Dispersion

Dispersion of the transmitted optical signal causes distortion for both digital and analog transmission along optical fibers. When considering the major implementation of optical fiber transmission which involves some form of digital modulation, then dispersion mechanisms within the fiber cause broadening of the transmitted light pulses as they travel along the channel. The phenomenon is illustrated in Figure 2.7, where it may be observed that each pulse broadens and overlaps with its neighbors, eventually becoming indistinguishable at the receiver input. The effect is known as intersymbol interference (ISI). Thus an increasing number of errors may be encountered on the digital optical channel as the ISI becomes more pronounced. The error rate is also a function of the signal attenuation on the link and the subsequent signal-to-noise ratio (SNR) at the receiver. For no overlapping of light pulses down on an optical fiber link the digital bit rate B_T must be less than the reciprocal of the broadened (through dispersion) pulse duration (2τ).

Hence:

$$B_T \leq \frac{1}{2\tau} \qquad (2.10)$$

The conversion of bit rate to bandwidth in hertz depends on the digital coding format used. For metallic conductors when a nonreturn-to-zero code is employed, the binary 1 level is held for the whole bit period τ. In this case there are two bit periods in one wavelength (i.e. 2 bits per second per hertz), as illustrated in Figure 2.8(a). Hence the maximum bandwidth B is one-half the maximum data rate or:

$$B_T(\max) = 2B \tag{2.12}$$

However, when a return-to-zero code is considered, as shown in Figure 2.8(b), the binary 1 level is held for only part (usually half) of the bit period. For this signaling scheme the data rate is equal to the bandwidth in hertz (i.e. 1 bit per second per hertz) and thus *B*T = *B*.

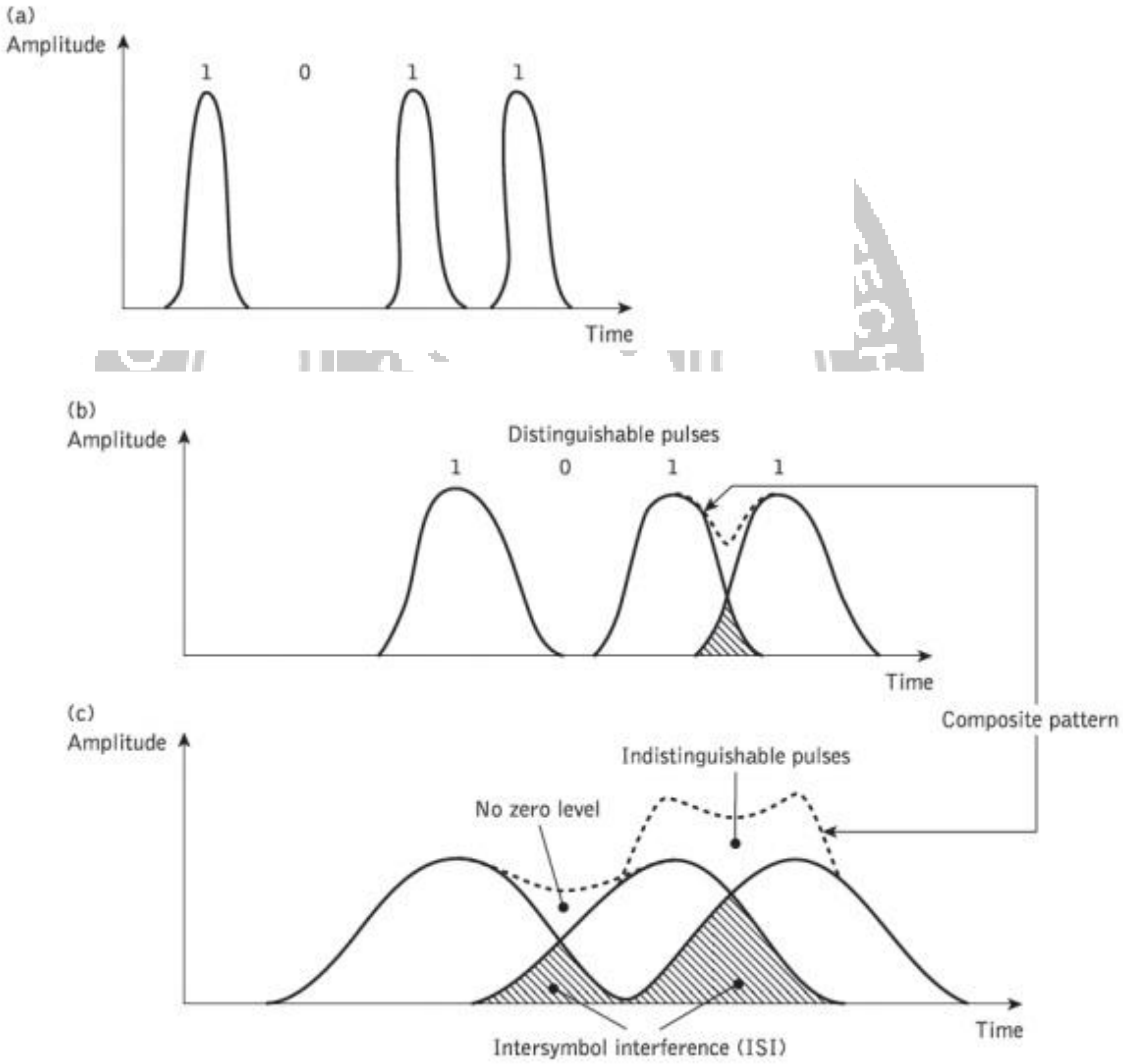

Figure 2.7 An illustration using the digital bit pattern 1011 of the broadening of light pulses as they are transmitted along a fiber: (a) fiber input; (b) fiber output at a distance $L1$; (c) fiber output at a distance $L2 > L1$

[Source: htpp://img.brainkart.com]

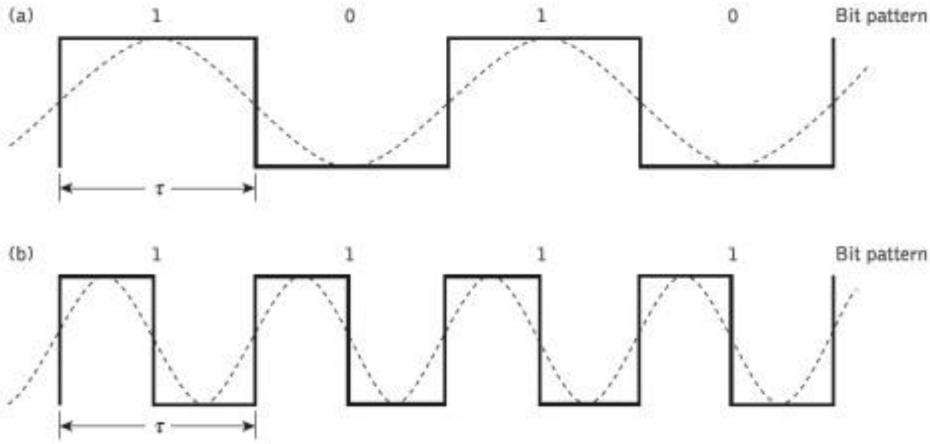

Figure 2.8 Schematic illustration of the relationships of the bit rate to wavelength for digital codes: (a) nonreturn-to-zero (NRZ); (b) return-to-zero (RZ)

[Source: htpp://img.brainkart.com]

The bandwidth *B* for metallic conductors is also usually defined by the electrical 3 Db points (i.e. the frequencies at which the electric power has dropped to one-half of its constant maximum value). However, when the 3 dB optical bandwidth of a fiber is considered it is significantly larger than the corresponding 3 dB electrical bandwidth. Hence, when the limitations in the bandwidth of a fiber due to dispersion are stated (i.e. optical bandwidth *B*opt), it is usually with regard to a return to zero code where the bandwidth in hertz is considered equal to the digital bit rate. Within the context of dispersion the bandwidths expressed in this chapter will follow this general criterion unless otherwise stated. when electro-optic devices and optical fiber systems are considered it is more usual to state the electrical 3 dB bandwidth, this being the more useful measurement when interfacing an optical fiber link to electrical terminal equipment.

Intramodal Dispersion

Chromatic or intramodal dispersion may occur in all types of optical fiber and results from the finite spectral linewidth of the optical source. Since optical sources do not emit just a single frequency but a band of frequencies (in the case of the injection laser corresponding to only a fraction of a percent of the center frequency, whereas for the LED it is likely to be a significant percentage), then there may be propagation delay differences between the different spectral components of the transmitted signal. This causes broadening of each transmitted mode and hence intramodal dispersion. The delay differences may be caused by the dispersive properties of the waveguide material (material dispersion) and also guidance effects within the fiber structure (waveguide dispersion).

1. Material Dispersion

Pulse broadening due to material dispersion results from the different group velocities of the various spectral components launched into the fiber from the optical source. It occurs when the phase velocity of a plane wave propagating in the dielectric medium varies nonlinearly with wavelength, and a material is said to exhibit material dispersion when the second differential of the refractive index with respect to wavelength is not zero.

Hence the group delay is given by:

$$\tau_g = \frac{d\beta}{d\omega} = \frac{1}{c}\left(n_1 - \lambda\frac{dn_1}{d\lambda}\right) \qquad (2.13)$$

Where n_1 is the refractive index of the core material. The pulse delay τ_m due to material dispersion in a fiber of length L is therefore:

$$\tau_m = \frac{L}{c}\left(n_1 - \lambda\frac{dn_1}{d\lambda}\right) \tag{2.14}$$

For a source with rms spectral width □□ and a mean wavelength □, the rms pulse broadening due to material dispersion □m may be obtained from the expansion of Eq. (2.14) in a Taylor series about □ where:

$$\sigma_m = \sigma_\lambda \frac{d\tau_m}{d\lambda} + \sigma_\lambda \frac{2d^2\tau_m}{d\lambda^2} + \ldots \tag{2.15}$$

As the first term in Eq. (2.15) usually dominates, especially for sources operating over the 0.8 to 0.9 μm wavelength range, then:

$$\sigma_m \simeq \sigma_\lambda \frac{d\tau_m}{d\lambda} \tag{2.16}$$

Hence the pulse spread may be evaluated by considering the dependence of τm on λ, where from Eq. (2.14):

$$\frac{d\tau_m}{d\lambda} = \frac{L\lambda}{c}\left[\frac{dn_1}{d\lambda} - \frac{d^2n_1}{d\lambda^2} - \frac{dn_1}{d\lambda}\right]$$

$$= \frac{-L\lambda}{c}\frac{d^2n_1}{d\lambda^2} \tag{2.17}$$

Therefore, substituting the expression obtained in Eq. (2.17) into Eq. (2.16), the rms pulse broadening due to material dispersion is given by:

$$\sigma_m \simeq \frac{\sigma_\lambda L}{c} \left| \lambda \frac{d^2 n_1}{d\lambda^2} \right| \tag{2.18}$$

The material dispersion for optical fibers is sometimes quoted as a value for $|\lambda^2(d^2 n_1/d\lambda^2)|$ or simply $|d^2 n_1/d\lambda^2|$.

However, it may be given in terms of a material dispersion parameter M which is defined as:

$$M = \frac{1}{L}\frac{d\tau_m}{d\lambda} = \frac{\lambda}{c}\left| \frac{d^2 n_1}{d\lambda^2} \right| \tag{2.19}$$

and which is often expressed in units of ps nm−1 km−1.

2. Waveguide Dispersion

The wave guiding of the fiber may also create chromatic dispersion. This results from the variation in group velocity with wavelength for a particular mode. Considering the ray theory approach, it is equivalent to the angle between the ray and the fiber axis varying with wavelength which subsequently leads to a variation in the transmission times for the rays, and hence dispersion. For a single mode whose propagation constant is β, the fiber exhibits waveguide dispersion when d2β/dλ2!=0 . Multimode fibers, where the majority of modes propagate far from cutoff, are almost free of waveguide dispersion and it is generally negligible compared with material dispersion (≈0.1 to 0.2 ns/ km). However, with single-mode fibers where the effects of the different dispersion mechanisms are not easy to separate, waveguide dispersion may be significant.

Intermodal Dispersion

Pulse broadening due to intermodal dispersion (sometimes referred to simply as modal or mode dispersion) results from the propagation delay differences between modes within a multimode fiber. As the different modes which constitute a pulse in a multimode fiber travel along the channel at different group velocities, the pulse width at the output is dependent upon the transmission times of the slowest and fastest modes. This dispersion mechanism creates the fundamental difference in the overall dispersion for the three types of fiber. Thus multimode step index fibers exhibit a large amount of intermodal dispersion which gives the greatest pulse broadening. However, intermodal dispersion in multimode fibers may be reduced by adoption of an optimum refractive index profile which is provided by the near-parabolic profile of most graded index fibers.

Hence, the overall pulse broadening in multimode graded index fibers is far less than that obtained in multimode step index fibers (typically by a factor of 100). Thus graded index fibers used with a multimode source give a tremendous bandwidth advantage over multimode step index fibers. Under purely single-mode operation there is no intermodal dispersion and therefore pulse broadening is solely due to the intramodal dispersion mechanisms. In theory, this is the case with single-mode step index fibers where only a single mode is allowed to propagate. Hence they exhibit the least pulse broadening and have the greatest possible bandwidths, but in general are only usefully operated with single-mode sources. In order to obtain a simple comparison for intermodal pulse broadening between multimode step index and multimode graded index fibers, it is useful to consider the geometric optics picture for the two types of fiber.

1. Multimode Step Index Fiber

Using the ray theory model, the fastest and slowest modes propagating in the step index fiber may be represented by the axial ray and the extreme meridional ray (which is incident at the core–cladding interface at the critical angle φ_c) respectively. The paths taken by these two rays in a perfectly structured step index fiber are shown in Figure 2.9. The delay difference between these two rays when traveling in the fiber core allows estimation of the pulse broadening resulting from intermodal dispersion within the fiber. As both rays are traveling at the same velocity within the constant refractive index fiber core, then the delay difference is directly related to their respective path lengths within the fiber. Hence the time taken for the axial ray to travel along a fiber of length L gives the minimum delay time TMin and:

$$T_{Min} = \frac{\text{distance}}{\text{velocity}} = \frac{L}{(c/n_1)} = \frac{Ln_1}{c} \quad (2.20)$$

where n_1 is the refractive index of the core and c is the velocity of light in a vacuum. The extreme meridional ray exhibits the maximum delay time T_{Max} where:

$$T_{Max} = \frac{L/\cos\theta}{c/n_1} = \frac{Ln_1}{c\cos\theta} \quad (2.21)$$

Using Snell's law of refraction at the core–cladding interface following Eq. (2.2):

$$\sin\phi_c = \frac{n_2}{n_1} = \cos\theta \quad (2.22)$$

where n_2 is the refractive index of the cladding. Furthermore, substituting into Eq. (2.21) gives:

$$T_{Max} = \frac{Ln_1^2}{cn_2} \quad (2.23)$$

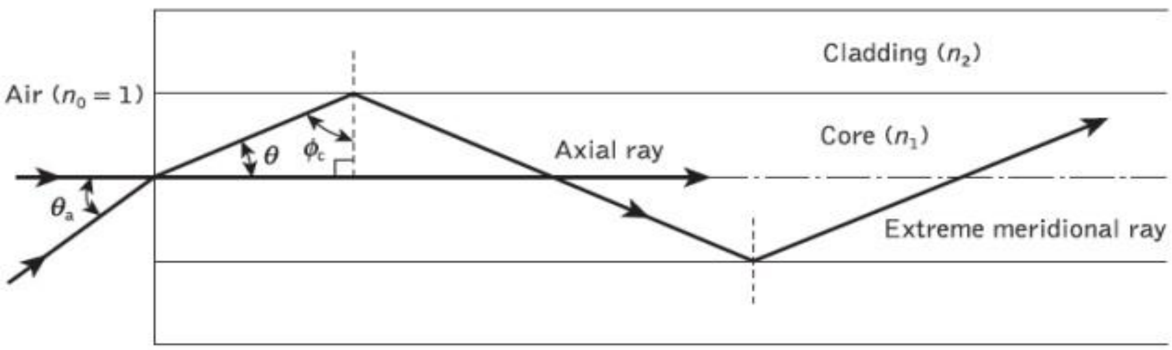

Figure 2.9 The paths taken by the axial and an extreme meridional ray in a perfect multimode step index fiber

[Source: htpp://img.brainkart.com]

The delay difference δT_s between the extreme meridional ray and the axial ray may be obtained by:

$$\delta T_s = T_{Max} - T_{Min} = \frac{Ln_1^2}{cn_2} - \frac{Ln_1}{c}$$

$$= \frac{Ln_1^2}{cn_2}\left(\frac{n_1 - n_2}{n_1}\right) \qquad (2.24)$$

$$\simeq \frac{Ln_1^2\Delta}{cn_2} \quad \text{when } \Delta \ll 1 \qquad (2.25)$$

where is the relative refractive index difference. However, when $\Delta \ll 1$, then from the definition given by Eq. (2.9), the relative refractive index difference may also be given approximately by:

$$\Delta \simeq \frac{n_1 - n_2}{n_2} \qquad (2.26)$$

Hence rearranging Eq. (3.24):

$$\delta T_s = \frac{Ln_1}{c}\left(\frac{n_1 - n_2}{n_2}\right) \simeq \frac{Ln_1\Delta}{c} \qquad (2.27)$$

Also substituting for Δ from Eq. (2.10) gives:

$$\delta T_s \simeq \frac{L(NA)^2}{2n_1c} \qquad (2.28)$$

where *NA* is the numerical aperture for the fiber. The approximate expressions for the delay difference given in Eqs (2.27) and (2.28) are usually employed to estimate the maximum pulse broadening in time due to intermodal dispersion in multimode step index fibers. Again considering the perfect step index fiber, another useful quantity with regard to intermodal dispersion on an optical fiber link is the rms pulse broadening resulting from this dispersion mechanism along the fiber. When the optical input to the fiber is a pulse $p_i(t)$ of unit area, as illustrated in Figure 2.10, then

$$\int_{-\infty}^{\infty} p_i(t)\,dt = 1 \quad (2.29)$$

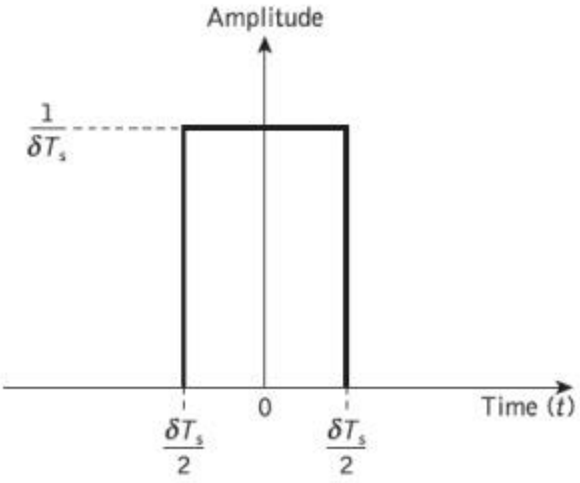

Figure 2.10 An illustration of the light input to the multimode step index fiber consisting of an ideal pulse or rectangular function with unit area

[Source: htpp://img.brainkart.com]

It may be noted that *p*i(*t*) has a constant amplitude of 1/*T*s over the range:

$$\frac{-\delta T_s}{2} \le p(t) \le \frac{\delta T_s}{2}$$

The rms pulse broadening at the fiber output due to intermodal dispersion for the multimode step index fiber (i.e. the standard deviation) may be given in terms of the variance s2:

$$\sigma_s^2 = M_2 - M_1^2 \quad (2.30)$$

Where *M*1 is the first temporal moment which is equivalent to the mean value of the pulse and *M*2, the second temporal moment, is equivalent to the mean square value of the pulse.

Hence:

$$M_1 = \int_{-\infty}^{\infty} t p_i(t)\, dt \tag{2.31}$$

And:

$$M_2 = \int_{-\infty}^{\infty} t^2 p_i(t)\, dt \tag{2.32}$$

The mean value $M1$ for the unit input pulse of Figure 2.10 is zero, and assuming this is maintained for the output pulse, then from Eqs (2.30) and (2.32):

$$\sigma_s^2 = M_2 = \int_{-\infty}^{\infty} t^2 p_i(t)\, dt \tag{2.33}$$

Integrating over the limits of the input pulse (Figure 3.12) and substituting for pi(t) in Eq. (2.33) over this range gives:

$$\sigma_s^2 = \int_{-\delta T_s/2}^{\delta T_s/2} \frac{1}{\delta T_s} t^2\, dt$$

$$= \frac{1}{\delta T_s}\left[\frac{t^3}{3}\right]_{-\delta T_s/2}^{\delta T_s/2} = \frac{1}{3}\left(\frac{\delta T_s}{2}\right)^2 \tag{2.34}$$

Hence substituting from Eq. (2.27) for δTs gives:

$$\sigma_s \simeq \frac{L n_1 \Delta}{2\sqrt{3} c} \simeq \frac{L(NA)^2}{4\sqrt{3} n_1 c} \tag{2.35}$$

Equation (2.35) allows estimation of the rms impulse response of a multimode step index fiber if it is assumed that intermodal dispersion dominates and there is a uniform distribution of light rays over the range The pulse broadening is directly proportional to the relative refractive index difference and the length of the fiber L. The latter emphasizes the bandwidth–length trade-off that exists, especially with multimode step index fibers, and which inhibits their use for wideband long-haul (between repeaters) systems. Furthermore, the pulse broadening is reduced by reduction of the relative refractive index difference for the fiber. Intermodal dispersion may be reduced by propagation mechanisms within practical fibers. For instance, there is differential attenuation of the various modes in a step index fiber. This is due to the greater field penetration of the higher order modes into the cladding of the waveguide. These slower modes therefore exhibit larger losses at any core–cladding irregularities, which tends to concentrate the transmitted optical power into the faster lower order modes. Thus the differential attenuation of modes reduces intermodal pulse broadening on a multimode optical link.

Another mechanism which reduces intermodal pulse broadening in nonperfect (i.e. practical) multimode fibers is the mode coupling or mixing. The coupling between guided modes transfers optical power from the slower to the faster modes, and vice versa. Hence, with strong coupling the optical power tends to be transmitted at an average speed, which is the mean of the various propagating modes. This reduces the intermodal dispersion on the link and makes it advantageous to encourage mode coupling within multimode fibers. The expression for delay difference given in Eq. (2.27) for a perfect step index fiber may be modified for the fiber with mode coupling among all guided modes to:

$$\delta T_{sc} \simeq \frac{n_1 \Delta}{c} (L L_c)^{\frac{1}{2}} \qquad (2.36)$$

2. Multimode Graded Index Fiber

Intermodal dispersion in multimode fibers is minimized with the use of graded index fibers. Hence, multimode graded index fibers show substantial bandwidth improvement over multimode step index fibers. The reason for the improved performance of graded index fibers may be observed by considering the ray diagram for a graded index fiber shown in Figure 2.11. The fiber shown has a parabolic index profile with a maximum at the core axis, as illustrated in Figure 2.11(a). Analytically, the index profile is given by:

$$n(r) = n_1[1 - 2\Delta(r/a)^2]^{\frac{1}{2}} \qquad r < a \text{ (core)}$$
$$= n_1(1 - 2\Delta)^{\frac{1}{2}} = n_2 \qquad r \geq a \text{ (cladding)} \qquad (2.37)$$

Figure 2.11(b) shows several meridional ray paths within the fiber core. It may be observed that apart from the axial ray, the meridional rays follow sinusoidal trajectories of different path lengths which result from the index grading. However, the local group velocity is inversely proportional to the local refractive index and therefore the longer sinusoidal paths are compensated for by higher speeds in the lower index medium away from the axis.

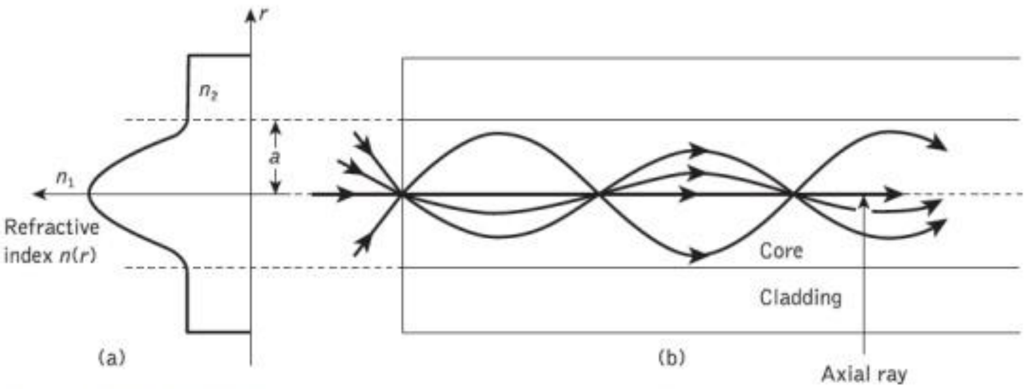

Figure 2.11 A multimode graded index fiber: (a) parabolic refractive index profile; (b) meridional ray paths within the fiber core

[Source: htpp://img.brainkart.com]

Hence there is an equalization of the transmission times of the various trajectories towards the transmission time of the axial ray which travels exclusively in the high-index region at the core axis, and at the slowest speed. As these various ray paths may be considered to represent the different modes propagating in the fiber, then the graded profile reduces the disparity in the mode transit times. The dramatic improvement in multimode fiber bandwidth achieved with a parabolic or near-parabolic refractive index profile is highlighted by consideration of the reduced delay difference between the fastest and slowest modes for this graded index fiber □*T*g. Using a ray theory approach the delay difference is given by:

$$\delta T_g \simeq \frac{L n_1 \Delta^2}{2c} \simeq \frac{(NA)^4}{8 n_1^3 c} \tag{2.38}$$

However, a more rigorous analysis using electromagnetic mode theory gives an absolute temporal width at the fiber output of :

$$\delta T_g = \frac{L n_1 \Delta^2}{8c} \tag{2.39}$$

which corresponds to an increase in transmission time for the slowest mode of 2/8 over the fastest mode. The expression given in Eq. (2.39) does not restrict the bandwidth to pulses with time slots corresponding to *T*g as 70% of the optical power is concentrated in the first half of the interval. Hence the rms pulse broadening is a useful parameter for assessment of intermodal dispersion in multimode graded index fibers. It may be shown that the rms pulse broadening of a near-parabolic index profile graded index fiber g is reduced compared with similar broadening for the corresponding step index fibers (i.e. with the same relative refractive index difference) following

$$\sigma_g = \frac{\Delta}{D} \sigma_s \tag{2.40}$$

where *D* is a constant between 4 and 10 depending on the precise evaluation and the exact optimum profile chosen. The best minimum theoretical intermodal rms pulse broadening for a graded index fiber with an optimum characteristic refractive index profile for the core α_{op} of

$$\alpha_{op} = 2 - \frac{12\Delta}{5} \tag{2.41}$$

is given by combining Eqs (2.27) and (2.40) as:

$$\sigma_g = \frac{L n_1 \Delta^2}{20\sqrt{3}c} \tag{2.42}$$