5.4 CONVOLUTION SUM

The convolution sum provides a concise, mathematical way to express the output of an LTI system based on an arbitrary discrete-time input signal and the system's response. The convolution sum is expressed as

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

1. Convolution is commutative

$$x[n] * h[n] = h[n] * x[n]$$

2. Convolution is Distributive

$$x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n]$$

3. System connected in cascade

$$y[n] = h_1[n] * [\, h_2[n] * x[n] \,] = [\, h_1[n] * h_2[n] \,] * x[n]$$

4. System connected in parallel\

$$y[n] = h_1[n] * x[n] + h_2[n] * x[n] \,] = [\, h_1[n] + h_2[n] \,] * x[n]$$

LTI Systems are said to be stable if ,

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty$$

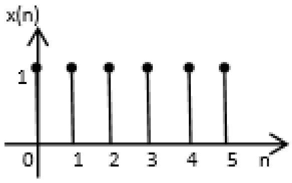
LTI system are causal if,

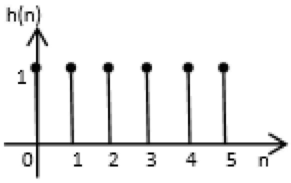
$$h(n) = 0\,, n < 0.$$

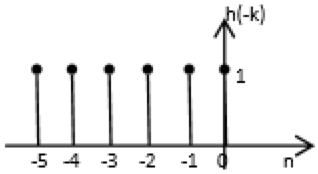
EXAMPLE 1: Convolve the following discrete time signals using graphical convolution $x(n) = h(n) = u(n)$.

Solution:

$$x(n) = u(n) = 1; n \geq 0$$
$$h(n) = u(n) = 1; n \geq 0$$

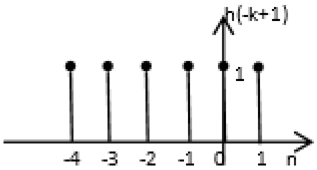




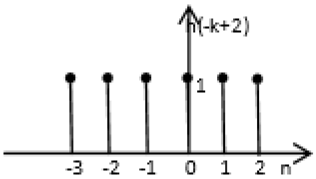


$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

$$when\ n = 0$$
$$y(0) = (1)(1) = 1$$
$$when\ n = 1$$
$$y\,(1) = (1)(1) + (1)(1) = 2$$



$$when\ n = 2$$
$$y\,(2) = (1)(1) + (1)(1) + (1)(1) = 3$$



$$\therefore y(n) = \{1,2,3,4,5,\ldots\}$$

Example 2: Compute linear convolution $x(n) = \{2,2,0,1,1\}$ $h(n) = \{1,2,3,4\}$.

Solution:

	2	2	0	1	1
1	2	2	0	1	1
2	4	4	0	2	2
3	6	6	0	3	3
4	8	8	0	4	4

$$y(n) = x(n) * h(n) = \{2,6,10,15,11,5,7,4\}$$