

2.2 Randomised Block Design (RBD)

Working Rule:

Set the null hypothesis H_0 : There is no significance difference between the treatments.

Step: 1 Find T = The total value of observations

Step: 2 Find the Correction Factor $C.F = \frac{T^2}{N}$

Step: 3 Calculate the total sum of squares and find the total sum of squares

$$TSS = (\sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \dots) - C.F$$

Step: 4 Find column sum of squares $SSC = \left(\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_2} + \frac{(\sum X_3)^2}{N_3} + \dots \right) - C.F$

Where N_i = Total number of observation in each column ($i = 1, 2, 3, \dots$)

Step: 5 Find Column sum of squares $SSR = \left(\frac{(\sum Y_1)^2}{N_1} + \frac{(\sum Y)^2}{N_2} + \frac{(\sum Y_3)^2}{N_3} + \dots \right) - C.F$

Where N_j = Total number of observation in each ROW ($j = 1, 2, 3, \dots$)

Step: 6 $SSE = TSS - (SSC + SSR)$

Step: 7 Prepare the ANOVA to calculate F – ratio

| Source of variation | Sum of Degrees | Degrees of Freedom | Mean Square | F - Ratio |
|---------------------|----------------|--------------------|--------------------------------|--|
| Between Columns | SSC | $c - 1$ | $MSC = \frac{SSC}{c-1}$ | $F_c = \frac{MSC}{MSE}$ if $MSC > MSE$ $F_c = \frac{MSE}{MSC}$ if $MSE > MSC$ |
| Between Rows | SSR | $r - 1$ | $MSR = \frac{SSR}{r-1}$ | $F_c = \frac{MSR}{MSE}$ if $MSR > MSE$ $F_c = \frac{MSE}{MSR}$ if $MSE > MSR$ |
| Error | SSE | $(r - 1)(c - 1)$ | $MSE = \frac{SSE}{(r-1)(c-1)}$ | |

Step: 8 Find the table value (use chi square table)

Step: 9 Conclusion:

Calculated value < Table value, then we accept null hypothesis.

Calculated value > Table value, then we reject null hypothesis.

PROBLEMS ON TWO WAY ANOVA TABLE

1. Three varieties A, B, C of a crop are tested in a randomized block design with four replication. The plot yields in pounds as follows.

| | | | |
|----|----|-----|----|
| A6 | C5 | A8 | B9 |
| C8 | A4 | B6 | C9 |
| B7 | B6 | C10 | A6 |

Analysis the experiment yield and state your conclusion.

Solution:

Set the null hypothesis H_0 : There is no significance difference between the rows and columns.

| Varieties | Yields | | | | Total |
|-----------|--------|----|----|----|-------|
| | 1 | 2 | 3 | 4 | |
| A | 6 | 4 | 8 | 6 | 24 |
| B | 7 | 6 | 6 | 9 | 28 |
| C | 8 | 5 | 10 | 9 | 32 |
| Total | 21 | 15 | 24 | 24 | 84 |

TEST STATISTIC:

| Varieties | | 1 | 2 | 3 | 4 | Total | X_1^2 | X_2^2 | X_3^2 | X_4^2 |
|-----------|---|-------|-------|-------|-------|-------|---------|---------|---------|---------|
| | | X_1 | X_2 | X_3 | X_4 | | | | | |
| Y_1 | A | 6 | 4 | 8 | 6 | 24 | 36 | 16 | 64 | 36 |
| Y_2 | B | 7 | 6 | 6 | 9 | 28 | 49 | 36 | 36 | 81 |
| Y_3 | C | 8 | 5 | 10 | 9 | 32 | 64 | 25 | 100 | 81 |
| Total | | 21 | 15 | 24 | 24 | 84 | 149 | 77 | 200 | 198 |

Step:1 Grand Total $T = 84$

Step: 2 Correction Factor $C.F = \frac{T^2}{N} = \frac{(84)^2}{12} = 588$

Step: 3 Calculate the total sum of squares and find the total sum of squares

$$TSS = (\sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \dots) - C.F$$

$$\begin{aligned}
 &= (149 + 77 + 200 + 198) - 588 \\
 &= 624 - 588 = 36
 \end{aligned}$$

Step: 4 Find column sum of squares $SSC = \left(\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_2} + \frac{(\sum X_3)^2}{N_3} + \dots \right) - C.F$

$$SSC = \left(\frac{(21)^2}{3} + \frac{(15)^2}{3} + \frac{(24)^2}{3} + \frac{(24)^2}{3} \right) - 588 = 18$$

Step: 5 Find Row sum of squares $SSR = \left(\frac{(\sum Y_1)^2}{N_1} + \frac{(\sum Y)^2}{N_2} + \frac{(\sum Y_3)^2}{N_3} + \dots \right) - C.F$

$$SSR = \left(\frac{(24)^2}{4} + \frac{(28)^2}{4} + \frac{(32)^2}{4} + \dots \right) - 588 = 8$$

Step: 6 SSE = Residual sum of squares

$$\begin{aligned}
 &= TSS - (SSC + SSR) \\
 &= 36 - (18 + 8) = 10
 \end{aligned}$$

Step: 7 Prepare the ANOVA to calculate F – ratio

| Source of variation | Sum of Degrees | Degrees of Freedom | Mean Square | F - Ratio |
|---------------------|----------------|--------------------------------------|--|-------------------------------|
| Between Columns | SSC=18 | $c - 1$ $= 4 - 1 = 3$ | $MSC = \frac{SSC}{c-1} =$ 6 | $F_c = \frac{MSC}{MSE} = 3.6$ |
| Between Rows | SSR=8 | $r - 1$ $= 3 - 1 = 2$ | $MSR = \frac{SSR}{r-1} =$ 4 | $F_R = \frac{MSR}{MSE} = 2.4$ |
| Error | SSE = 10 | $(r - 1)(c - 1)$ $2 \times 3 = 6$ | $MSE =$ $\frac{SSE}{(r - 1)(c - 1)}$ 1.667 | |

Step: 8 d.f for (3, 6) at 5% level of significance is 4.76

d.f for (2, 6) at 5% level of significance is 5.14

Step: 9 Conclusion:

Calculated value $F_c <$ Table value, then we accept null hypothesis.

There is no significance difference between the columns.

Calculated value $F_R <$ Table value, then we accept null hypothesis.

There is no significance difference between the rows.

2. Four varieties A, B, C, D of a fertilizer are tested in a randomized block design with four replication. The plot yields in pounds as follows.

| | | | |
|------|------|------|------|
| A 12 | D 20 | C 16 | B 10 |
| D 18 | A 14 | B 11 | C 14 |
| B 12 | C 15 | D 19 | A 13 |
| C 16 | B 11 | A 15 | D 20 |

Analysis the experimental yield.

Solution:

Set the null hypothesis H_0 : There is no significance difference between the rows and columns.

| Varieties | Yields | | | | |
|-----------|--------|----|----|----|---------|
| | 1 | 2 | 3 | 4 | Total |
| A | 12 | 14 | 15 | 13 | 54 |
| B | 12 | 11 | 11 | 10 | 44 |
| C | 16 | 15 | 16 | 14 | 61 |
| D | 18 | 20 | 19 | 20 | 77 |
| Total | 58 | 60 | 61 | 57 | 236 (T) |

TEST STATISTIC:

| Varieties | | 1 | 2 | 3 | 4 | Total | X_1^2 | X_2^2 | X_3^2 | X_4^2 |
|-----------|---|-------|-------|-------|-------|-------|---------|---------|---------|---------|
| | | X_1 | X_2 | X_3 | X_4 | | | | | |
| Y_1 | A | 12 | 14 | 15 | 13 | 54 | 144 | 196 | 225 | 169 |
| Y_2 | B | 12 | 11 | 11 | 10 | 44 | 144 | 121 | 121 | 100 |
| Y_3 | C | 16 | 15 | 16 | 14 | 61 | 256 | 225 | 256 | 196 |
| Y_4 | D | 18 | 20 | 19 | 20 | 77 | 324 | 400 | 361 | 400 |
| Total | | 58 | 60 | 61 | 57 | 236 | 868 | 942 | 963 | 865 |

Step:1 Grand Total $T = 236$

Step: 2 Correction Factor $C.F = \frac{T^2}{N} = \frac{(236)^2}{16} = 3481$

Step: 3 Calculate the total sum of squares and find the total sum of squares

$$\begin{aligned}
 \text{TSS} &= (\sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \dots) - C.F \\
 &= (868 + 942 + 963 + 865) - 3481 \\
 &= 3638 - 3481 = 157
 \end{aligned}$$

Step: 4 Find column sum of squares $\text{SSC} = \left(\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_2} + \frac{(\sum X_3)^2}{N_3} + \dots \right) - C.F$

$$\begin{aligned} \text{SSC} &= \left(\frac{(58)^2}{4} + \frac{(60)^2}{4} + \frac{(61)^2}{4} + \frac{(57)^2}{4} \right) - 3481 \\ &= 841 + 900 + 930 + 812 - 3481 = 2 \end{aligned}$$

Step: 5 Find Row sum of squares $\text{SSR} = \left(\frac{(\sum Y_1)^2}{N_1} + \frac{(\sum Y)^2}{N_2} + \frac{(\sum Y_3)^2}{N_3} + \dots \right) - C.F$

$$\begin{aligned} \text{SSR} &= \left(\frac{(54)^2}{4} + \frac{(44)^2}{4} + \frac{(61)^2}{4} + \frac{(77)^2}{4} \right) - 3481 \\ &= 729 + 484 + 930.25 + 1482.25 - 3481 \\ &= 144.5 \end{aligned}$$

Step: 6 SSE = Residual sum of squares

$$\begin{aligned} &= \text{TSS} - (\text{SSC} + \text{SSR}) \\ &= 157 - (2 + 144.5) = 10.5 \end{aligned}$$

Step: 7 Prepare the ANOVA to calculate F – ratio

| Source of variation | Sum of Degrees | Degrees of Freedom | Mean Square | F - Ratio |
|---------------------|----------------|--|--|---|
| Between Columns | SSC=2 | $c - 1$ $= 4 - 1 = 3$ | $\text{MSC} = \frac{\text{SSC}}{c-1}$ $= 0.666$ | $F_c = \frac{\text{MSE}}{\text{MSC}} = 1.74$ |
| Between Rows | SSR=144.5 | $r - 1$ $= 4 - 1 = 3$ | $\text{MSR} = \frac{\text{SSR}}{r-1} =$ 48.16 | $F_R = \frac{\text{MSR}}{\text{MSE}} = 41.51$ |
| Error | SSE = 10.5 | $(r - 1)(c - 1)$ $= 3 \times 3 = 9$ | $\text{MSE} =$ $\frac{\text{SSE}}{(r - 1)(c - 1)} =$ 1.6 | |

Step: 8 d.f for (9, 3) at 5% level of significance is 8.82

d.f for (3, 9) at 5% level of significance is 3.86

Step: 9 Conclusion:

Calculated value $F_c < \text{Table value}$, then we accept null hypothesis.

There is no significance difference between the columns.

Calculated value $F_R > \text{Table value}$, then we reject null hypothesis.

There is a significance difference between the rows.