ROHINI COLLEGE OF ENGINEERING & TECHNOLOGY
Near Anjugramam Junction, Kanyakumari Main Road, Palkulam, Variyoor P.O - 629401
Kanyakumari Dist, Tamilnadu., E-mail : admissions@rcet.org.in, Website : www.rcet.org.in
DEPARTMENT OF MECHANICAL ENGINEERING

ME3491 THEORY OF MACHINES

COURSE MATERIAL

KINEMATICS OF LINKAGE MECHANISMS

1.1Displacement, velocity and acceleration analysis in simple mechanisms:

Important Concepts in Velocity Analysis

1. The absolute velocity of any point on a mechanism is the velocity of that point with reference to ground.
2. Relative velocity describes how one point on a mechanism moves relative to another point on the mechanism.
3. The velocity of a point on a moving link relative to the pivot of the link is given by the equation: $V = \omega r$, where = angular velocity of the link and r = distance from pivot.

Acceleration Components

- ***Normal Acceleration:*** $\mathbf{A}^n$ = Points toward the centre of rotation
- ***Tangential Acceleration:*** $\mathbf{A}^t$ = In a direction perpendicular to the link
- ***Coriolis Acceleration:*** $\mathbf{A}^c$ = In a direction perpendicular to the link
- ***Sliding Acceleration:*** $\mathbf{A}^s$ = In the direction of sliding.

A rotating link will produce normal and tangential acceleration components at any point a distance, r, from the rotational pivot of the link. The total acceleration of that point is the vector sum of the components. A slider attached to ground experiences only sliding acceleration.

The total acceleration of a point is the vector sum of all applicable acceleration components:

$$\mathbf{A} = \mathbf{A}^n + \mathbf{A}^t + \mathbf{A}^c + \mathbf{A}^s$$

These vectors and the above equation can be broken into x and y components by applying sines and cosines to the vector diagrams to determine the x and y components of each vector. In this way, the x and y components of the total acceleration can be found.

1.2Graphical Method, Velocity and Acceleration polygons:

*** Graphical velocity analysis:**

It is a very short step (using basic trigonometry with sines and cosines) to convert the graphical results into numerical results. The basic steps are these:

4. Set up a velocity reference plane with a point of zero velocity designated.
5. Use the equation, $V = \omega r$, to calculate any known linkage velocities.
6. Plot your known linkage velocities on the velocity plot. A linkage that is rotating about ground gives an absolute velocity. This is a vector that originates at the zero-velocity point and runs perpendicular to the link to show the direction of motion. The vector, **V**A, gives the velocity of point A.
7. Plot all other velocity vector directions. A point on a grounded link (such as point B) will produce an absolute velocity vector passing through the zero-velocity point and perpendicular to the link. A point on a floating link (such as B relative to point A) will produce a relative velocity vector. This vector will be perpendicular to the link AB and pass through the reference point (A) on the velocity diagram.
8. One should be able to form a closed triangle (for a 4-bar) that shows the vector equation: **V**B = **V**A + **V**B/A where **V**B = absolute velocity of point B, **V**A = absolute velocity of point A, and **V**B/A is the velocity of point B relative to point A.

2.2 Velocity and Acceleration analysis of mechanisms (Graphical Methods):

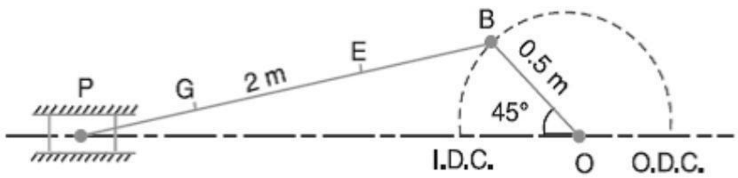
Velocity and acceleration analysis by vector polygons: Relative velocity and accelerations of particles in a common link, relative velocity and accelerations of coincident particles on separate link, Coriolis component of acceleration.

Velocity and acceleration analysis by complex numbers: Analysis of single slider crank mechanism and four bar mechanism by loop closure equations and complex numbers.

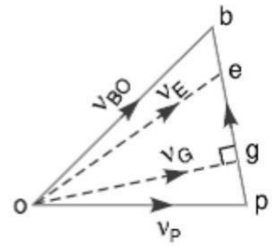
2.3 Coincident points, Coriolis Acceleration:

- ***Coriolis Acceleration:*** $\mathbf{A}^c$ = 2 (dr/dt). In a direction perpendicular to the link. A slider attached to ground experiences only sliding acceleration.

Example: 1 The crank and connecting rod of a steam engine are 0.5m and 2 m long. The crank makes 180 rpm in CW direction. When it turned 45º from IDC determine 1. Velocity of piston. 2.Angular velocity of connecting rod, 3. Velocity of point E on connecting rod, 1.5 m from gudgeon pin 4.Velocities of rubbing at the pins of crank shaft, crank and crosshead when the diameters of the pins are 50mm, 60mm, 30mm respectively



(*a*) Space diagram.



(*b*) Velocity diagram.

vector $oa = v_{AO} = v_A = 1.76$ m/s

v_D = vector $od = 1.6$ m/s

v_{DB} = vector $bd = 1.7$ m/s

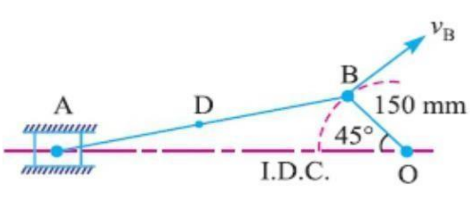
$$\omega_{BD} = \frac{v_{DB}}{BD} = \frac{1.7}{0.046} = 36.96 \text{ rad/s (Clockwise about } B)$$

Example *The crank of a slider crank mechanism rotates clockwise at a constant speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long. Determine : 1. linear velocity and acceleration of the midpoint of the connecting rod, and 2. angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead centre position.*

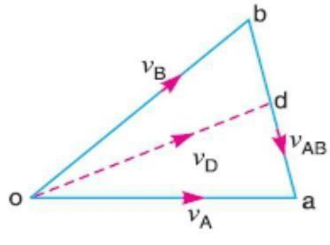
Solution. Given : $N_{BO} = 300$ r.p.m. or $\omega_{BO} = 2\pi \times 300/60 = 31.42$ rad/s; $OB = 150$ mm 0.15 m ; $BA = 600$ mm = 0.6 m

We know that linear velocity of B with respect to O or velocity of B,

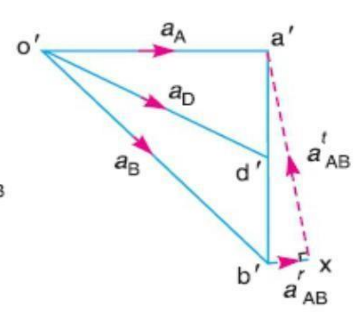
$$v_{BO} = v_B = \omega_{BO} \times OB = 31.42 \times 0.15 = 4.713 \text{ m/s}$$



(*a*) Space diagram.



(*b*) Velocity diagram.



(*c*) Acceleration diagram.

$$\text{vector } ob = v_{BO} = v_B = 4.713 \text{ m/s}$$

$$v_{AB} = \text{vector } ba = 3.4 \text{ m/s}$$

Velocity of A, $v_A = \text{vector } oa = 4 \text{ m/s}$

$$v_D = \text{vector } od = 4.1 \text{ m/s}$$

$$a^r_{BO} = a_B = \frac{v^2_{BO}}{OB} = \frac{(4.713)^2}{0.15} = 148.1 \text{ m/s}^2$$

$$a^r_{AB} = \frac{v^2_{AB}}{BA} = \frac{(3.4)^2}{0.6} = 19.3 \text{ m/s}^2$$

$$\text{vector } o'b' = a^r_{BO} = a_B = 148.1 \text{ m/s}^2$$

$$a_D = \text{vector } o'd' = 117 \text{ m/s}^2$$

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{3.4}{0.6} = 5.67 \text{ rad/s}^2 \text{ (Anticlockwise about } B\text{)}$$

$$a^t_{AB} = 103 \text{ m/s}^2$$

$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{103}{0.6} = 171.67 \text{ rad/s}^2 \text{ (Clockwise about } B\text{)}$$

An engine mechanism is shown in Fig. 8.5. The crank CB = 100 mm and the connecting rod BA = 300 mm with centre of gravity G, 100 mm from B. In the position shown, the crankshaft has a speed of 75 rad/s and an angular acceleration of 1200 rad/s². Find: **1.** *velocity of G and angular velocity of AB, and* **2.** *acceleration of G and angular acceleration of AB.*

Solution. Given : $\omega_{BC} = 75$ rad/s ; $\alpha_{BC} = 1200$ rad/s^2, $CB = 100$ mm $= 0.1$ m; $BA = 300$ mm $= 0.3$ m

We know that velocity of B with respect to C or velocity of B,

$$v_{BC} = v_B = \omega_{BC} \times CB = 75 \times 0.1 = 7.5 \text{ m/s} \qquad \text{...(Perpendicular to } BC\text{)}$$

Since the angular acceleration of the crankshaft, $\alpha_{BC} = 1200$ rad/s^2, therefore tangential component of the acceleration of B with respect to C,

$$a^t_{BC} = \alpha_{BC} \times CB = 1200 \times 0.1 = 120 \text{ m/s}^2$$

Note: When the angular acceleration is not given, then there will be no tangential component of the acceleration.

1. *Velocity of G and angular velocity of AB*

First of all, draw the space diagram, to some suitable scale, as shown in Fig. 8.6 (*a*). Now the velocity diagram, as shown in Fig. 8.6 (*b*), is drawn as discussed below:

1. Draw vector cb perpendicular to CB, to some suitable scale, to represent the velocity of B with respect to C or velocity of B (*i.e.* v_{BC} *or* v_B), such that

$$\text{vector } cb = v_{BC} = v_B = 7.5 \text{ m/s}$$

2. From point b, draw vector ba perpendicular to BA to represent the velocity of A with respect to B *i.e.* v_{AB}, and from point c, draw vector ca parallel to the path of motion of A (which is along AC) to represent the velocity of A *i.e.* v_A. The vectors ba and ca intersect at a.

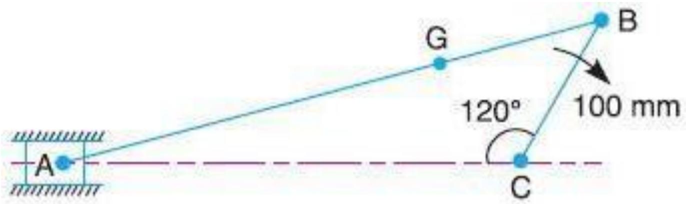
3. Since the point G lies on AB, therefore divide vector ab at g in the same ratio as G divides AB in the space diagram. In other words,

$$ag / ab = AG / AB$$

The vector cg represents the velocity of G.

By measurement, we find that velocity of G,

$$v_G = \text{vector } cg = 6.8 \text{ m/s} \quad \textbf{Ans.}$$

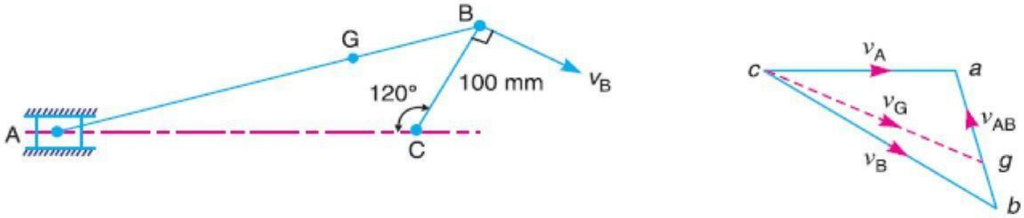


From velocity diagram, we find that velocity of A with respect to B,

$$v_{AB} = \text{vector } ba = 4 \text{ m/s}$$

We know that angular velocity of $A\,B$,

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{4}{0.3} = 13.3 \text{ rad/s (Clockwise)} \quad \textbf{Ans.}$$



2. ***Acceleration of G and angular acceleration of AB***

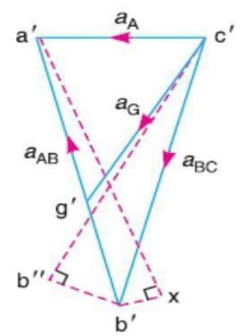
We know that radial component of the acceleration of B with respect to C,

$$* \; a^r_{BC} = \frac{v^2_{BC}}{CB} = \frac{(7.5)^2}{0.1} = 562.5 \text{ m/s}^2$$

and radial component of the acceleration of A with respect to B,

$$a^r_{AB} = \frac{v^2_{AB}}{BA} = \frac{4^2}{0.3} = 53.3 \text{ m/s}^2$$

Now the acceleration diagram, as shown in Fig. 8.6 (c), is drawn as discussed below:



(c) Acceleration diagram.

Fig. 8.6

1. Draw vector $c'\,b''$ parallel to CB, to some suitable scale, to represent the radial component of the acceleration of B with respect to C, *i.e.* a^r_{BC}, such that

$$\text{vector } c'\,b'' = a^r_{BC} = 562.5 \text{ m/s}^2$$

2. From point b'', draw vector $b''\,b'$ perpendicular to vector $c'\,b''$ or CB to represent the tangential component of the acceleration of B with respect to C *i.e.* a^t_{BC}, such that

$$\text{vector } b''b' = a^t_{BC} = 120 \text{ m/s}^2 \qquad \text{... (Given)}$$

3. Join $c'b'$. The vector $c'b'$ represents the total acceleration of B with respect to C *i.e.* a_{BC}.

4. From point b', draw vector $b'\,x$ parallel to $B\,A$ to represent radial component of the acceleration of A with respect to B *i.e.* a^r_{AB} such that

$$\text{vector } b'x = a^r_{AB} = 53.3 \text{ m/s}^2$$

5. From point x, draw vector xa' perpendicular to vector $b'x$ or $B\,A$ to represent tangential component of the acceleration of A with respect to B *i.e.* a^t_{AB}, whose magnitude is not yet known.

6. Now draw vector $c'a'$ parallel to the path of motion of A (which is along $A\,C$) to represent the acceleration of A *i.e.* a_A. The vectors xa' and $c'a'$ intersect at a'. Join $b'\,a'$. The vector $b'\,a'$ represents the acceleration of A with respect to B *i.e.* a_{AB}.

7. In order to find the acceleratio of G, divide vector $a'\,b'$ in g' in the same ratio as G divides $B\,A$ in Fig. 8.6 (a). Join $c'\,g'$. The vector $c'\,g'$ represents the acceleration of G.

By measurement, we find that acceleration of G,

$$a_G = \text{vector } c'\,g' = 414 \text{ m/s}^2 \quad \textbf{Ans.}$$

From acceleration diagram, we find that tangential component of the acceleration of A with respect to B,

$$a^t_{AB} = \text{vector } xa' = 546 \text{ m/s}^2 \qquad \text{...(By measurement)}$$

$\therefore$ Angular acceleration of $A\,B$,

$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{546}{0.3} = 1820 \text{ rad/s}^2 \text{ (Clockwise)} \quad \textbf{Ans.}$$

Example 8.3. *In the mechanism shown in Fig. 8.7, the slider C is moving to the right with a velocity of 1 m/s and an acceleration of 2.5 m/s².*

The dimensions of various links are AB = 3 m inclined at 45° with the vertical and BC = 1.5 m inclined at 45° with the horizontal. Determine: **1.** *the magnitude of vertical and horizontal component of the acceleration of the point B, and* **2.** *the angular acceleration of the links AB and BC.*

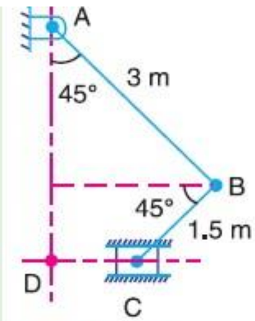


Fig. 8.7

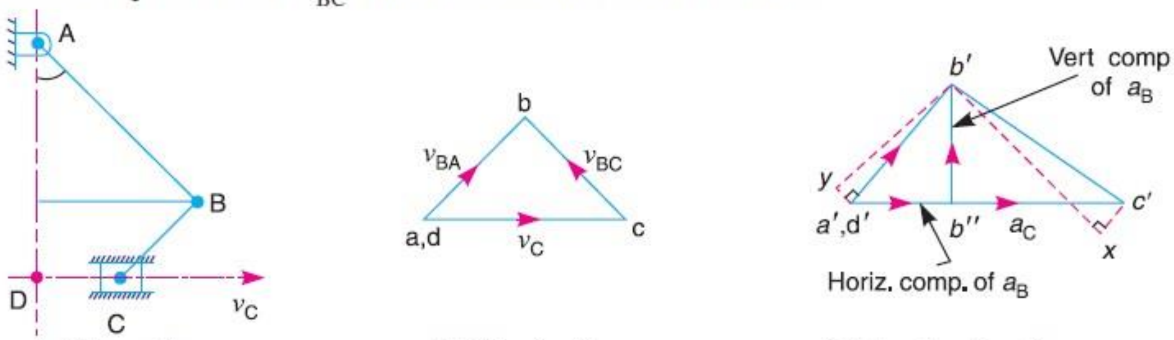
Solution. Given : $v_C = 1$ m/s ; $a_C = 2.5$ m/s^2; $AB = 3$ m ; $BC = 1.5$ m

First of all, draw the space diagram, as shown in Fig. 8.8 (*a*), to some suitable scale. Now the velocity diagram, as shown in Fig. 8.8 (*b*), is drawn as discussed below:

1. Since the points *A* and *D* are fixed points, therefore they lie at one place in the velocity diagram. Draw vector *dc* parallel to *DC*, to some suitable scale, which represents the velocity of slider *C* with respect to *D* or simply velocity of *C*, such that

$$\text{vector } dc = v_{CD} = v_C = 1 \text{ m/s}$$

2. Since point *B* has two motions, one with respect to *A* and the other with respect to *C*, therefore from point *a*, draw vector *ab* perpendicular to *AB* to represent the velocity of *B* with respect to *A*, *i.e.* v_{BA} and from point *c* draw vector *cb* perpendicular to *CB* to represent the velocity of *B* with respect to *C* *i.e.* v_{BC}. The vectors *ab* and *cb* intersect at *b*.



(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

Fig. 8.8

By measurement, we find that velocity of *B* with respect to *A*,

By measurement, we find that velocity of *B* with respect to *A*,

$$v_{BA} = \text{vector } ab = 0.72 \text{ m/s}$$

and velocity of *B* with respect to *C*,

$$v_{BC} = \text{vector } cb = 0.72 \text{ m/s}$$

Example 8.4. *PQRS is a four bar chain with link PS fixed. The lengths of the links are PQ = 62.5 mm ; QR = 175 mm ; RS = 112.5 mm ; and PS = 200 mm. The crank PQ rotates at 10 rad/s clockwise. Draw the velocity and acceleration diagram when angle QPS = 60° and Q and R lie on the same side of PS. Find the angular velocity and angular acceleration of links QR and RS.*

Solution. Given : $\omega_{QP} = 10$ rad/s; $PQ = 62.5$ mm $= 0.0625$ m ; $QR = 175$ mm $= 0.175$ m ; $RS = 112.5$ mm $= 0.1125$ m ; $PS = 200$ mm $= 0.2$ m

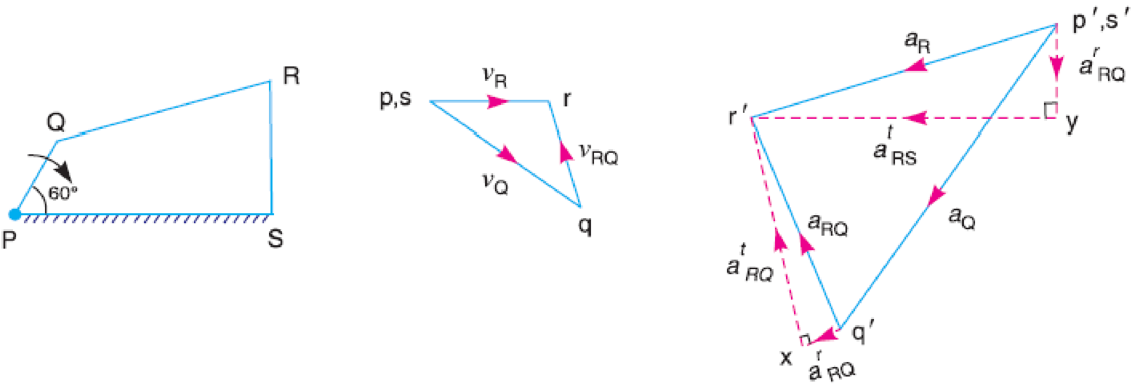
We know that velocity of Q with respect to P or velocity of Q,

$$v_{QP} = v_Q = \omega_{QP} \times PQ = 10 \times 0.0625 = 0.625 \text{ m/s}$$

...(Perpendicular to PQ)

Angular velocity of links QR and RS

First of all, draw the space diagram of a four bar chain, to some suitable scale, as shown in Fig. 8.9 (*a*). Now the velocity diagram as shown in Fig. 8.9 (*b*), is drawn as discussed below:



(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

Fig. 8.9

1. Since P and S are fixed points, therefore these points lie at one place in velocity diagram. Draw vector pq perpendicular to PQ, to some suitable scale, to represent the velocity of Q with respect to P or velocity of Q *i.e.* v_{QP} or v_Q such that

$$\text{vector } pq = v_{QP} = v_Q = 0.625 \text{ m/s}$$

2. From point q, draw vector qr perpendicular to QR to represent the velocity of R with respect to Q (*i.e.* v_{RQ}) and from point s, draw vector sr perpendicular to SR to represent the velocity of R with respect to S or velocity of R (*i.e.* v_{RS} or v_R). The vectors qr and sr intersect at r. By measurement, we find that

$$v_{RQ} = \text{vector } qr = 0.333 \text{ m/s, and } v_{RS} = v_R = \text{vector } sr = 0.426 \text{ m/s}$$

We know that angular velocity of link QR,

$$\omega_{QR} = \frac{v_{RQ}}{RQ} = \frac{0.333}{0.175} = 1.9 \text{ rad/s (Anticlockwise)} \textbf{ Ans.}$$

Angular acceleration of links QR and RS

Since the angular acceleration of the crank PQ is not given, therefore there will be no tangential component of the acceleration of Q with respect to P.

We know that radial component of the acceleration of Q with respect to P (or the acceleration of Q),

$$a_{QP}^r = a_{QP} = a_Q = \frac{v_{QP}^2}{PQ} = \frac{(0.625)^2}{0.0625} = 6.25 \text{ m/s}^2$$

Radial component of the acceleration of R with respect to Q,

$$a_{RQ}^r = \frac{v_{RQ}^2}{QR} = \frac{(0.333)^2}{0.175} = 0.634 \text{ m/s}^2$$

and radial component of the acceleration of R with respect to S (or the acceleration of R),

$$a_{RS}^r = a_{RS} = a_R = \frac{v_{RS}^2}{SR} = \frac{(0.426)^2}{0.1125} = 1.613 \text{ m/s}^2$$

The acceleration diagram, as shown in Fig. 8.9 (*c*) is drawn as follows :

1. Since P and S are fixed points, therefore these points lie at one place in the acceleration diagram. Draw vector $p'q'$ parallel to PQ, to some suitable scale, to represent the radial component of acceleration of Q with respect to P or acceleration of Q *i.e* a_{QP}^r or a_Q such that

$$\text{vector } p'q' = a_{QP}^r = a_Q = 6.25 \text{ m/s}^2$$

2. From point q', draw vector $q'x$ parallel to QR to represent the radial component of acceleration of R with respect to Q *i.e.* a_{RQ}^r such that

$$\text{vector } q'x = a_{RQ}^r = 0.634 \text{ m/s}^2$$

3. From point x, draw vector xr' perpendicular to QR to represent the tangential component of acceleration of R with respect to Q *i.e* a_{RQ}^t whose magnitude is not yet known.

4. Now from point s', draw vector $s'y$ parallel to SR to represent the radial component of the acceleration of R with respect to S *i.e.* a_{RS}^r such that

$$\text{vector } s'y = a_{RS}^r = 1.613 \text{ m/s}^2$$

5. From point y, draw vector yr' perpendicular to SR to represent the tangential component of acceleration of R with respect to S *i.e.* a_{RS}^t.

6. The vectors xr' and yr' intersect at r'. Join $p'r$ and $q'r'$. By measurement, we find that

$$a_{RQ}^t = \text{vector } xr' = 4.1 \text{ m/s}^2 \text{ and } a_{RS}^t = \text{vector } yr' = 5.3 \text{ m/s}^2$$

We know that angular acceleration of link QR,

$$\alpha_{QR} = \frac{a_{RQ}^t}{QR} = \frac{4.1}{0.175} = 23.43 \text{ rad/s}^2 \text{ (Anticlockwise) } \textbf{Ans.}$$

and angular acceleration of link RS,

$$\alpha_{RS} = \frac{a_{RS}^t}{SR} = \frac{5.3}{0.1125} = 47.1 \text{ rad/s}^2 \text{ (Anticlockwise) } \textbf{Ans.}$$

Example 8.6. *In the mechanism, as shown in Fig. 8.12, the crank OA rotates at 20 r.p.m. anticlockwise and gives motion to the sliding blocks B and D. The dimensions of the various links are OA = 300 mm; AB = 1200 mm; BC = 450 mm and CD = 450 mm.*

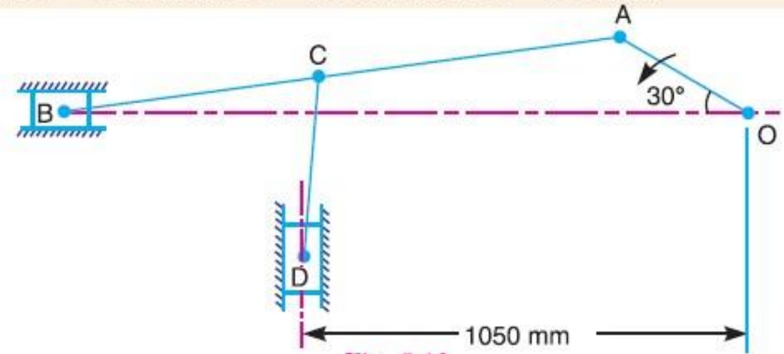


Fig. 8.12

*For the given configuration, determine : **1.** velocities of sliding at B and D, **2.** angular velocity of CD, **3.** linear acceleration of D, and **4.** angular acceleration of CD.*

Solution. Given : $N_{AO} = 20$ r.p.m. or $\omega_{AO} = 2\pi \times 20/60 = 2.1$ rad/s ; $OA = 300$ mm $= 0.3$ m ; $AB = 1200$ mm $= 1.2$ m ; $BC = CD = 450$ mm $= 0.45$ m

U

1. *Velocities of sliding at B and D*

First of all, draw the space diagram, to some suitable scale, as shown in Fig. 8.13 (*a*). Now the velocity diagram, as shown in Fig. 8.13 (*b*), is drawn as discussed below:

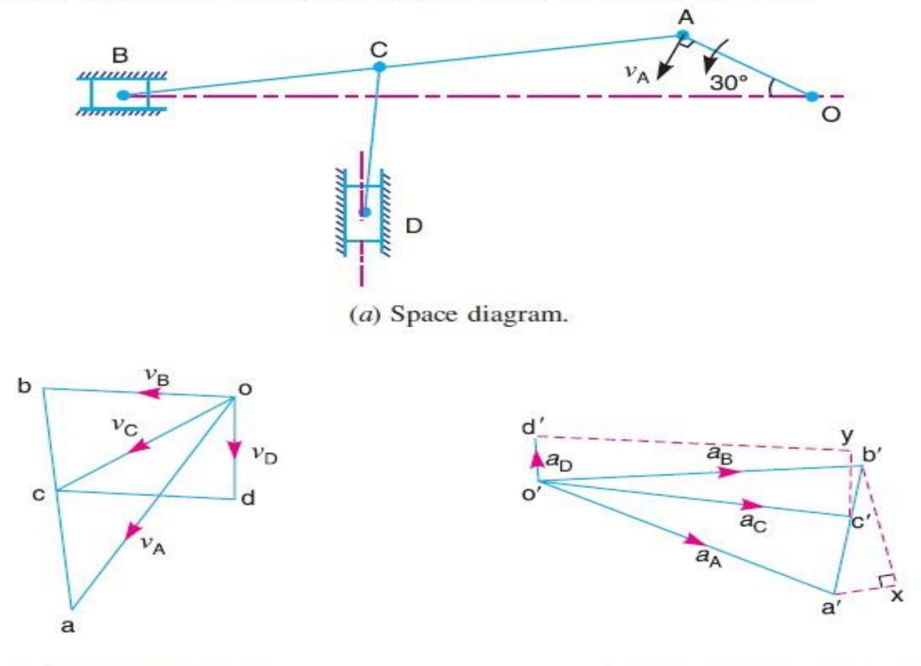


Fig. 8.13

1. Draw vector *oa* perpendicular to *OA*, to some suitable scale, to represent the velocity of *A* with respect to *O* (or simply velocity of *A*), such that

$$\text{vector } oa = v_{AO} = v_A = 0.63 \text{ m/s}$$

2. From point *a*, draw vector *ab* perpendicular to *AB* to represent the velocity of *B* with respect to *A* (*i.e.* v_{BA}) and from point *o* draw vector *ob* parallel to path of motion *B* (which is along *BO*) to represent the velocity of *B* with respect to *O* (or simply velocity of *B*). The vectors *ab* and *ob* intersect at *b*.

3. Divide vector *ab* at *c* in the same ratio as *C* divides *AB* in the space diagram. In other

4. Now from point c, draw vector cd perpendicular to CD to represent the velocity of D with respect to C (*i.e.* v_{DC}) and from point o draw vector od parallel to the path of motion of D (which along the vertical direction) to represent the velocity of D.

By measurement, we find that velocity of sliding at B,

$$v_B = \text{vector } ob = 0.4 \text{ m/s} \quad \textbf{Ans.}$$

and velocity of sliding at D,

$$v_D = \text{vector } od = 0.24 \text{ m/s} \quad \textbf{Ans.}$$

2. Angular velocity of CD

By measurement from velocity diagram, we find that velocity of D with respect to C,

$$v_{DC} = \text{vector } cd = 0.37 \text{ m/s}$$

$\therefore$ Angular velocity of CD,

$$\omega_{CD} = \frac{v_{DC}}{CD} = \frac{0.37}{0.45} = 0.82 \text{ rad/s (Anticlockwise). } \textbf{Ans.}$$

3. Linear acceleration of D

We know that the radial component of the acceleration of A with respect to O or acceleration of A,

$$a^r_{AO} = a_A = \frac{v^2_{AO}}{OA} = \omega^2_{AO} \times OA = (2.1)^2 \times 0.3 = 1.323 \text{ m/s}^2$$

Radial component of the acceleration of B with respect to A,

$$a^r_{BA} = \frac{v^2_{BA}}{AB} = \frac{(0.54)^2}{1.2} = 0.243 \text{ m/s}^2$$

...(By measurement, $v_{BA} = 0.54$ m/s)

Radial component of the acceleration of D with respect to C,

$$a^r_{DC} = \frac{v^2_{DC}}{CD} = \frac{(0.37)^2}{0.45} = 0.304 \text{ m/s}^2$$

Now the acceleration diagram, as shown in Fig. 8.13 (*c*), is drawn as discussed below:

1. Draw vector $o'a'$ parallel to OA, to some suitable scale, to represent the radial component of the acceleration of A with respect to O or simply the acceleration of A, such that

$$\text{vector } o'a' = a^r_{AO} = a_A = 1.323 \text{ m/s}^2$$

2. From point a', draw vector $a'x$ parallel to AB to represent the radial component of the acceleration of B with respect to A, such that

$$\text{vector } a'x = a^r_{BA} = 0.243 \text{ m/s}^2$$

3. From point x, draw vector xb' perpendicular to AB to represent the tangential component of the acceleration of B with respect to A (*i.e.* a^t_{BA}) whose magnitude is not yet known.

4. From point o', draw vector $o'b'$ parallel to the path of motion of B (which is along BO) to represent the acceleration of B (a_B). The vectors xb' *and* $o'b'$ intersect at b'. Join $a'b'$. The vector $a'b'$ represents the acceleration of B with respect to A.

5. Divide vector $a'b'$ at c' in the same ratio as C divides AB in the space diagram. In other words,

$$BC/BA = b'c'/b'a'$$

6. From point c', draw vector $c'y$ parallel to CD to represent the radial component of the acceleration of D with respect to C, such that

4. Angular acceleration of CD

From the acceleration diagram, we find that the tangential component of the acceleration of D with respect to C,

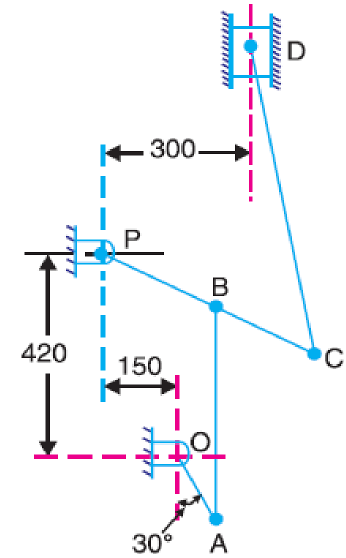
$$a^t_{DC} = \text{vector } yd' = 1.28 \text{ m/s}^2 \qquad \text{...(By measurement)}$$

∴ Angular acceleration of CD,

$$\alpha_{CD} = \frac{a^t_{DC}}{CD} = \frac{1.28}{0.45} = 2.84 \text{ rad/s}^2 \text{ (Clockwise) } \textbf{Ans.}$$

Example 8.7. *Find out the acceleration of the slider D and the angular acceleration of link CD for the engine mechanism shown in Fig. 8.14.*

The crank OA rotates uniformly at 180 r.p.m. in clockwise direction. The various lengths are: OA = 150 mm ; AB = 450 mm; PB = 240 mm ; BC = 210 mm ; CD = 660 mm.



All dimensions in mm.

Fig. 8.14

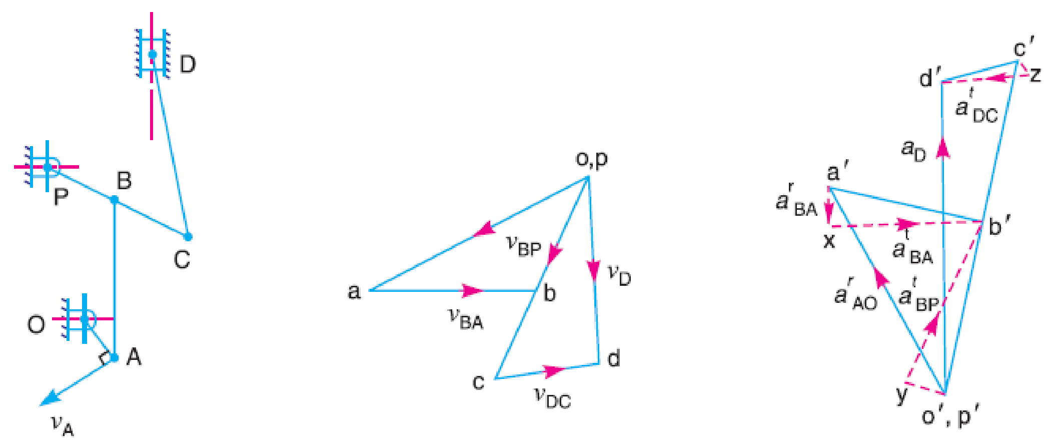
Solution. Given: $N_{AO} = 180$ r.p.m., or $\omega_{AO} = 2\pi \times 180/60 = 18.85$ rad/s ; $OA = 150$ mm $= 0.15$ m ; $AB = 450$ mm $= 0.45$ m ; $PB = 240$ mm $= 0.24$ m ; $CD = 660$ mm $= 0.66$ m

We know that velocity of A with respect to O or velocity of A,

$$v_{AO} = v_A = \omega_{AO} \times OA$$
$$= 18.85 \times 0.15 = 2.83 \text{ m/s}$$

...(Perpendicular to OA)

First of all draw the space diagram, to some suitable scale, as shown in Fig. 8.15 (*a*). Now the velocity diagram, as shown in Fig. 8.15 (*b*), is drawn as discussed below:



(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

Fig. 8.15

1. Since O and P are fixed points, therefore these points lie at one place in the velocity diagram. Draw vector oa perpendicular to OA, to some suitable scale, to represent the velocity of A with respect to O or velocity of A (*i.e.* v_{AO} or v_A), such that

$$\text{vector } oa = v_{AO} = v_A = 2.83 \text{ m/s}$$

2. Since the point B moves with respect to A and also with respect to P, therefore draw vector ab perpendicular to AB to represent the velocity of B with respect to A *i.e.* v_{BA}, and from point p draw vector pb perpendicular to PB to represent the velocity of B with respect to P or velocity of B (*i.e.* v_{BP} or v_B). The vectors ab and pb intersect at b.

3. Since the point C lies on PB produced, therefore divide vector pb at c in the same ratio as C divides PB in the space diagram. In other words, $pb/pc = PB/PC$.

4. From point c, draw vector cd perpendicular to CD to represent the velocity of D with respect to C and from point o draw vector od parallel to the path of motion of the slider D (which is vertical), to represent the velocity of D, *i.e.* v_D.

By measurement, we find that velocity of the slider D,

$$v_D = \text{vector } od = 2.36 \text{ m/s}$$

Velocity of D with respect to C,

$$v_{DC} = \text{vector } cd = 1.2 \text{ m/s}$$

Velocity of B with respect to A,

$$v_{BA} = \text{vector } ab = 1.8 \text{ m/s}$$

and velocity of B with respect to P, $v_{BP} = \text{vector } pb = 1.5$ m/s

Acceleration of the slider D

We know that radial component of the acceleration of A with respect to O or acceleration of A,

$$a^r_{AO} = a_A = \omega^2_{AO} \times AO = (18.85)^2 \times 0.15 = 53.3 \text{ m/s}^2$$

Radial component of the acceleration of B with respect to A,

$$a^r_{BA} = \frac{v^2_{BA}}{AB} = \frac{(1.8)^2}{0.45} = 7.2 \text{ m/s}^2$$

Radial component of the acceleration of B with respect to P,

$$a^r_{BP} = \frac{v^2_{BP}}{PB} = \frac{(1.5)^2}{0.24} = 9.4 \text{ m/s}^2$$

Radial component of the acceleration of D with respect to C,

$$a^r_{DC} = \frac{v^2_{DC}}{CD} = \frac{(1.2)^2}{0.66} = 2.2 \text{ m/s}^2$$

Now the acceleration diagram, as shown in Fig. 8.15 (c), is drawn as discussed below:

1. Since O and P are fixed points, therefore these points lie at one place in the acceleration diagram. Draw vector $o'a'$ parallel to OA, to some suitable scale, to represent the radial component of the acceleration of A with respect to O or the acceleration of A (*i.e.* a^r_{AO} or a_A), such that

$$\text{vector } o'a' = a^r_{AO} = a_A = 53.3 \text{ m/s}^2$$

2. From point a', draw vector $a'x$ parallel to AB to represent the radial component of the acceleration of B with respect to A (*i.e.* a^r_{BA}), such that

$$\text{vector } a'x = a^r_{BA} = 7.2 \text{ m/s}^2$$

3. From point x, draw vector xb' perpendicular to the vector $a'x$ or AB to represent the tangential component of the acceleration of B with respect to A *i.e.* a^t_{BA} whose magnitude is yet unknown.

4. Now from point p', draw vector $p'y$ parallel to PB to represent the radial component of the acceleration of B with respect to P (*i.e.* a^r_{BP}), such that

$$\text{vector } p'y = a^r_{BP} = 9.4 \text{ m/s}^2$$

5. From point y, draw vector yb' perpendicular to vector $b'y$ or PB to represent the tangential component of the acceleration of B, *i.e.* a^t_{BP}. The vectors xb' and yb' intersect at b'. Join $p'b'$. The vector $p'b'$ represents the acceleration of B, *i.e.* a_B.