INTRODUCTION

Random Process:

Consider a random experiment with a sample space S. If a time function X(t, s) is assigned to each outcome $s \in S$ and where $t \in T$, then the family of all such functions, denoted by $\{X(t, s)\}$, where $s \in S$, $t \in T$ is called a random process. In other words, a random process is a collection of random variables together with time.

Note: A random process is also called stochastic process.

Classification of Random Process:

Classify a random process according to the characteristic of T and the state space S. We shall consider only 4 cases based on T and S.

i) Continuous random process
ii) Continuous random sequence
iii) Discrete random process
iv) Discrete random sequence

Continuous random Process:

If both S and T are continuous, then the random process is called continuous Random process.

Continuous Random Sequence:

If S is continuous and T is discrete, then the random process is called continuous random sequence.

Discrete Random Process:

If S is discrete and T is continuous, then the random process is called discrete random process.

Discrete Random Sequence:

If both S and T are discrete, then the random process is called discrete random process.

Deterministic Random Process:

A random process is called a deterministic random process if all the future values are predicted from past observation.

Non Deterministic Random Process:

A random process is called a non - deterministic random process if the future values of any sample function cannot be predicted from the past observation.

Wide Sense Stationary Process (WSS);

A process $\{X(t)\}$ is said to be Wide Sense Stationary Process if

(i)$Mean = E[X(t)] = constant$

(ii) $Auto\ correlation\ \ R_{XX}(\tau) = E[X(t)X(t+\tau)]$ depends on τ

Note:

A WSS process is also called as Weak Sense Stationary Process.

A SSS process is also called a strongly stationary process.

For stationary process mean and variance are constants.

A random process, which is not stationary in any sense, is called evolutionary.

Formulae:

Wide Sense Stationary (WSS):

(i)$Mean = E[X(t)] = constant$

(ii) $Auto\ correlation\ \ R_{XX}(\tau) = E[X(t)X(t+\tau)]$ depends on τ

Stationary Process:

(i)$E[X(t)] = constant$

(ii)$Var[X(t)] = constant$

Strict Sense Stationary (SSS):

$$E[X^n(t)]\ is\ a\ constant\ for\ every\ n$$

Joint Wide Sense Stationary (JWSS):

(i)$E[X(t)] = constant$

(ii)$E[Y(t)] = constant$

(iii)$R_{XX}(t, t+\tau\ ) = E[X(t)Y(t+\tau)]\ \ depends\ on\ \tau$

Mean Ergodic:

Time average, $\overline{X_T} = \frac{1}{2T}\int_{-T}^{T} X(t)dt$

$$E[X(t)] = \lim_{T\to\infty} \overline{X_T}$$

Correlation Ergodic:

$$\overline{X_T} = \frac{1}{2T}\int_{-T}^{T} X(t)\ X(t+\tau\ )dt$$

$$R_{XX}(t, t+\tau\ ) = \lim_{T\to\infty} \overline{X_T}$$

If $Y(t) = X(t+a) - X(t-a)$, prove that $R_{YY}(\tau) = \langle 2R_{XX}(\tau) - R_{XX}(\tau + 2a) - R_{XX}(\tau - 2a)$

Solution:

Given $Y(t) = X(t+a) - X(t-a)$

$R_{YY}(t) = E[Y(t_1)Y(t_2)]$

$= E\left[\left(X(t_1+a) - X(t_1-a)\right)\left(X(t_2+a) - X(t_2-a)\right)\right]$

$= E[(X(t_1+a)(X(t_2+a) - X(t_1+a)X(t_2-a) - X(t_1-a)(X(t_2 \quad +a) + X(t_1-a)X(t_2-a))]$

$= E[X(t_1+a)(X(t_2+a)] - E[X(t_1+a)X(t_2-a)] - E[X(t_1 \quad -a)(X(t_2+a)] + E[X(t_1-a)X(t_2-a)]$

$= R_{XX}(t_1+a, t_2+a) - R_{XX}(t_1+a, t_2-a) - R_{XX}(t_1-a, t_2+a) + R_{XX}(t_1-a, t_2-a)$

$= R_{XX}(t_1+a-t_2-a) - R_{XX}(t_1+a-t_2+a) - R_{XX}(t_1-a-t_2-a) + R_{XX}(t_1-a-t_2+a)$

$= R_{XX}(t_1-t_2) - R_{XX}(t_1-t_2+2a) - R_{XX}(t_1-t_2-2a) + R_{XX}(t_1-t_2)$

$= R_{XX}(\tau) - R_{XX}(\tau+2a) - R_{XX}(\tau-2a) + R_{XX}(\tau)$

$R_{YY}(\tau) = 2R_{XX}(\tau) - R_{XX}(\tau+2a) - R_{XX}(\tau-2a)$

The following formulas are very useful to solve problems under stationary process.

> If X is a RV with mean zero, then $\mathrm{Var}(X) = E(X^2)$

> $1 + 2x + 3x^2 + \cdots = (1-x)^{-2}$

> $1 + 4x + 9x^2 + \cdots = (1+x)(1-x)^{-3}$

> If A and B are RV's and λ is a constant, then

$E[A\cos\lambda t + B\sin\lambda t] = E(A)\cos\lambda t + E(B)\sin\lambda t$

> $\therefore E(\cos\lambda\tau) = \cos\lambda\tau$, since λ and τ are constants.