Problems on Wide Sense Stationary (WSS):

1. Show that the random process $(t) = A\ \mathbf{co}(mt + \theta)$ is WSS, where $A\ and\ m$ are constants and θ is uniformly distributed on the interval $(0, 2\pi)$

Solution:

Given, $(t) = A\cos(\omega t + \theta)$

θ is uniformly distributed on the interval $(0, 2\pi)$

$$f(\theta) = \frac{1}{b-a}, a < \theta < b$$

$$f(\theta) = \frac{1}{2\pi}, 0 < \theta < 2\pi$$

To Prove (t) is WSS.

(i)$Mean = E[X(t)] = constant$

(ii) $Auto\ correlation\ R(r) = E[X(t)X(t+r)]$ depends on r

(i) $[(t)] = \int_{-\infty}^{\infty} X(t) f(\theta) d\theta$

$$= \int_0^{2\pi} A\cos(\omega t + \theta) \frac{1}{2\pi} d\theta$$

$$= \frac{A}{2\pi} [\sin(\omega t + \theta)]_0^{2\pi}$$

$$= \frac{A}{2\pi} [\sin(\omega t + 2\pi) - \sin(\omega t + 0)]$$

$$= \frac{A}{2\pi} [\sin \omega t - \sin \omega t] = 0$$

$[X(t)] = 0\ is\ constant.$

(ii)$R_{XX}(r) = E[X(t)X(t+r)]$

$$= [A\cos(\omega t + \theta)\, A\cos(\omega(t+r) + \theta)]$$

$$= [A^2][\cos(\omega t + \theta)\cos(\omega(t+r) + \theta)]$$

$$= A^2 \frac{1}{2}[\cos(\omega t + \theta + \omega t + \omega r + \theta) \cos(\omega t + \theta - \omega t - \omega r - \theta)]$$

$$\cos(-\theta) = \cos\theta$$

$$\cos\ A \cos\ B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$$

$$= A^2 \frac{1}{-}[\cos(2\omega t + 2\theta + \omega r)\cos(-\omega r)]2$$

$$= \frac{A^2}{2}[\cos(2\omega t + 2\theta + \omega r)\cos(\omega r)]$$

$$= \frac{A^2}{2}[\cos\ \omega r + \int_0^{2\pi}\cos(2\omega t + 2\theta + \omega r)\frac{1}{2\pi}d\theta]$$

$$= \frac{A^2}{2}\cos\ \omega r + \frac{A^2}{4\pi}\left[\frac{\sin\ (2\omega t + 2\theta + \omega c)}{2}\right]_0^{2\pi}$$

$$= \frac{A^2}{2}\cos\ \omega r + \frac{A^2}{8\pi}[\sin(2\omega t + \omega r + 4\pi) - \sin(2\omega t + \omega r)]$$

$$= \frac{A^2}{2}\cos\ \omega r + \frac{A^2}{8\pi}[\sin(2\omega t + \omega r) - \sin(2\omega t + \omega r)]$$

$$= \frac{A^2}{2}\cos\ \omega r + \frac{A^2}{8\pi}[0]$$

$$R_{XX}(r) = \frac{A^2}{2}\cos\ \omega r$$

Hence (t) is WSS process.

2.Show that the random process $X(t) = A\cos\lambda t + B\ sin\ \lambda t$ where λ is a constant, A and B are random variables, is WSS if (i) $E[A] = E[B] = 0$ (ii) $[A^2] = [B^2]$ and (iii) $E[AB] = 0$

Solution:

Given, $(t) = A\cos\ \lambda t\ + B\ sin\ \lambda t$

$[A] = [B] = 0\ ,\ E[A^2] = E[B^2]\ ,\ E[AB] = 0$

To Prove (t) is WSS.

(i)$Mean = E[X(t)] = constant$

(ii) $Auto\ correlation\ R(r) = E[X(t)X(t + r)]$ depends on r

(i)$[X(t)] = [A\cos\ \lambda t + B\ sin\ \lambda t\ ]$

$= [A]\cos\ \lambda t\ + [B]\ sin\ \lambda t$

$= 0 * \cos\ \lambda t + 0 *\ sin\ \lambda t$

$[X(t)] = 0\ is\ constant.$

(ii)$R_{XX}(r) = E[X(t)X(t + r)]$

$= [(A\cos\ \lambda t + B\ sin\ \lambda t)(A\cos\ \lambda(t + r)\ + B\ sin\ \lambda(t + r))]$

$= [A^2\cos\ \lambda t\ \cos\ (t + r) + AB\ \cos\ \lambda t\ sin\ \lambda(t + r)$

$+ AB\ sin\ \lambda t\ \cos\ (t + r) + B^2 sin\ \lambda t\ sin\ (t + r)]$

$= [A^2\cos\ \lambda t\ \cos\ (t + r)\ ] + E[AB\ \cos\ \lambda t\ sin\ \lambda(t + r)] +$

$[AB\ sin\ \lambda t\ \cos\ (t + r)\ ] + [B^2 sin\ \lambda t\ sin\ \lambda(t + r)]$

$= [A^2\cos\ \lambda t\ \cos\ (t + r)\ ] + E[B^2 sin\ \lambda t\ sin\ \lambda(t + r)]$

$= [A^2]\cos\ \lambda t\ \cos\ (t + r)\ + E[B^2] sin\ \lambda t\ sin\ \lambda(t + r)$

$$= k\cos\lambda t\ \cos(t+r) + k\, sin\, \lambda t\, sin\,(t+r)$$

$$= [\cos\lambda t\ \cos(t+r) + \ sin\, \lambda t\, sin\, \lambda(t+r)]$$

$$= [\cos(\lambda t - \lambda t - \lambda r)]$$

$\cos(-\theta) = \cos\theta$ $\cos A\cos B + \sin A\sin B = \cos(A-B)$

$$= [\cos(-\lambda r)]$$

$$= [\cos(\lambda r)]$$

Hence (t) is WSS process.

3. Given a random variable y with characteristic function $(m) =$ $[e^{imy}]$ and a random process $(t) = \cos(\lambda t + y)$. Show that (t) isstationary in the wide sense if $(1) = \varphi(2) = 0$

Solution:

Given, $(t) = \cos(\lambda t + y)$

$$(\omega) = [e^{i\omega y}] = E[\cos\omega y + i\sin\omega y]$$

$$= [\cos\omega y] + i\ \mathrm{E}[\sin\omega y]$$

Given, $(1) = 0$

$$\Rightarrow 0 = [\cos\ y] + i\ \mathrm{E}[\sin\ y]$$

$$[\cos y] = 0;\ \mathrm{E}[\sin y] = 0$$

Given, $(2) = 0$

$$\Rightarrow 0 = [\cos\ 2y] + i\,\mathrm{E}[\sin 2y]$$

$$[\cos\ 2y] = 0;\ \mathrm{E}[\sin\ 2y] = 0$$

To Prove (t) is WSS.

(i)$Mean = E[X(t)] = constant$

(ii) $Auto\ correlation\ R(r) = E[X(t)X(t + r)]$ depends on r

(i)$[X(t)] = [\cos(\lambda t + y)\ ]$

$$= [\cos \lambda t\ \cos y\ \ - \sin \lambda t\ sin\ y]$$

$\mathrm{Cos}(A + B) = \cos\ A \cos\ B - \sin A \sin B$
$\cos\ A \cos\ B = \frac{1}{2}[\cos(A + B) + \cos(A - B)]$

$$= \cos \lambda t\ \ [\cos\ \ ] - \sin \lambda t\, E[\sin y]$$

$$= \cos \lambda t * 0 - \sin \lambda t * 0$$

$[X(t)] = 0\ is\ constant.$

(ii)$R_{XX}(r) = E[X(t)X(t + r)]$

$$= [\cos\ (\lambda t + y)\ \cos\ ((t + r) + y)\ ]$$

$$= [\cos\ (\lambda t + y)\ \cos\ (\lambda t + \lambda r + y)\ ]$$

$$= \frac{1}{}[\cos(\lambda t + y + \lambda t + \lambda r + y) + \cos(\lambda t + y - \lambda t - \lambda r - y)]2$$

$$= \frac{1}{-}[\cos(2\lambda t + 2y + \lambda r) + \cos(-\lambda r)]2$$

$$= \frac{1}{2}\cos\lambda r + \frac{1}{2}[\cos(2\lambda t + \lambda r)\cos 2y - \sin(2\lambda t + \lambda r)\sin 2y]$$

$$= \frac{1}{2}\cos\lambda r + \frac{1}{2}\cos(2\lambda t + \lambda r)\,E[\cos 2y] - \frac{1}{2}\sin(2\lambda t + \lambda r)\,E[\sin 2y]$$

$$= \frac{1}{2}\cos\lambda r + \frac{1}{2}(0)$$

$$R(r) = \frac{1}{2}\cos\lambda r$$

Hence $\{(t)\}$ is WSS process.

4. Show that the process $X(t) = Y\cos mt + Z\sin mt$ where Y and Z independent RV's which follows $N(0, \sigma^2)$ and m is a constant, is wide sense stationary.

Solution:

Given $(t) = Y\cos\omega t + Z\sin\omega t$, where Y and Z are independent(i)$(Y) = E(Z) = 0$

(ii) $(YZ) = 0$

(iii) $(Y^2) = (Z^2) = \sigma^2$

To prove $\{(t)\}$ is a WSS process,

(1) $E[X(t)]$ is a constant

(2) $R(t_1, t_2)$ is a function of r

(1) $[(t)] = E[Y\cos \omega t + Z\sin \omega t]$

$= (Y)\cos \omega t + E(Z)\sin \omega t$

$= 0 + 0 = 0$ From (i)

$\therefore$ $[(t)]$ is a constant

(2) $R(t_1, t_2) = E[X(t_1)X(t_2)]$

$= [(Y\cos \omega t_1 + Z\sin \omega t_1)(Y\cos \omega t_2 + Z\sin \omega t_2)]$

$= [Y^2 \cos \omega t_1 \cos \omega t_2 + YZ \sin \omega t_2 \cos \omega t_1 + ZY\sin \omega t_1 \cos \omega t_2$

$+Z^2\sin \omega t_1 \sin \omega t_2]$

$= (Y^2)\cos \omega t_1 \cos \omega t_2 + E(YZ)\sin \omega t_2 \cos \omega t_1 + E(YZ)\sin \omega t_1 \cos \omega t_2$

$+(Z^2)\sin \omega t_1 \sin \omega t_2$

$= \sigma^2\cos \omega t_1 \cos \omega t_2 + 0 + 0 +$

$\sigma^2\sin \omega t_1 \sin \omega t_2$ From (ii) & (iii)

$= \sigma^2[\cos \omega t_1 \cos \omega t_2 + \sin \omega t_1 \sin \omega t_2]$

$= \sigma^2\cos(\omega t_1 - \omega t_2)$

$= \sigma^2\cos[\omega(t_1 - t_2)]$

$R_{XX}(t_1, t_2) = \sigma^2\cos \omega r$

$R(t_1, t_2)$ is a function of r.

Since the conditions (1) and (2) for WSS are satisfied.

Hence, $\{(t)\}$ is a WSS Process.

5. If $(t) = Y\cos t + Z\sin t$, where Y and Z are independent binary random variables each of which assumes the values -1 and $+2$ with probabilities $\frac{2}{3}$ and $\frac{1}{3}$ respectively. Prove that $\{(t)\}$ is WSS.

Solution:

Given $(t) = Y\cos t + Z\sin t$, where Y and Z are independent binary random variables.

The probability distribution of Y and Z are given by

y	-1	2
P(y)	2/3	1/3

z	-1	2
P(z)	2/3	1/3

$$(Y) = \sum (y) = (-1)\left(\frac{2}{3}\right) + 2\left(\frac{1}{3}\right)$$

$$(Y) = 0$$

$$(Y^2) = \sum y^2 (y) = 1\left(\frac{2}{3}\right) + 4\left(\frac{1}{3}\right)$$

$$= \frac{2}{3} + \frac{4}{3}$$

$$= \frac{6}{3} = 2$$

Similarly, $(Z) = 0, (Z^2) = 2.$

Since, Y and Z are independent, $(YZ) = (Y)E(Z) = 0$

To prove $\{(t)\}$ is a WSS process

(1) $E[X(t)]$ is a constant

(2) $R(t_1, t_2)$ is a function of n of r

(1) $[(\text{t})] = E[Y\cos t + Z\sin t]$

$$= (Y)\cos t + E(Z)\sin t = 0$$

$\therefore$ $[(t)]$ is a constant.

(2) $R(t_1, t_2) = E[X(t_1)X(t_2)]$

$$= [(Y\cos t_1 + Z\sin t_1)(Y\cos t_2 + Z\sin t_2)]$$

$$= [Y^2\cos t_1\cos t_2 + YZ\sin t_2\cos t_1 + YZ\sin t_1\cos t_2 + Z^2\sin t_1\sin t_2]$$

$$= (Y^2)\cos t_1\cos t_2 + E(YZ)\sin t_2\cos t_1 + E(YZ)\sin t_1\cos t_2$$

$$+(Z^2)\sin t_1\sin t_2$$

$$= 2\cos t_1\cos t_2 + 2\sin t_1\sin t_2$$

$$= 2\cos(t_1 - t_2) = 2\cos r$$

$R(t_1, t_2)$ is a function of r.

Since the conditions (1) and (2) for WSS are satisfied.

Hence, $\{(t)\}$ is a WSS Process.

6. Consider the process $(t) = \sum^{n}{}_{i=1}(A(A_i\cos p_i t + B_i\sin p_i t)$ where A_i and B_i are uncorrelated R.v's with mean' 0 ' & variance σ_i^2. Prove that $\{(t)\}$ is a WSS process.

Solution:

Given $(t) = \sum^{n}{}_{i=1}(A_i\cos p_i t + B_i\sin p_i t)$

A_i and B_i are Rv's

Given Means of A_i and $B_i = 0 \Rightarrow [A_i] = 0$ & $[B_i] = 0$ and

$\text{Var}[A_i] = \text{Var}[B_i] = \sigma_i^2$

$\Rightarrow [A^2_i] = [B^2_i] = \sigma^2_i$

Also A_i and B_i are uncorrelated $\therefore [A_i B_j] = 0$ for all i, j

$\therefore [A_i A_j] = [B_i B_j] = 0$ for $i \neq j$

To prove $\{(t)\}$ is a WSS process

(1) $E[X(t)]$ is a constant

(2) $R(t_1, t_2)$ is a function of r.

(1) $[(t)] = E[\sum^{n}{}_{i=1}(A_i\cos p_i t + B_i\sin p_i t)]$

$$= \sum_{i=1}^{n}[(A_i)\cos p_i t + E(B_i)\sin p_i t] = 0$$

$\therefore [(t)]$ is a constant.

2) The ACF of $\{(t)\}$ is given by $(r) = E[X(t_1)X(t_2)]$

$= [\sum_{i=1}^{n} (A_i\cos p_i t_1 + B_i\sin p_i t_1)\sum_{j=1}^{n} (A_j\cos p_j t_2 + B_j\sin p_j t_2)$

$= [\sum_{i=1}^{n}\sum_{j=1}^{n} (A_i\cos p_i t_1 + B_i\sin p_i t_1)(A_j\cos p_j t_2 + B_j\sin p_j t_2)]$

$=E[\sum_{i=1}^{n}\sum_{j=1}^{n} [[A_iA_j\cos p_i t_1\cos p_j t_2 + A_iB_j\cos p_i t_1\sin p_j t_2 +$

$A_jB_i\sin p_i t_1\cos p, t_2 + B_iB_j\sin p_i t_1\sin p_j t_2)]$

$=\sum_{i=1}^{n}\sum_{j=1}^{n} [[(A_iA\ )\cos p_i t_1 \cos p_j t_2 + E(A_iB_j) \cos p_i t_1 \sin p_j t_2 +$

$(A_jB_i)\sin p_i t_1\cos p, t_2 + E(B_iB_j)\sin p_i t_1\sin p_j t_2)]$

$=\sum_{i=1}^{n}\sum_{j=1}^{n} [(A_iA\ )\cos p_i t_1 \cos p_j t_2 + E(B_iB_j)\sin p_i t_1\sin p_j t_2)]$

$=\sum_{i=1}^{n} [(A\ )^2\cos p_i t_1 \cos p_j t_2 + E(B_i)^2\sin p_i t_1\sin p_j t_2)]$

$= \sum_{=1}^{n} \sigma_i^2(\cos p_i t_1\cos p_i t_2 + \sin p_i t_1\sin p_i t_2) = \sum_{i=1}^{n} \sigma^2\cos(p_i t_1 - p_i t_2)$

$(r) = \sum_{i=1}^{n} \sigma^2\cos p_i r$

$R_{XX}(t_1, t_2)$ is a function of r.

Since the conditions (1) and (2) for WSS are satisfied.

Hence, $\{(t)\}$ is a WSS Process.

7. Let $(t) = B\text{si}(100t + \theta)$, where B and θ are independent RV' s such that θ is uniform distributed over $(-\pi, \pi)$ and B has mean '0' and variance

‘1’. Find mean and auto correlation function of {X(t)}

Solution:

Given $(t) = B\sin(100t + \theta)$, where B and θ are independent RV' s.

θ is uniform distributed over $(-\pi, \pi)$.

$$(\theta) = \frac{1}{2\pi}, -\pi < \theta < \pi$$

Mean of $B = 0 \Rightarrow [B] = 0$

variance of B= $1 \Rightarrow E[B^2] = 1$

The mean of (t) is given by

$$[X(t)] = [B \sin(100t + \theta)]$$

$$= [B][\sin(100t + \theta)$$

$$= 0 \times [\sin(100t + \theta)]$$

$$[X(t)] = 0$$

Mean of $(t) = 0$

The ACF of $\{(t)\}$ is given by

$$R_{XX}(t_1, t_2) = E[X(t_1)X(t_2)]$$

$$= [B\sin(100t_1 + \theta)B\sin(100t_2 + \theta)]$$

$$= [B^2\sin(100t_1 + \theta)\sin(100t_2 + \theta)]$$

$$= [B^2][\sin(100t_1 + \theta)\sin(100t_2 + \theta)]$$

$$= 1 \times \frac{1}{2}E[\cos(100t_1 + \theta - 100t_2 - \theta) - \cos(100t_1 + \theta + 100t_2 + \theta)]$$

$= \frac{1}{2}[E[\cos(100t_1 - 100t_2) - \cos(100t_1 + 100t_2 + 2\theta)]]$

$= \frac{1}{2}[E(\cos 100r - \cos(100t_1 + 100t_2 + 2\theta))]$

$= \frac{1}{2}\cos 100r - \frac{1}{2}E[\cos(100t_1 + 100t_2 + 2\theta)]$

$= \frac{1}{2}\cos 100r - \frac{1}{2}\int_{-\pi}^{\pi} \cos(100t_1 + 100t_2 + 2\theta) f(\theta)\, d\theta$

$= \frac{1}{2}\cos 100r - \frac{1}{2}\int_{-\pi}^{\pi} \cos(100t_1 + 100t_2 + 2\theta) \frac{1}{2\pi}\, d\theta$

$= \frac{1}{2}\cos 100r - \frac{1}{4\pi}\left[\frac{\sin(100t_1+100t_2+2\theta)}{2}\right]_{-\pi}^{\pi}$

$= \frac{1}{2}\cos 100r - \frac{1}{8\pi}[\sin(100t_1 + 100t_2 + 2\pi) - \sin(100t_1 + 100t_2 + 2\pi)]$

$= \frac{1}{2}\cos 100r - \frac{1}{8\pi}(0) \quad = \frac{1}{2}\cos 100r$

Auto Correlation function of X(t) = $\frac{1}{2}\cos 100r$

8. Let $(t) = A\cos \lambda t + B\sin \lambda t$, where λ is a constant and $A\&B$ are independent R 's with mean $'0'$ & variance 1. Prove that $\{(t)\}$ is covariance stationary.

Solution:

Given $(t) = A\cos \lambda t + B\sin \lambda t$, where $A\&B$ are RV's and λ is a constant.Given $(A) = (B) = 0$.

Also given A and B are independent RV's.

$\therefore [AB] = [A]E[B] = 0$

Also given $\text{Var}(A) = \text{Var}(B) = 1$

$\Rightarrow [A^2] = [B^2] = 1$

To prove $\{(t)\}$ is a covariance stationary process

(1) $E[X(t)]$ is a constant

(2) $C(t_1, t_2)$ is a function of r

(1) E[X(t)] $= E[A\cos\lambda t + B\sin\lambda t]$

$= [A]\cos\lambda t + E[B]\sin\lambda t$

$[X(t)] = 0$

$\therefore [(t)]$ is a constant.

(2) $C_{XX}(t_1, t_2) = E[X(t_1)X(t_2)] - E[X(t_1)]E[X(t_2)]$

= $[(A\cos\lambda t_1 + B\sin\lambda t_1)(A\cos\lambda t_2 + B\sin\lambda t_2)] - 0 \times 0$

= $[A^2\cos\lambda t_1\cos\lambda t_2 + AB\cos\lambda t_1\sin\lambda t_2 +$

$AB\sin\lambda t_1\cos\lambda t_2 + B^2\sin\lambda t_1\sin\lambda t_2]$

= $[A^2]\cos\lambda t_1\cos\lambda t_2 + E[AB]\cos\lambda t_1\sin\lambda t_2 + E[AB]\sin\lambda t_1\cos\lambda t_2 +$

$[B^2]\sin\lambda t_1\sin\lambda t_2$

$=\cos\lambda t_1\cos\lambda t_2 + 0 + 0 + \sin\lambda t_1\sin\lambda t_2$

$= \cos(t_1 - t_2)$

$$C(t_1, t_2) = \cos \lambda r$$

$\therefore C(t_1, t_2)$ is a function of r

Since the conditions (1) and (2) for Covariance Stationary Process are {X(t)} is a covariance stationary process.

Problems under SSS process:

For a SSS process, $[X(t)]$ $\boldsymbol{is\ a\ constant\ for\ every\ n}$.

1. Verify whether the sine wave $(t) = Y \cos \omega t$, where Y is a random variable uniformly distributed over $(0, 1)$ is a SSS process or not.

Solution:

Given, $(t) = Y \cos \omega t$, where Y is a random variable uniformly distributed over (0,1)

$$\therefore (y) = 1 ; 0 < y < 1$$

For a SSS process, $[X(t)]$ $is\ a\ constant\ for\ every\ n.$

$$E[Y] = \int_0^1 y\, f_Y(y)\, dy$$

$$= \int_0^1 y\, dy = \left[\frac{y^2}{2}\right]_0^1 = \frac{1}{2}$$

$$[X(t)] = [Y \cos \omega t]$$

$$= [Y] \cos \omega t$$

$= \frac{1}{2} \cos \omega t$

$[(t)]$ is not a constant.

Hence $\{(t)\}$ is not a SSS process.

2. Consider random process X((t) defined by $X(t) = Ucost + Vsint$, where U and V are independent random variables each of which assumes the values -2 and 1 with probabilities 1/3and 2/3 respectively. Show that {X(t)} is a wide sense stationary process and not a strict sense stationary process(SSS)

Solution:

Given $(t) = Ucost + Vsint$ where U and V are R.V'S with the following probability distributions

u	-2	1
P(u)	1/3	2/3

v	-2	1
P(v)	1/3	2/3

$$(U) = \sum up(u) = (-2 \times \frac{1}{3}) + (1 \times \frac{2}{3}) = -\frac{2}{3} + \frac{2}{3} = 0$$

$$(U^2) = \sum u^2 p(u) = (4 \times \frac{1}{3}) + (1 \times \frac{2}{3}) = \frac{4}{3} + \frac{2}{3} = 2$$

$$(U^3) = \sum u^3 (u) = (-8 \times \frac{1}{3}) + (1 \times \frac{2}{3}) = -\frac{8}{3} + \frac{2}{3} = -2$$

Similarly $(V) = 0, (V^2) = 2, E(V^3) = -2$

Since U and V are independent R.V'S , follow that

$$(UV) = (U)E(V) = 0 \times 0 = 0$$

$$(U^2V) = (U^2)E(V) = 2 \times 0 = 0$$

$$(UV^2) = (U)E(V^2) = 0 \times 2 = 0$$

To Prove (t) is WSS.

(i)$Mean = E[X(t)] = constant$

(ii) $Auto\ correlation\ R(r) = E[X(t_1)X(t_2)]$ depends on r

(i)$[X(t)] = [U \cos t + V\ sint\ ]$

$= [U] \cos t + [V] \sin\ t$

$= 0 + 0 = 0$

$E[X(t)]\ is\ a\ constant.$

(ii)$R(t_1, t_2) = E[X(t_1)X(t_2)]$

$= [(U \cos t_1 + V\ sin\ t_1)(U \cos\ t_2 + V\ sint_2)]$

$= [U^2 \cos\ t_1 \cos\ t_2 + UV\ \cos\ t_1\ sin\ t_2 + UV\ sin\ t_1 \cos\ t_2 +$

$V^2 sin\ t_1\ sin\ t_2]$

$= [U^2 \cos\ t_1 \cos\ t_2\ ] + E[UV\ \cos t_1\ sin\ t_2] + E[UV\ sin\ t_1\ \cos\ t_2\ ] +$

$[V^2 sin\ t_1\ sin\ t_2]$

$= [U^2 \cos\ t_1\ \cos\ t_2\ ] + E[V^2 sin\ t_1\ sin\ t_2]$

$= [U^2] \cos\ t_1\ \cos t_2 + [V^2] sin\ t_1\ sin\ t_2$

$= 2\cos\ t_1 \cos\ t_2 + 2\, sin\, t_1\, sin\, t_2$

$= 2[\cos\ t_1\ \cos\ \ t_2 +\ sin\, t_1\, sin\ \ t_2]$

$= 2[\cos(\ t_1 -\ t_2)]$

$\cos(-\theta) = \cos\theta$

$\cos A \cos\ B + \sin A \sin B\ = \cos(A - B)$

$= 2[\cos\ r]$

Since the conditions (1) and (2) for WSS are satisfied , $X(t)$ is WSS process.

To check {X(t)} is Strict Sense Stationary

$[X^3(t)] = [(U\cos t + V\ sint)^3]$

$= (U^3cos^3t\ +\ 3U^2Vcos^2tsint\ +\ 3UV^2costsin^2t\ +\ V^3sin^3t)$

$= (U^3)os^3t\ +\ 3E(U^2V)cos^2tsint\ +\ 3E(UV^2)costsin^2t\ +\ E(V^3)sin^3t$

$= -2cos^3t + 0 + 0 - 2sin^3t = -2(cos^3t + sin^3t)$

Which depends on t.

Hence {X(t)} is not a strict sense stationary process.

Cross Correlation Function:

Let {X(t)} and {Y(t)}be two random processes. Then cross correlation function of X(t) and Y(t) is $R_{XY}(t_1, t_2\ ) = E[X(t_1)Y(t_2)]$