Maxwell-Ampere Law

The progress in the theory of displacement current can be traced back to a famous physicist named James Clerk Maxwell. Maxwell is well known for Maxwell's Equations. The combination of four equations demonstrates the fundamentals of electricity and magnetism. For displacement current, we will be focusing on one of these equations known as the Maxwell-Ampere law.

Before Maxwell, Andre-Marie Ampere had developed the famous equation known as Ampere's law. This law relates the magnetic field (B) surrounding a closed loop to the conduction current (I) traveling through that loop multiplied by a constant known as the permeability of free space (μ_0).

$$\int B \cdot ds = \mu_0 I$$

Whenever there is continuous conduction current Ampere's law holds true, but there are cases when problems arise in the law as it's written. For example, a circuit with a capacitor in it. When the capacitor is charging and discharging, current flows through the wires creating a magnetic field, but between the plates of the capacitor, there is no presence of current flow. According to Ampere's law, there can be no magnetic field created by the current here, but we know that a magnetic field does exist. Maxwell realized this discrepancy in Ampere's law and modified it in order to resolve the issue.

$$\int B \cdot ds = \mu_0 (I + \varepsilon_0 (d\varphi_E /dt))$$

This final form of the equation is known as the Maxwell-Ampere law. The part Maxwell added to it is known as displacement current (Id), and the formula is,

$$Id = \varepsilon_0 (d\varphi_E /dt)$$

The above equation consists of two terms multiplied together. The first is known as the permittivity of free space (ε_0), and the second is the derivative with respect to time and electric flux (φ_E). Electric flux is the rate of flow of an electric field through a given area. By taking its derivative with respect to time, we consider the change in that rate of flow over time.

Need for Displacement Current

Ampere's circuital law for conduction of current during charging of a capacitor was found inconsistent. Therefore, Maxwell modified Ampere's circuital law by introducing the concept of displacement current. Displacement currents play a central role in the propagation of electromagnetic radiation, such as light and radio waves through empty space. It is required to make the conduction current lead in the circuit. Conduction current in wires can be made to lead the voltage by means of displacement current inside the capacitor and it has vast uses in induction motors, industrial appliances, and in our day-to-day life.

Solved Examples on Displacement Current

Example 1: Instantaneous displacement current of 2.0 A is set up in the space between two parallel plates of a 1 μF capacitor. find the rate of change in potential difference across the capacitor.

Solution:

In a Capacitor

$V = q/C$

$dV/dt = i/C$

$dV/dt = 2.0A / 1\mu F$

$= 2 \times 106\ V/s$

The rate of change in potential difference across the capacitor is, 2 x 106 V/s

Example 2: A parallel plate capacitor with plate area A and separation between the plates d, is charged by a constant current I. Consider a plane surface of area A/4 parallel to the plates and drawn between the plates. What is the displacement current through this area?

Solution:

Electric field between the plates of Capacitor

$E = q/A\varepsilon 0 = It / A\varepsilon 0$

Electric flux through area A/4 is,

$\varphi E = (A/4)E = It / 4\varepsilon 0$

Then displacement current is,

$ID = \varepsilon 0 \, (d\varphi E / dt)$

$= \varepsilon 0 \, d/dt \, (It / 4\varepsilon 0)$

$= I/4$

Hence, the displacement current through the required area is I/4.

Example 3: A coil that has 700 turns develops an average induced voltage of 50 V. What must be the change in the magnetic flux that occur to produce such a voltage if the time interval for this change is 0.7 seconds?

Solution:

Given:

Number of Turn (N) = 700

Induced Voltage (e) = 50 V

Time Interval (dt) = 0.7 s

By using Faraday's Law,

$e = N \, (d\varphi/dt)$

Therefore, Change in flux ($d\varphi$) is,

$d\varphi = e \times dt / N$

$= 50 \times 0.7 / 700$

$d\varphi = 0.05 \text{ Wb}$