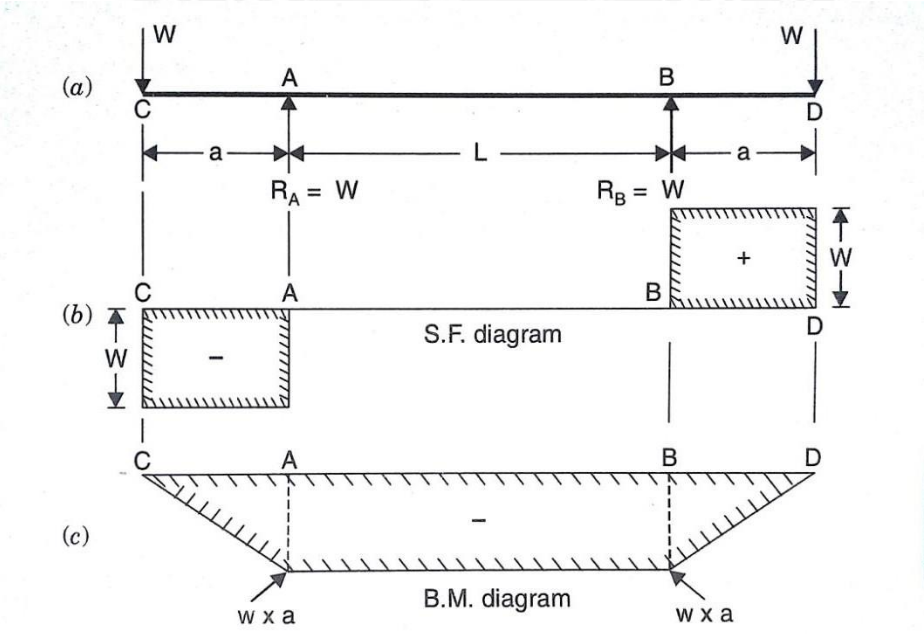
BENDING AND SHEAR STRESS IN BEAM

2.5. BENDING STRESS IN BEAM

When some external load acts on a beam, the shear force and bending moments are set up at all sections of the beam. Due to the shear force and bending moment, the beam undergoes certain deformation. The stresses introduced by bending moment is known as bending stresses.

2.5.1. PURE BENDING OR SIMPLE BENDING

If a length of a beam is subjected to a constant bending moment and no shear force (i .e, zero shear force), then the stresses will be set up in that length of the beam due to B.M. only and that length of beam is known as pure bending or simple bending. The stresses set up in that length of beam are known as bending stresses.



A beam simply supported at A and B and overhanging by same length at each support. A point load W is applied at each end of the overhanging portion. It is clear that there is no shear force between A and B but the B.M. between A and B is known as pure bending or simple bending.

2.5.2. THEORY OF SIMPLE BENDING WITH ASSUMPTIONS MADE

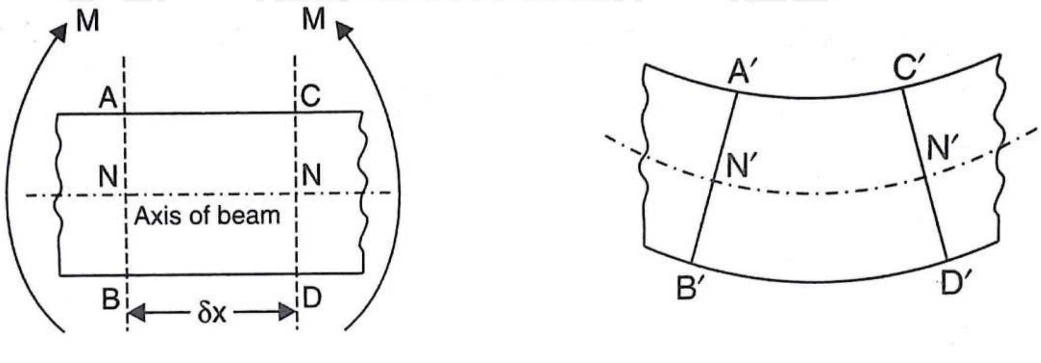
Assumptions made in the theory of simple bending. The following are the important assumptions:

1. The material of the beam is homogeneous and isotropic.

2. The value of young's modulus of elasticity is the same in tension and compression.
3. The transverse sections which were plane before bending , remain plane after bending also.
4. The beam is initially straight and all longitudinal filaments bend into circular areas with a common centre of curvature.
5. The radius of curvature is large compared with the dimensions of the cross section.
6. Each layer of the beam is free to expand or contract, independently of the layer, above or below it.

Theory of Simple Bending

The fig. shows A part of a beam is subjected to simple bending. Consider a small length d x of this part of beam. Consider two sections AB and CD which are normal to the axis of the beam N-N. Due to the action of the bending moment, the part of length d x will be deformed.



(a) Before bending (b) After bending

The top layer such as AC has deformed to the shape A'C'. This layer has been elongated. It is clear that some of the layers have been shortened while some of them are elongated. At a level between the top and bottom of the beam there will be a layer which is neither shortened nor elongated. This layer is known as neutral layer or neutral surface.

This layer is N'-N'. The line of intersection of the neutral layer on a cross section of the beam is known as the neutral axis.

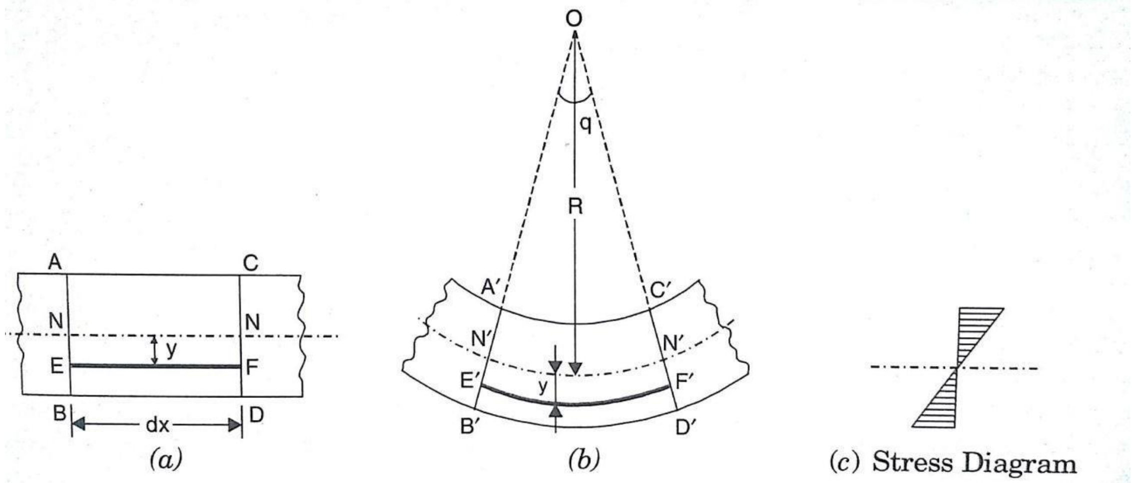
Hence the amount by which a layer increases or decreases in length, depends upon the position of the layer with respect to N-N. This theory of bending is known as theory of simple bending.

2.5.3. EXPRESSION FOR BENDING STRESS

A small length d x of a beam subjected to a simple bending. Due to the action of bending, the part of length d x will be deformed. Let A'B' and C'D' meet at O.

Let R= Radius of neutral layer N'N'

θ=Angle subtended at O by A'B' and C''D' produced.



(a) (b) (c) Stress Diagram

2.20.1. Strain

Variation along the Depth of Beam

Consider a layer EF at a distance y below the neutral layer NN. After bending this layer will be elongated to E'F'.

Original length of layer EF=d x.

Also length of neutral layer NN=d x.

After bending, the length of neutral layer N'N' will remain unchanged. But the length of layer E'F' will increase.

N'N'=NN=d x.

N'N'=R $\times$ θ

E'F'= (R + y)$\times$ θ

But N'N'=NN=d x

Hence d x=R$\times$ θ

$\therefore$ Increase in length of layer EF=E'F'-EF=(R+ y)θ -R$\times$ θ

$= y \times \theta$

$\therefore$ Strain in layer EF $= \frac{Increase\ in\ length}{original\ length}$

$= y \times \frac{\theta}{EF} \quad = \frac{y \times \theta}{R \times \theta} \qquad = \frac{y}{R}$

2.5.3.1. Stress Variation

Let σ = stress in layer EF

E = young's modulus of beam

E = stress in layer EF/strain in layer EF

= σ/(y/R)

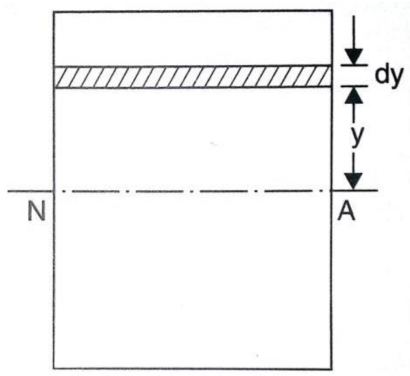
$$\sigma = E \times \frac{y}{R} = \frac{E}{R} \times y$$

It can also be written as $\frac{\sigma}{y} = \frac{E}{R}$

2.5.4. NEUTRAL AXIS AND MOMENT OF RESISTANCE

The neutral axis of any transverse section of beam is defined as the line of intersection of the neutral layer with the transverse section. It is written as NA.

The stress at a distance y from the neutral axis is given by



$$\sigma = \frac{E}{R} \times y$$

Let d A = Area of layer

Force on layer = stress on layer ×area of layer

$$= \sigma \times dA$$

$$= \frac{E}{R} \times y \times d\,A$$

Total force on beam section $= \int \frac{E}{R} \times y \times d\,A$

$$= \frac{E}{R} \int y \times d\,A$$

But for pure bending there is no force on the section of beam

$$\therefore \quad \frac{E}{R} \int y \times d\,A = 0$$

$$\int y \times d\,A = 0$$

Hence $\int y \times d\,A$ represents the moment of entire area of the section about neutral axis.

2.5.4.1. Moment of Resistance

Due to pure bending the layers above the NA are subjected to compressive stresses whereas the layers below the NA are subjected to tensile stresses. Due to these stresses forces acts on the layers.

The force on the layer at a distance y from neutral axis.

$$\text{Force on layer} = \frac{E}{R} \times y \times dA$$

Moment of this force about NA

$$= \text{force on layer} \times y$$

$$= \frac{E}{R} \times y \times d\,A \times y$$

$$= \frac{E}{R} \times y^2 \times d\,A$$

Total moment of the forces on the section of beam $= \int \frac{E}{R} \times y^2 \times d\,A$

$$= \frac{E}{R} \int y^2 \times dA$$

Let M = External moment applied on the beam section.

$$M = \frac{E}{R} \int y^2 \times dA$$

But the expression $\int y^2 \times dA$ represent the moment of inertia of the area of the section about the neutral axis I

Then $M = \frac{E}{R} \times I$

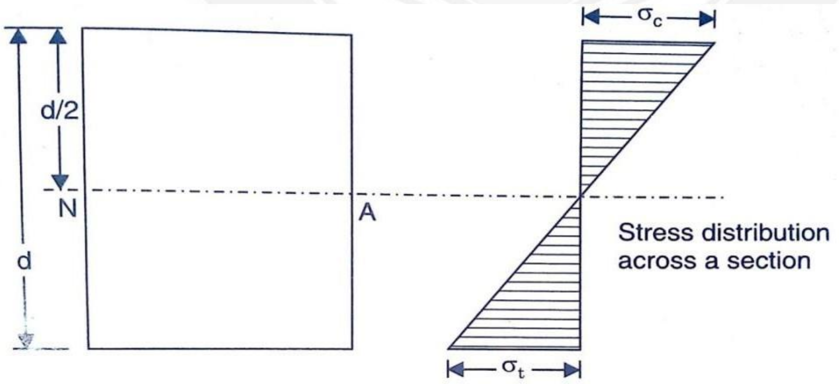
We know that $\frac{\sigma}{y} = \frac{E}{R}$

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

It is known as the bending equation.

2.5.5. BENDING STRESSES IN SYMMETRICAL SECTIONS

The neutral axis of a symmetrical section lies at a distance of $d/2$ from the outermost layer of the section where d is the diameter or depth. There is no stress at the neutral axis. But the stress at a point is directly proportional to its distance from the neutral axis. The maximum shear stress takes place at the outermost layer. For a simply supported beam, there is a compressive stress above the neutral axis and a tensile stress below it. If we plot these stresses, we will get a figure shown below.



Problem 2.13. A steel plate of width 120mm and of thickness 20mm is bent into a circular arc of radius 10m. Determine the maximum stress induced and the bending moment which will provide the maximum stress. Take E = $2 \times 10^5 N/mm^2$.

Given:

Width b= 120mm

Thickness t=20mm

Moment of inertia $I = \frac{bt^3}{12} = 120 \times \frac{20^3}{12} = 8 \times 10^4 mm^4$

Radius of curvature R =10m = 10×10^3mm

Young's modulus E = 2×10^5 N/mm^2

Solution:

Wkt $\frac{\sigma}{y} = \frac{E}{R}$

$y_{max} = \frac{t}{2} = \frac{20}{2} = 10\text{mm}$

$\sigma_{max} = \frac{E}{R}\times\ y_{max}$

$= \frac{2\times10^5}{10\times10^3}\times 10$

$= 200$ N/mm^2

Wkt $\frac{M}{I} = \frac{E}{R}$

$M = \frac{E}{R}\times I$

$= \frac{2\times10^5}{10\times10^3}\times\times 8\times 10^4$

$= 16\times10^5$Nmm = **1.6KNm**

Problem 2.13. Calculate the maximum stress induced in a cast iron pipe of external diameter 40mm of internal diameter 20mm and of length 4m when the pipe is supported on its ends and carries a point load of 80N at its centre.

Given

Ext dia. D = 40mm

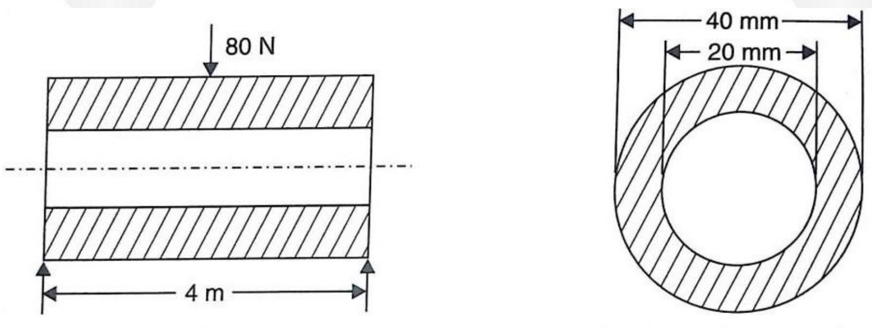
Int dia. d = 20mm

Length L= 4m = 4×1000 = 4000mm

Point load W = 80 N

Solution:

In this case of simply supported beam carrying a point load at the centre, the maximum bending moment is at the centre of the beam.



(a) (b) Area of cross-section

Max bending moment BM $= \frac{W\times L}{4}$

$$= 80\times4000/4 = 8\times 10^4 \text{Nmm}$$

Moment of inertia of hollow pipe I = $\frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}(40^4 - 20^4)$

$$= 117809.7\ mm^4$$

Now $\frac{M}{I} = \frac{\sigma}{y}$

$$y_{max} = D/2 = 40/2 = 20\text{mm}$$

$$\frac{M}{I} = \frac{\sigma_{max}}{y_{max}}$$

$$\sigma_{max} = \frac{M}{I}\times y_{max}$$

$$= 8 \times 10^4 \times \frac{20}{117809.7}$$

$$= \mathbf{13.58\ N}/mm^2$$

2.5.5. SECTION MODULUS

Section modulus is defined as the ratio of moment of inertia of a section about the neutral axis to the distance of the outermost layer from the neutral axis.

$$Z = \frac{1}{y_{max}}$$

Where I = moment of inertia about neutral axis

y_{max} = distance of the outermost layer from the neutral axis

Wkt $\frac{M}{I} = \frac{\sigma}{y}$

$$\frac{M}{I} = \frac{\sigma_{max}}{y_{max}}$$

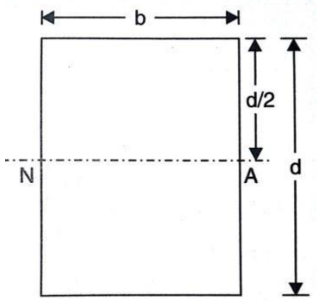
$$M = \sigma_{Max} \times \frac{I}{y_{max}}$$

But $\frac{I}{y_{max}} = Z$

∴ $M = \sigma_{max} \times Z$

2.5.6. SECTION MODULUS FOR VARIOUS SHAPES OR BEAM SECTIONS

1. Rectangular Section



Moment of inertia I = bd^3/12

Distance of outermost layer from NA

$$y_{max} = d/2$$

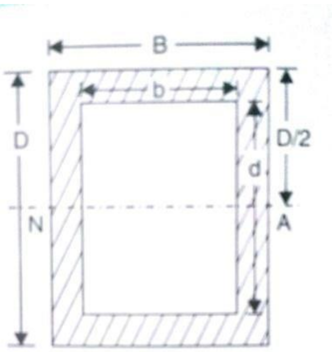
∴ Section modulus $Z = \frac{I}{y_{max}} = \frac{b\,d^3/12}{d/2} = \frac{bd^2}{6}$

2. Hollow Rectangular Section

$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$= \frac{1}{12}(BD^3 - bd^3)$$

$$y_{max} = D/2$$



$$\therefore \quad Z = \frac{I}{y_{max}}$$

$$= \frac{\frac{1}{12}(BD^3 - bd^3)}{D/2}$$

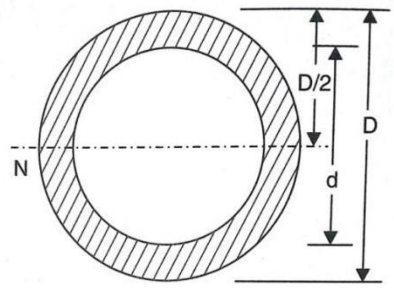
$$= \frac{1}{6D}(BD^3 - bd^3)$$

3. Circular Section

$$I = \frac{\pi}{64} d^4 \quad \text{and} \quad y_{max} = d/2$$

$$\therefore \quad Z = \frac{I}{y_{max}} = \frac{\frac{\pi}{64} d^4}{d/2} = \frac{\pi}{32} d^3$$

4. Hollow Circular Section



$$I = \frac{\pi}{64}(D^4 - d^4)$$

$$y_{max} = D/2$$

$$\therefore \quad Z = I/y_{max}$$

$$= \frac{\pi}{32\,D}(D^4 - d^4)$$

Problem 2.14. A cantilever of length 2m fails when a load of 2kN is applied at a free end. If the section of the beam is 40mm×60mm. Find the stress at the failure.

Given

Length	L = 2m
Load	W = 2Kn
Width	b = 40mm
Depth	d = 60mm

Solution:

Section modulus of rectangular section $Z = bd^2/6$

$$= 40 \times 60^2/6$$

$$= 24000 mm^3$$

Max bending moment for cantilever at fixed end M = W×L

$$= 2000 \times 2 \times 10^3$$

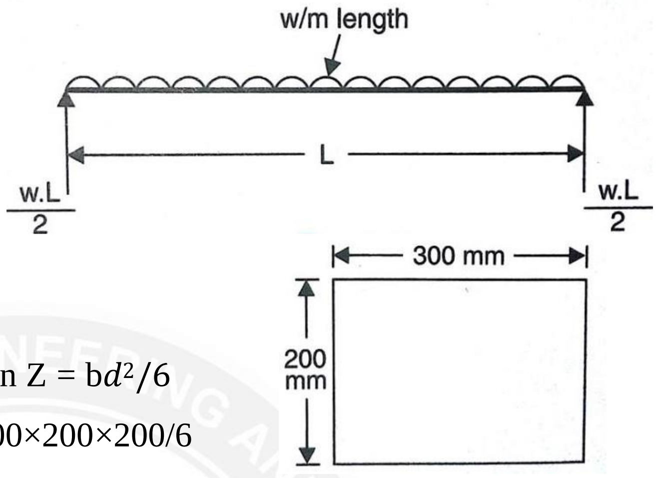
$$= 4 \times 10^6 Nm$$

$$M = Z \times \sigma_{max}$$

$$\sigma_{max} = M/Z = \mathbf{166.67\ N}/mm^2$$

Problem 2.15. A rectangular beam 200mm deep and 300 mm wide is simply supported over a span of 8m. What uniformly distributed load per metre the beam is carrying if the bending stress is not to exceed 120N/mm^2

Given



Depth d = 200mm

Width b = 300mm

Length L = 8m

Max bending stress σ_{max} =120N/mm^2

Section modulus for rectangular section Z = b$d^2/6$

= 300×200×200/6

= 2000000mm^3

Max bending moment for simply supported beam carrying uniform load M

= w× $L^2/8$

= 8w Nm = 8w×1000Nmm

= 8000w Nmm

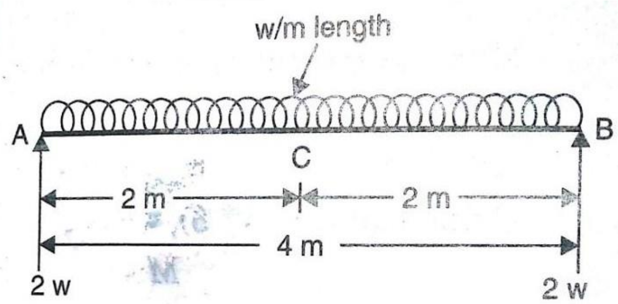
M = Z× σ_{max}

8000w = 120×2000000

w = **30KN/m**

Problem 2.16: A rectangular beam 300 mm deep is simply supported over a span of 4m. Determine the uniformly distributed load per metre which the beam may carry if the bending stress should not exceed 120N/mm^2. Take I = 8×$10^6$$mm^4$

Given



Depth d = 300mm

Span L = 4m

Max bending stress M = 120N/mm^2

Moment of inertia I = 8× $10^6$$mm^4$

Max BM = 2w×2-2w×1

= 4w-2w

= 2wNm

= 2w×1000 = 2000w Nmm

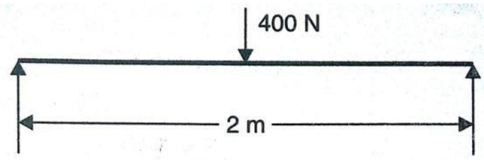
M = 2000w Nmm

$M = \sigma_{max} \times Z$

$Z = I/y_{max} = 8\times 10^6/150$

$\therefore$ $2000w = 120\times Z$ $\quad$ $\boldsymbol{w = 3200}$ **N/m**

Problem 2.17: A square beam 20mm×20mm in section and 2m long is supported at the ends. The beam fails when a point load of 400N is applied at the centre of the beam. What uniformly distributed load per metre length will break a cantilever of the same material 40mm wide 60mm deep and 3m long.



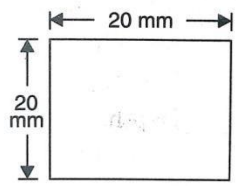
Given

Depth d = 20mm

Width b = 20mm

Length L =2m

Point load W = 400N



Solution:

Section modulus $Z = bd^3/6$

$= 4000/3\ mm^3$

Max bending moment $M = w\times L/4 = 400\times 2/4$

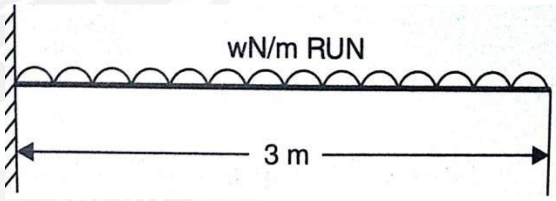
$= 200$Nm

$= 200\times 1000 = 200000$ Nmm

$M = Z\times \sigma_{max}$

$20000 = \sigma_{max} \times (4000/3)$

$\sigma_{max} = 150\ N/mm^2$

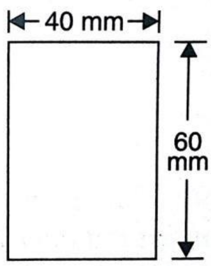


Let w = uniformly distributed load per m run

Let b = 40 mm

d = 60 mm

L = 3m



Section modulus $Z = bd^2/6 = 40\times 60^2/6 = 24000\ mm^3$

Max bending moment $= wL^2/2$

$M = 4.5\times 1000w$ Nmm

Wkt $M = \sigma_{max} \times Z$

$4.5\times 1000w = 150\times 24000$

$w =$ **800N/m**

Problem 2.18: A beam is simply supported and carries a uniformly distributed load of 40KN/m run over the whole span. The section of beam is rectangular having depth as 500mm. If the max stress in the material of beam is 120N/mm^2 and a moment of inertia of the section is 7× $10^8 mm^4$. Find the span of beam

Given

U.D.L w = 40KN/m

Depth d = 500mm

Max stress σ_{max} = 120 N/mm^2

M.O.I I = 7× $10^8 mm^4$

Section modulus Z = I/Y_{max}

y_{max} = d/2 = 500/2 = 250 mm

Z = 7× 10^8/250 = 28× $10^5 mm^3$

Max B.M for a simply supported beam carrying U.D.L = w× $L^2/8$

M = 40000× L^2/8

= 5000L^2 ×1000Nmm

Wkt M = σ_{max} × Z

5000×1000× L^2 = 120×28× 10^5

L = $\sqrt{2.4 * 28}$ = **8.197m**

Problem 2.19: A timber beam of rectangular section is to support a load of 20 kN uniformly distributed over a span of 3.6 m when beam is simply supported. If the depth of the section is to be twice the breadth and the stress in the timber is not to exceed 7N/mm^2. Find the dimensions of how would you modify the cross section of the beam if it carries a concentrated load of 20 k N placed at the centrevwith the same ratio of breadth to depth.

Given

Total load W = 20 k N

Span L = 3.6m

Max stress σ_{max} = 7N/mm^2

Depth d = 2b mm

Section modulus Z = b$d^2/6$

Z = b×(2b)×(2b)/6 = $\frac{2b^3}{6} mm^3$

Max B.M = w$L^2/8$ or WL/8

M = WL/8 = 9000 Nm = 9000×1000Nmm

W k t $M = Z\times \sigma_{max}$

$9000\times1000 = 7\times \frac{2b^3}{3}$

$b^3 = 1.92857\times10^6$

b = 124.5mm

d = 2b = 2×124.5 = 249 mm

B.M is maximum at the centre and is equal to WL/4

M = WL/4 = 20000×3.6/4 = 18000 N m

= 18000×1000 Nmm

$\sigma_{max} = 7\ N/mm^2$

$Z = 2b^3/3$

We get $M = Z\times \sigma_{max}$

$18000\times1000 = 7\times 2b^3/3$

b = **156.82mm**

d = 2×156.82 = **313.64mm**

Problem 2.20: A timber beam of rectangular section of length 8m is simply supported. The beam carries a U.D.L of 12kN/m run over the entire length and a point load of 10k N at 3m from the left support. If the depth is two times the width and the stress in the timber is not to exceed 8N/mm^2. Find the suitable dimensions of the section.

Given

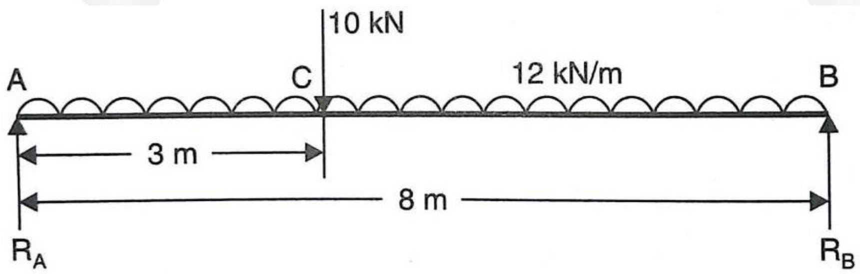
Length L = 8m

UDL w = 12000N/m

Point load W = 10000N

Depth of beam d = 2b

$\sigma_{max} = 8N/mm^2$



Solution:

Taking moment about A

We get, $R_B \times 8 = [(12000\times8)\times\frac{8}{2}]+(10000\times3)$ scoop

R_B = 51750 N

R_A = total load- R_B

= (12000×8)+10000 -5175 = 54250N

S.F at B = -R_B = -51750N

S.F at C = -51750+ (12000×5)= +8250N (without PL)

S.F at C = -51750+ (12000×5) +10000= - 18250N (with PL)

S.F at A = +R_A = +54250N

Let SF is zero at x metre from B

Equating (12000 × x) - R_B= 0

(120000× x) – 51750 = 0

∴ x = 4.3125m

∴ Max BM will occur at 4.3125m from B

∴ Max BM M = R_B ×4.3125 – 12000 × 4.3125 ×$\frac{4.3125}{2}$

=111585.9375×10^3Nmm

Section modulus for rectangular beam Z = $\frac{bd^2}{6} = \frac{b\,(2b)^2}{6} = \frac{2b^3}{3}$

Wkt M = Z× σ_{max}

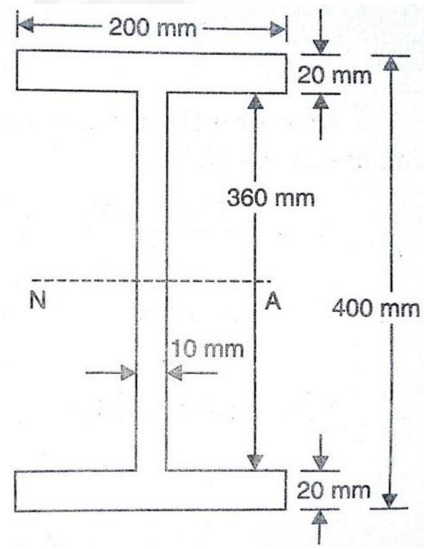
111586.9375×10^3 = 8× $\frac{2b^3}{3}$

b^3 = 20.9223× 10^6

b = **275.5mm**

d = 2×275.5 = **551mm**

Problem 2.21: A rolled steel joist of I section has the dimensions as shown. This beam of I section carries a UDL of 40 kN/m run on a span of 10m. Calculate the max stress produced due to bending



Given

UDl w= 40 K N/m = 40000N/m

Span L = 10m

Solution:

Moment of inertia about the neutral axis

= 200× $400^3/12$ - (200-10)× $360^3/12$

= 327946666mm^4

Max BM. M = w× $L^2/8 = \frac{40000 \times 10^2}{8}$ = 500000 Nm

$= 5\times 10^8$ N mm

Now using the relation $\frac{M}{I} = \frac{\sigma}{y}$

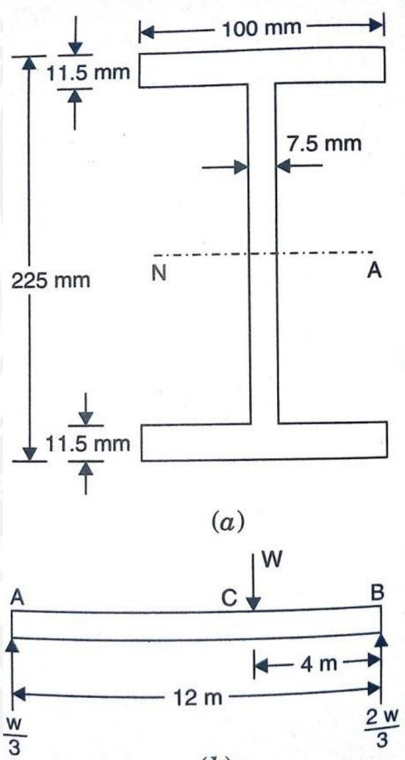
∴ $\sigma = \frac{M}{I} \times y$

$\sigma_{max} = \frac{M}{I} \times y_{max}$

$= 5\times \frac{10^8}{327946666} \times 200$

= **304.92 N/mm^2**

Problem 2.22: An I section shown in figure is simply supported over a span of 12m. If the max permissible bending stress is 80 N/mm^2. What concentrated load can be carried at a distance of 4m from one support?



Given

Bending stress σ_{max} = 80 N/mm^2

Load W = 4m from support B in N

Solution:

Taking moments about A, we get

$R_B \times 12 = W\times 8$

∴ $R_B = \frac{8W}{12} = \frac{2}{3}W$

and $R_A = W - R_B = W - \frac{2}{3}W = \frac{W}{3}$

B.M at C = $R_A \times 8$ = 8/3 W Nm

But B.M at C is maximum

∴ Max B.M $M_{max} = \frac{8}{3}$W Nm

$= \frac{8}{3}$ W×1000 N mm

$= \frac{8000}{3}$ W Nmm

Now find the MOI of the given I section about the N.A

∴ $I = \frac{100\times 225^3}{12} - \frac{(100-7.5)\times(225-2\times 11.5)^3}{12}$

= 31386647.45mm^4

Now using the relation, $\frac{M}{I} = \frac{\sigma}{y}$

$y_{max} = \frac{225}{2}$ = 112.5 mm

Now substituting the values, we get

$$=\frac{(\frac{8000}{3}W)}{31386647.45}=\frac{80}{112.5}$$

W = **8369.77 N**

Problem 2.23: Two circular beams where one is solid of dia. D and other is a hollow 0f outer dia. D_0 and the inner dia. D_i are of same length, same material and of same weight. Find the ratio of section modulus of these circular beams.

Given

Dia. Of solid beam = D

Dia. Of hollow beam = D_0 and D_i

Let L be the length W be the weight and p be the density

Solution:

Weight of solid beam = ρ ×g ×area of section × L

$$= \rho \times g \times L \times \pi/4 D^2$$

Weight of hollow beam = ρ ×g ×area of section × L

$$= \rho \times g \times L \times \frac{\pi}{4}(D_0^2 - D_i^2)$$

But the weights are same $\rho \times g \times L \times \pi/4 D^2 = \rho \times g \times L \times \frac{\pi}{4}(D_0^2 - D_i^2)$

$$D^2 = (D_0^2 - D_i^2) \quad \text{……………..(1)}$$

Now the section modulus of solid section Z = $\frac{\pi}{32}D^3$

Section modulus of hollow section

$$Z_1 = \frac{\pi}{32 D_0}(D_0^4 - D_i^4)$$

$$= \frac{\pi}{32 D_0}(D_0^2 - D_i^2)(D_0^2 + D_i^2)$$

$$\therefore \frac{\text{Section modulus of solid section}}{\text{Section Modulus of hollow section}}$$

$$= \frac{\frac{\pi}{32}D^3}{\frac{\pi}{32D_0}(D_0^2 - D_i^2)(D_0^2 + D_i^2)}$$

$$= \frac{D^3 \times D_0}{(D_0^2 - D_i^2)(D_0^2 + D_i^2)} = \frac{D \times D_0 \times D^2}{(D_0^2 - D_i^2)(D_0^2 + D_i^2)}$$

Substitute eqn.1 in the above calculation, then

$$= \frac{D \times D_0 \times (D_0^2 - D_i^2)}{(D_0^2 - D_i^2)(D_0^2 + D_i^2)}$$

$$= \frac{D \times D_0}{D_0^2 + D_i^2} \quad \text{……………..(2)}$$

Also from eqn.1

$$D^2 = D_0^2 - D_i^2 \qquad \text{or} \qquad D_i^2 = D_0^2 - D^2$$

Substituting the value D_i^2 in the section modules ratio

$$\frac{\text{Section modulus of solid section}}{\text{Section Modulus of hollow section}} = \frac{D \times D_0}{D_0^2 + D^2 - D_0^2} = \frac{D \times D_0}{2D_0^2 - D^2}$$

or

$$\frac{\text{Section modulus of hollow section}}{\text{Section Modulus of solid section}} = \frac{2D_0^2 - D^2}{D \times D_0} = \frac{2D_0^2}{D \times D_0} - \frac{D^2}{D \times D_0}$$

$$= \frac{2D_0}{D} - \frac{D}{D_0}$$

Problem 2.24: A water main of 500mm internal dia and 20mm thick is running full. The water main is of cast iron and is supported at two points 10m apart. Find the max stress in the metal. The cast iron and water weigh 72000 N/m^3 and 10000 N/m^3respectively.

Given:

Internal diameter D_i = 500mm = 0.5m

Thickness of pipe t = 20mm 0.02

Outer diameter = $D_0 = D_i + 2\times t = 0.5+2\times0.02 = 0.54$m

Length of pipe L = 10m

Weight density of cast iron = 72000 N/m^3

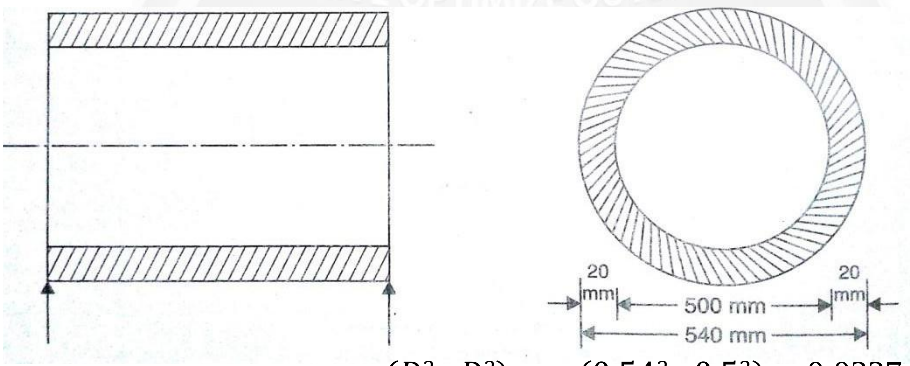
Weight density of water = 10000 N/m^3

Solution:

Internal area of pipe $= \frac{\pi}{4} D_i^2 = \frac{\pi}{4} \times 0.5^2 = 0.1960\ m^2$

This is equal to the Area of water section

∴ Area of water section = 0.1960 m^2



Outer area of pipe $= \frac{\pi}{4} D_0^2 = \frac{\pi}{4}$

Area of pipe section $= \frac{\pi}{4} D_0^2 - \frac{\pi}{4} D_i^2$

$$= \frac{\pi}{4}(D_0^2 - D_i^2) = \frac{\pi}{4}(0.54^2 - 0.5^2) = 0.0327 m^2$$

Moment of Inertia of pipe section about the neutral axis

$$I = \frac{\pi}{64}(D_0^4 - D_i^4) = \frac{\pi}{64}(0.54^4 - 0.5^4) = 1.105 \times 10^9 \ m^4$$

Let us now find the weight of pipe and weight of water for one meter length,

Weight of pipe for one meter length

= weight density of cast iron × volume of pipe

= 72000×area of pipe section ×length

= 72000×0.0327×1 (∴ length = 1m)

= 2354 N

Weight of water for one meter run

= wt. density of water × volume of water

= 10000× area of water section × length

= 10000×0.196×1 = 1960N

∴ Total weight on pipe for one meter run = 2354+1960 =4314N

Hence the above weight is the UDL on the pipe. the maximum bending moment due to UDL is $\frac{w \times L^2}{8}$, where w = rate of UDL = 4314N per meter length of beam.

∴ Max bending moment due to UDL $M = \frac{w \times L^2}{8}$

$$= \frac{4314 \times 10^2}{8} = 53925 \text{ Nm} = 5.392 \times 10^7 \text{ Nmm}$$

Now using $\frac{M}{I} = \frac{\sigma}{y}$

∴ $\sigma = \frac{M}{I} \times y$

to find the maximum maxim um stress it will acts at y is maximum

$$y = \frac{D0}{2} = \frac{540}{2} = 270\text{mm}$$

∴ $y_{max} = 270$ mm

∴ $\sigma_{max} = \frac{5.392 \times 10^7}{1.105 \times 10^9} \times 270 =$ **13.18N/mm^2**

2.5.7. BENDING STRESS IN UNSYMMETRICAL SECTION

In case of symmetrical section, the neutral axis passes through the geometrical centre of the section. but in case of unsymmetrical section such as L, T section, the neutral axis does not passes through the geometrical centre of the section. Hence the value of y for the topmost layer or bottom layer of the section from neutral axis will not be same. For finding the bending stress in the beam, the bigger value of y is used. As the neutral axis passes through the centre of gravity of the section, hence in unsymmetrical section, first the centre of gravity is calculated and followed to find other values.

Problem 2.25: A cast iron bracket subject to bending has the cross section of I form with unequal flanges. The dimensions of the sections are shown. Find the position of the neutral axis and M.O.I. of the section about the neutral axis. If the max bending moment on the section is 40 MN mm. Determine the max bending stress. What is the nature of the stress.

Given

Max. BM M = 40MN mm

Wkt, centroid about x axis $\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$

Where

A_1 = Area of bottom flange = $b_1 \times d_1$ = 130×50 = 6500mm^2

y_1 = distance of C.G of A_1 from bottom f ace = $\frac{50}{2}$ = 25mm

A_2 = area of web = $b_2 \times d_2$ = 50×200 = 10000 mm^2

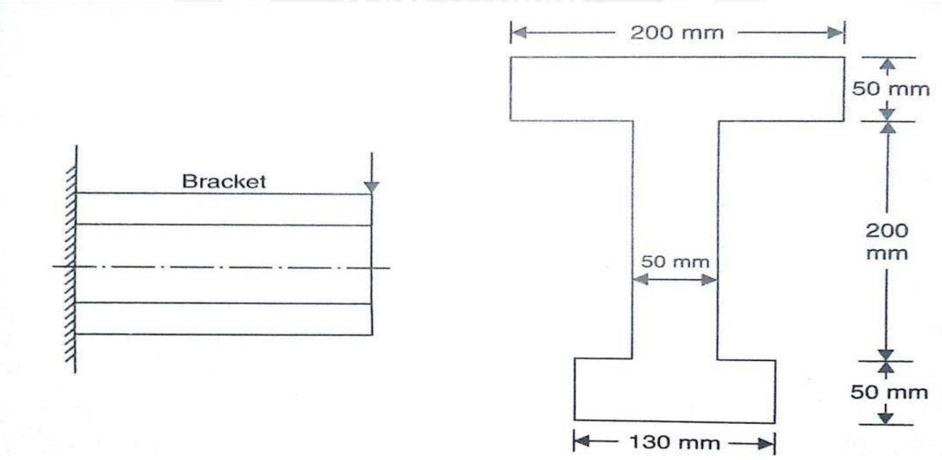
y_2 = distance of C.G of A_2 from bottom face = 50 + $\frac{200}{2}$ = 150 mm

A_3 = area of top flange = $b_3 \times d_3$ = 200×50 = 10000mm^2

y_3 = dist of C.G of A_3 from bottom face = 50+200+$\frac{50}{2}$ = 275mm

Then,

$$\bar{y} = \frac{6500 \times 25 + 10000 \times 150 + 10000 \times 275}{6500 + 10000 + 10000} = 166.51\text{mm}$$



Hence the neutral axis is at a distance of 166.51mm from the bottom face

M.O.I of the section about the neutral axis I = $I_1 + I_2 + I_3$

Where I_1 = M.O.I of bottom flange about N.A

$= I_{C.G} + A_1 \times (distance\ of\ its\ C.G.\ from\ N.A.)^2$

$= \frac{b_1 \times d_1^3}{12} + A_1(\bar{y} - y_1)^2$

$$= \frac{130\times50^3}{12} + 6500 \times (166.51 - 25)^2 = 131517186.6mm^4$$

Similarly $$I_2 = \frac{b_2\times d_2^3}{12} + A_2(\bar{y} - y_2)^2$$

$$= \frac{50\times200^3}{12} + 10000 \times (166.51 - 150)^2$$

$$= 3360913.43\ mm^4.$$

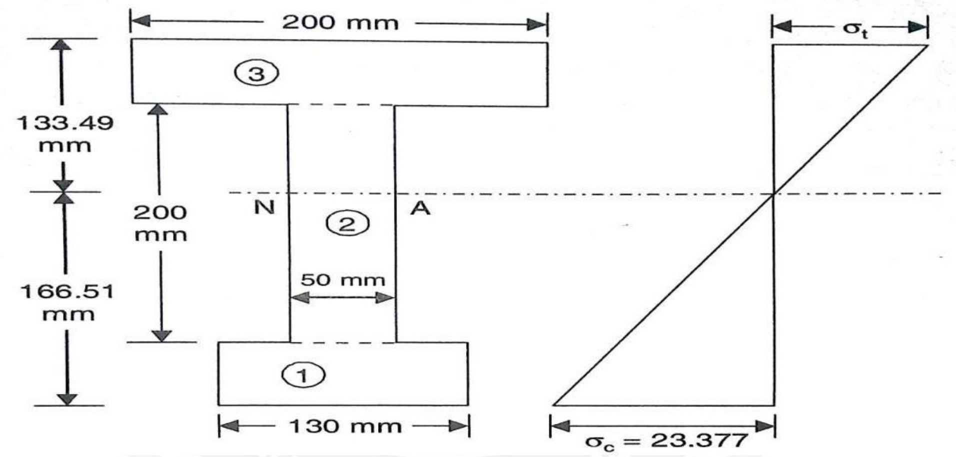
$$I_3 = \frac{b3\times d3^3}{12} + A_3(\bar{y} - y_3)^2$$

$$= \frac{200\times50^3}{12} + 10000 \times (166.51 - 275)^2$$

$$= 119784134.3mm^4.$$

∴ $$I = I_1 + I_2 + I_3 = 131517186.6 + 3360913.43 + 119784134.3$$

$$= \mathbf{284907234.9}mm^4$$



Now distance of C.G. from the upper top fibre

$$= 300 - \bar{y} = 300 - 166.51 = 133.49mm$$

Distance of C.G. from the bottom fibre = 166.51mm

Hence we shall take the value of y = 166.51mm for maximum bending stress.

Now using bending equation

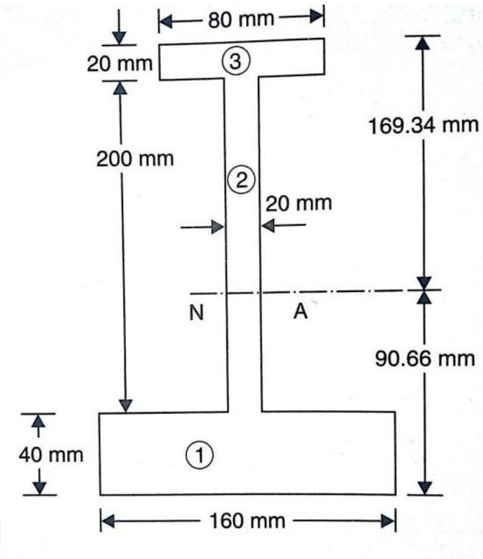
$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma = \frac{M}{I} \times y = \frac{40\times10^6}{284907234.9} \times 166.51 = 23.377\ N/mm^2$$

∴Max bending stress = **23.377N/**mm^2

Problem 2.26: A cast iron beam is of I section. The beam is simply supported on a span of 5m. If the tensile stress is not to exceed 20N/mm^2. Find the safe uniformly load which the beam can carry. Find also the max compressive stress.

Given



Length L = 5m

Max tensile stress σ_t = 20N/mm^2

Solution:

Wkt, centroid about x axis $\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$

Where

A_1 = Area of bottom flange

$= b_1 \times d_1 = 160 \times 40$

$= 6400 mm^2$

y_1 = distance of C.G of A_1 from bottom f ace

$= \frac{40}{2} = 20$mm

A_2 = area of web $= b_2 \times d_2 = 20 \times 200 = 4000\ mm^2$

y_2 = distance of C.G of A_2 from bottom face = 40 + $\frac{200}{2}$ = 140 mm

A_3 = area of top flange $= b_3 \times d_3 = 80 \times 20 = 1600 mm^2$

y_3 = dist of C.G of A_3 from bottom face = 40+200+$\frac{20}{2}$ = 250mm

Then,

$$\bar{y} = \frac{6400 \times 20 + 4000 \times 140 + 1600 \times 250}{6400 + 4000 + 1600} = 90.66\text{mm}$$

Hence the neutral axis is at a distance of 166.51mm from the bottom face or 260-90.66 = 169.34 mm from the top face.

M.O.I of the section about the neutral axis I = $I_1 + I_2 + I_3$

Where I_1 = M.O.I of bottom flange about N.A

$= I_{C.G} + A_1 \times (distance\ of\ its\ C.G.\ from\ N.A.)^2$

$= \frac{b_1 \times d_1^3}{12} + A_1 (\bar{y} - y_1)^2$

$= \frac{160 \times 40^3}{12} + 6400 \times (90.66 - 20)^2 = 32807481.17 mm^4$

Similarly $I_2 = \frac{b_2 \times d_2^3}{12} + A_2 (\bar{y} - y_2)^2$

$= \frac{20 \times 200^3}{12} + 4000 \times (90.66 - 140)^2$

$= 23071075.73 mm^4$

$I_3 = \frac{b3 \times d3^3}{12} + A_3 (\bar{y} - y_3)^2$

$= \frac{80 \times 20^3}{12} + 1600 \times (90.66 - 250)^2$

$= 40676110.29 mm^4$

$\therefore \quad I = I_1 + I_2 + I_3 = 32807481.17 + 23071075.73 + 40676110.29$

$= 96554667.21 mm^4$

For a simply supported beam the tensile stress will be at the extreme bottom and the compressive stress will be at the extreme top fibre

Here max. tensile stress = 20N/mm^2

Hence for max tensile stress y = 90.66mm

Using $\frac{M}{I} = \frac{\sigma}{y}$

$M = \frac{\sigma}{y} \times I$

$= \frac{20}{90.66} \times 96554667.21$

$= 21300389.85$Nmm

Let w = uniformly distributed load in N/m on the simply supported beam

Max bending moment is at the centre and equal to wL^2/8

$\therefore \quad M = w \times 5^2/8$ Nm $= w \times 25 \times 1000/8$ N mm $= 3125w$ N mm

Equating the values of M

3125w = 21300389.85

w = **6816.125N/m**

Max compressive stress

Distance of extreme top fibre from N.A

$y_c = 169.34$mm

M = 21300389.85

I = 96554667.21

Let σ_c = max compressive stress

Using the relation $\frac{M}{I} = \frac{\sigma}{y}$

$\therefore \quad \sigma = \frac{M}{I} \times y = \frac{21300389.85}{96554667.21} \times 169.34$

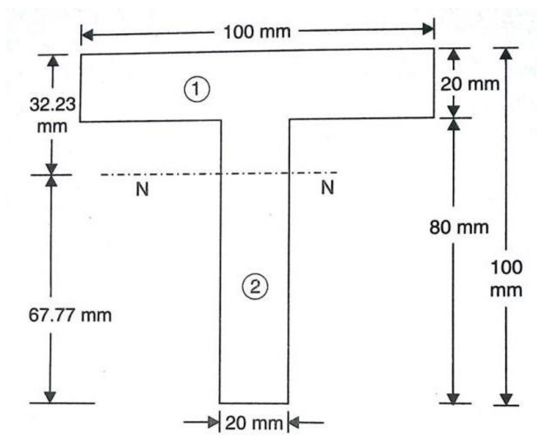
$\sigma_c =$ **37.357 N/mm^2**

Problem 2.27: A cast iron beam is of T section as shown. The beam is simply supported on a span of 8m. The beam carries a uniformly distributed load of 1.5kN/m length on the entire span. Determine the max tensile and max compressive stress.

Given

Length L = 8m

U.D.L w = 1.5KN/m



Let y = distance of C.G. of section from the bottom

$Y = (A_{1F_1} + A_{2F_2})/(A_1 + A_2)$

=(100×20)×(100×20)×(80+(20/2))+80×20×80/2)/(100×2) + (80×20)

= 67.77mm

N.A lies at the distance of 67.77mm from the bottom face or 100 – 67.77 = 32.23mm from the top face

Moment of inertia I = $I_1 + I_2$

Where I_1 = M.O.I of top flange about N.A

$= \frac{100\times20^3}{12} + (100\times20)\times(32.23-10)^2$

$= 1055012.5 mm^4$

I_2 = M.O.I of web

$= 20\times 80^3/12 + (80\times20)\times(67.77-40)^2$

$= 2087209.9 mm^4$

$I = I_1 + I_2 = 1055012.5 + 2087209.9 mm^4$

$= 3142222.4 mm^4$

For simply supported beam the tensile stress will be at the extreme bottom and the compressive stress will be at the extreme top

Max B.M $M = w\times L^2/8 = 1500\times 8^2/8 = 12000$ Nm

= 12000000 Nmm

Now using relation $\frac{M}{I} = \frac{\sigma}{y}$ € $\sigma = \frac{M}{I}\times y$

1. For max tensile stress y = 67.77mm

$\sigma = \frac{12000000}{3142222.4}\times 67.77 = \mathbf{258.81\ N/mm^2}$

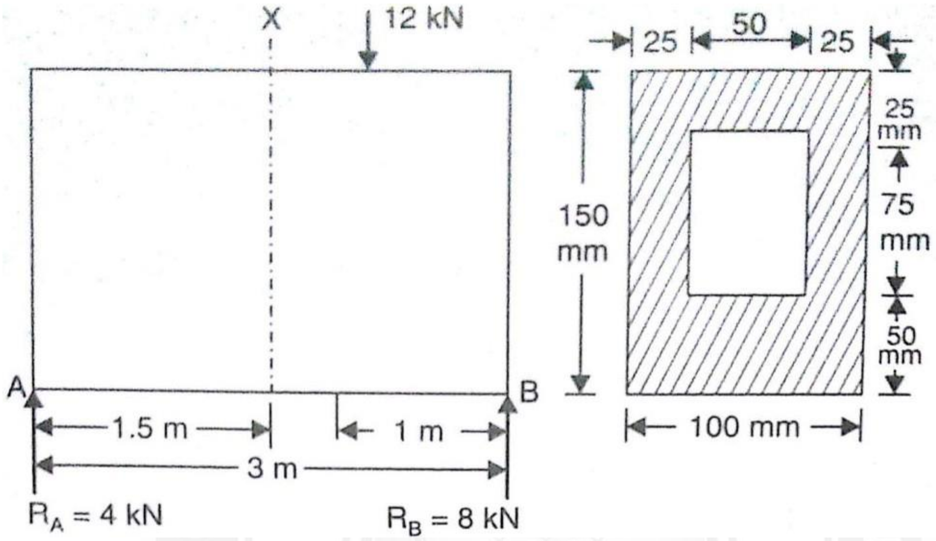
2. for max compressive stress y = 32.33mm

$\sigma = \frac{12000000}{3142222.4}\times 32.23 = \mathbf{123.08\ N/mm^2}$

Problem 2.28: A simply supported beam of length 3m carries a point load of 12k N at a distance of 2m from left support. The cross section of the beam is shown. Determine the max tensile and compressive stress at X-X

Given

Point load w = 12 K N = 1200N



Solution:

First find the B.M at X-X.

Taking moments about A

$R_B \times 3 = 12 \times 2$

R_B = 8k N

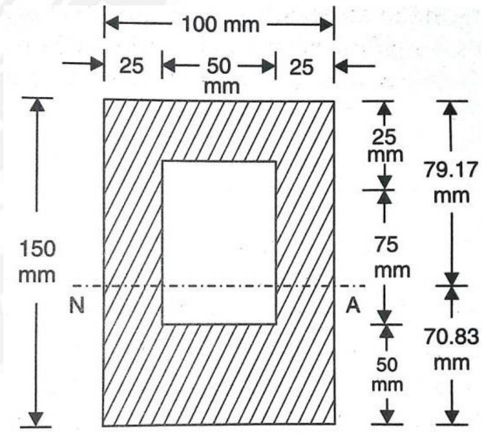
R_A = W- R_B

= 4k N

B.M. at X-X = $R_A \times 1.5 = 4 \times 1.5$ = 6k Nm

= 6000×1000 N mm

M = 6000,000 N mm



Let y = distance of C.G. of the section from the bottom edge

$= (A_{1F_1} - A_{2F_2})/(A_1 - A_2)$

= (150×100) × 75 -(75×50) ×(50+ (75/2))/(150×100-75×50)

= 70.83mm

Hence N.A will lie at a distance of 70.83mm from the bottom edge or 150 – 70.83 = 79.17mm from the top edge

M.O.I of section I = I_1 - I_2

Where I_1 =M.O.I of outer rectangle about N.A

= M.O.I of rectangle 100×150 about its C.G + A_1

= 100× $150^3/12$ +100×150× $(75 - 70.83)^2$

= 28385833.5mm^4

I_2 = M.O.I of cut out part about N.A

= 50× $75^3/12$ +50×75×(50+75/2 – 70.83)

= 2799895.875mm^4

I = 28385833.5 – 2799895.875 = 25585937.63mm^4

The bottom edge of the section will be subjected to tensile stress whereas the top edge will be subjected to compressive stress. the top edge is at 79.17 mm from N.A whereas bottom edge is 70.83 mm from N.A

Now using relation $\frac{M}{I} = \frac{\sigma}{y}$ € $\sigma = \frac{M}{I} \times y$

For max tensile stress y = 70.83mm

Max tensile stress $\sigma = \frac{6000000}{25585937.63} \times 70.83 =$ **16.60 N/**mm^2

For max compressive stress y = 79.17mm

$\sigma = \frac{6000000}{25585937.63} \times 79.17 =$ **18.56 N/** mm^2

2.5.8. STRENGTH OF A SECTION

The strength of a section means the moment of resistance offered by the section and moment of resistance is given by

M = Z× σ

Where M = moment of resistance

σ = bending stress

Z = section modulus

Problem 2.29: Three beams have the same length same allowable bending stress and the same bending moment. The cross section of the beam are a square rectangle with depth twice the width and a circle. Find the radius of weights of the circular and the rectangular beams with respect to square beams.

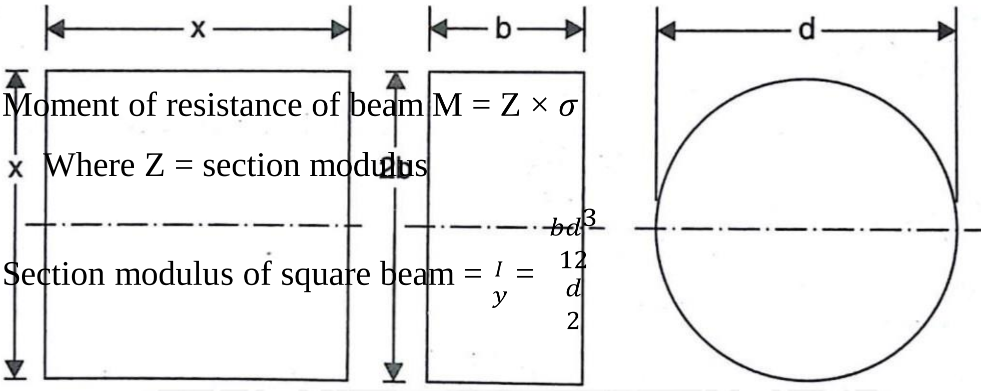
Given

Let x = side of square beam

b = width of rectangular beam

2b = depth of rectangular beam

D = diameter of circular section



Solution:

Moment of resistance of beam M = Z × σ

Where Z = section modulus

Section modulus of square beam = $\frac{I}{y} = \frac{\frac{bd^3}{12}}{\frac{d}{2}}$

$= \frac{x \times x^3}{12} \times \frac{2}{x}$

$= \frac{x^3}{6}$

Section modulus of rectangular beam = $\frac{\frac{b \times (2b)^3}{12}}{\frac{2b}{2}}$

$= \frac{2}{3} b^3$

Section modulus of a circular beam = $\frac{\frac{\pi d^4}{64}}{\frac{d}{2}} = \frac{\pi d^3}{32}$

Equating the section modulus of square beam with that of rectangular beam

$$\frac{x^3}{6} = \frac{2}{3} b^3$$

$$b^3 = 0.25x^3$$

$$b = 0.63x$$

Equating the section modulus of square beam with that of a circular beam

$$\frac{x^3}{6} = \frac{\pi d^3}{32}$$

$$d^3 = \frac{32x^3}{6\pi}$$

$$d = 1.1927x$$

Weight of beams are proportional to their cross sectional areas. Hence

$$\frac{\text{Weight of rectangular beam}}{\text{weight of square beam}} = \frac{\text{Area of rectangular beam}}{\text{Area of square beam}}$$

$$\frac{b \times 2b}{x \times x} = \frac{0.63x \times 2 \times 0.63x}{x \times x} = \mathbf{0.7938}$$

$$\frac{\text{Weight of rectangular beam}}{\text{weight of square beam}} = \frac{\text{Area of rectangular beam}}{\text{Area of square beam}}$$

$$= \frac{\frac{\pi d^2}{4}}{x^2} = \frac{\pi d^2}{4\,x^2} \qquad (\therefore d = 1.1927\,x)$$

$$= \frac{\pi\,(1.1927\,x)^2}{4\,x^2} = \mathbf{1.1172}$$