1.1 INTRODUCTION

SIGNAL

Signal is one that carries information and is defined as a physical quantity that varies with one or more independent variable.

Example: Music, speech

The signal may depend on one or more independent variables. If a signal depends on only one variable, then it is known as one dimensional signal. Example: AC power signal, speech signal, ECG etc.

When a signal is represented as a function of two or more variable, it is said to be multidimensional signal Example: An image represented as F(x,y). Here x & y represents the horizontal and vertical co-ordinates. The intensity of the image varies at each co-ordinate.

SIGNAL MODELING

The representation of a signal by mathematical expression is known as signal modeling.

ANALOG SIGNAL

A signal that is defined for every instants of time is known as analog signal. Analog signals are continuous in amplitude and continuous in time. It is denoted by x(t). It is also called as Continuous time signal. Example for Continuous time signal is shown in Figure 1.1.1

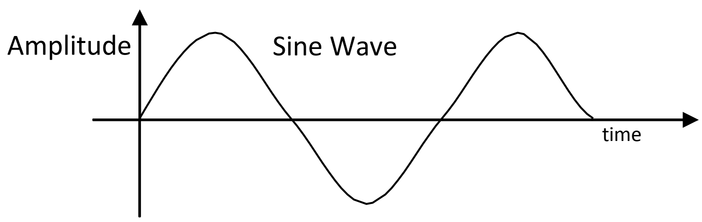


Figure 1.1.1 Continuous time signal

[https://drive.google.com/file/d/1baseARsD-geFLoR-QrEbV5hVsYKKLzGM/view]

DIGITAL SIGNAL

The signals that are discrete in time and quantized in amplitude is called digital signal (Figure 1.1.2)

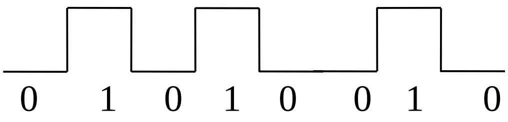


Figure 1.1.2 Digital Signal

[https://drive.google.com/file/d/1baseARsD-geFLoR-QrEbV5hVsYKKLzGM/view]

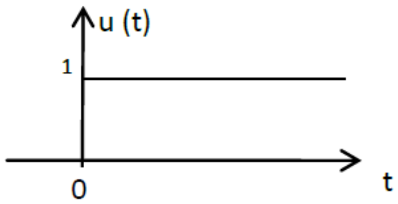
BASIC (ELEMENTARY OR STANDARD) CONTINUOUS TIME SIGNALS

Step signal

Unit Step signal is defined as

$$u(t) = 1 \;\; for\ t \geq 0$$

$$= 0\ for\ t < 0$$



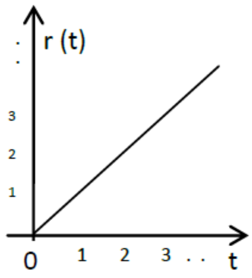
Unit step signal

Ramp signal

Unit ramp signal is defined as

$$r(t) = t \;\; for\ t \geq 0$$

$$= 0\ for\ t < 0$$

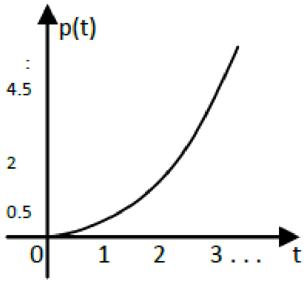


Unit ramp signal

Parabolic signal

Unit Parabolic signal is defined as

$$x(t) = \frac{t^2}{2} \quad for\ t \geq 0$$
$$= 0\ for\ t < 0$$



Unit Parabolic signal

Relation between Unit Step signal, Unit ramp signal and Unit Parabolic signal:

- Unit ramp signal is obtained by integrating unit step signal

$$i.e.,\int u(t)dt = \int 1dt = t = r(t)$$

- Unit Parabolic signal is obtained by integrating unit ramp signal

$$i.e., \int r(t)dt = \int t\ dt = \frac{t^2}{2} = p(t)$$

- Unit step signal is obtained by differentiating unit ramp signal

$$i.e., \frac{d}{dt}(r(t)) = \frac{d}{dt}(t) = 1 = u(t)$$

- Unit ramp signal is obtained by differentiating unit Parabolic signal

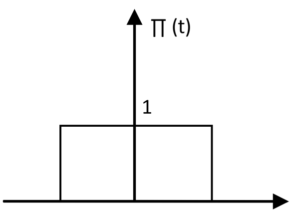
$$i.e., \frac{d}{dt}(p(t)) = \frac{d}{dt}\left(\frac{t^2}{2}\right) = \frac{1}{2}(2t) = t = r(t)$$

Unit Pulse Signal

Unit Pulse signal is defined as

$$\Pi(\mathrm{t}) = 1 \text{ for } |\mathrm{t}| \leq \frac{1}{2}$$

$$= 0 \text{ elsewhere}$$

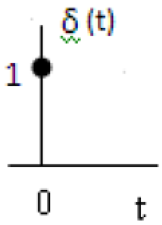


Unit pulse signal

Impulse signal

Unit Impulse signal is defined as

$$\delta(t) = 0 \; for \; t \neq 0$$
$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$



Unit Impulse signal

Properties of Impulse signal:

Property 1:

$$\int_{-\infty}^{\infty} \boldsymbol{x(t)\delta(t)\,dt = x(0)}$$

$$\int_{-\infty}^{\infty} x(t)\delta(t)\,dt = x(0)\delta(0) = x(0) \qquad [\because \delta(t) \; exists \; only \; at \; t = 0 \; and \; \delta(0) = 1]$$

Hence proved

Property 2:

$$\int_{-\infty}^{\infty} x(t)\delta(t - t_o)dt = x(t_o)$$

proof:

$$\int_{-\infty}^{\infty} x(t)\delta(t - t_o)dt = x(t_o)\delta(t_o - t_o) = x(t_o)\delta(0) = x(t_o)$$

$$\therefore [\delta(t - t_o) \; exists \; onlyat \; t = t_o \; and \; \delta(0) = 1]$$

Hence proved

Sinusoidal signal

Cosinusoidal signal is defined as

$$x(t) = A cos(\Omega t + \Phi)$$

Sinusoidal signal is defined as

$$x(t) = A sin(\Omega t + \Phi)$$

Where $\Omega = 2\pi f = \frac{2\pi}{T}$ *and* Ω *is the angular frequency in rad/sec* f is frequency in cycles/sec or Hertz and

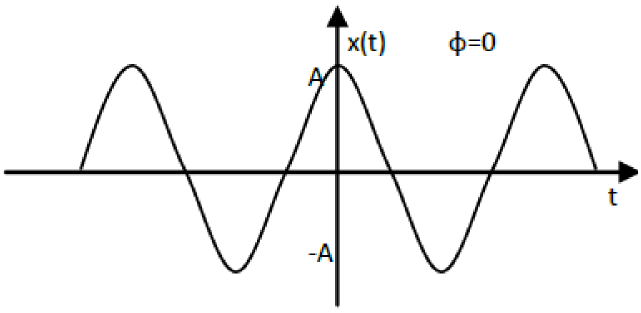
A is amplitude

T is time period in seconds

Φ is phase angle in radians

Cosinusoidal signal

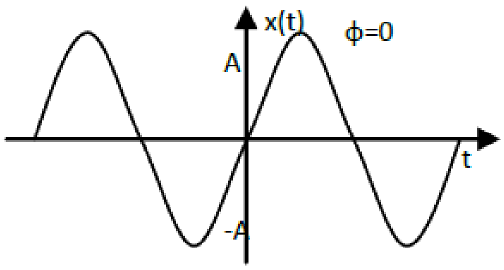
$$when\ \phi = 0,\ x(t) = A\cos(\Omega t)$$



Cosinusoidal signal

Sinusoidal signal

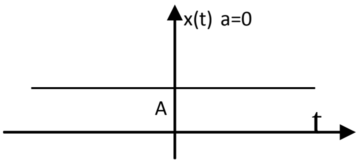
$$when\ \phi = 0,\ x(t) = A\sin(\Omega t)$$



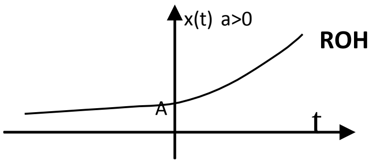
Sinusoidal signal

Exponential signal

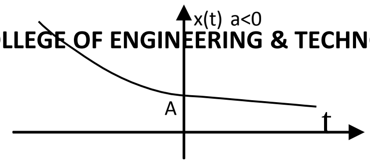
Real Exponential signal is defined as $x(t) = Ae^{at}$,where A is amplitude Depending on the value of 'a' we get dc signal or growing exponential signal or decaying exponential signal



DC signal



Exponentially growing signal



Exponentially decaying signal

Complex exponential signal is defined as

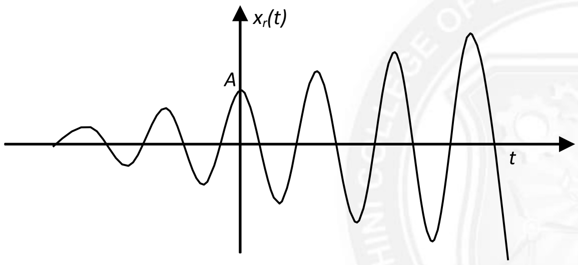
$$x(t) = Ae^{st}$$

where A is amplitude, s is complex variable and $s = \sigma + j\Omega$

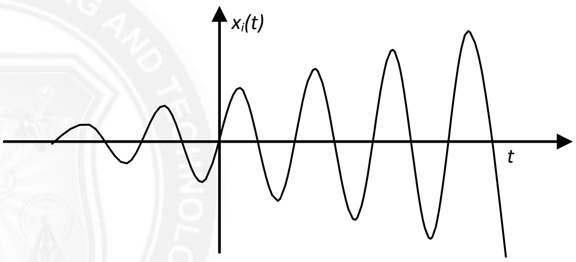
$$x(t) = Ae^{st} = Ae^{(\sigma+j\Omega)t} = Ae^{\sigma t}\, e^{j\Omega t} = Ae^{\sigma t}\,(cos\Omega t + jsin\Omega t)$$

$when\ \sigma = +ve,\ tten\ x(t) = Ae^{\sigma t}\,(cos\Omega t + jsin\Omega t),$

$where\ x_r(t) = Ae^{\sigma t}\, cos\Omega t\ and\ x_i(t) = Ae^{\sigma t}\, sin\Omega t$



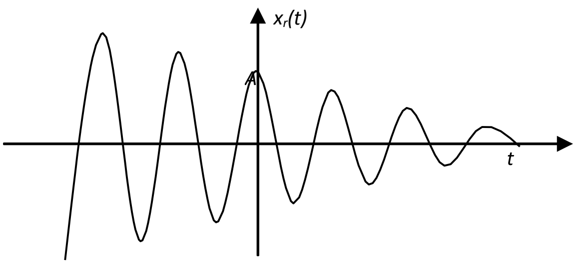
Exponentially growing Cosinusoidal signal



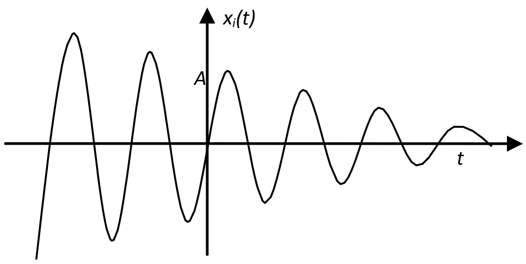
Exponentially growing sinusoidal signal

$when\ \sigma = -ve,\ then\ x(t) = Ae^{-\sigma t}\,(cos\Omega t + jsin\Omega t),$

$where\ x_r(t) = Ae^{-\sigma t}\, cos\Omega t\ and\ x_i(t) = Ae^{-\sigma t}\, sin\Omega t$



Exponentially decaying Cosinusoidal signal



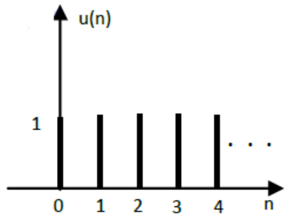
Exponentially decaying sinusoidal signal

BASIC (ELEMENTARY OR STANDARD) DISCRETE TIME SIGNALS

Step signal

Unit Step signal is defined as

$$u(n) = 1 \ \ for\ n \geq 0$$
$$= 0\ for\ n < 0$$

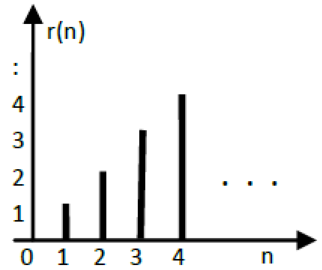


Unit step signal

Unit Ramp signal

Unit Ramp signal is defined as

$$r(n) = n \ \ for\ n \geq 0$$
$$= 0 \quad for\ n < 0$$

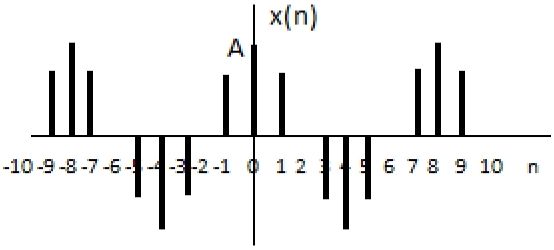


Unit Ramp signal

Sinusoidal signal

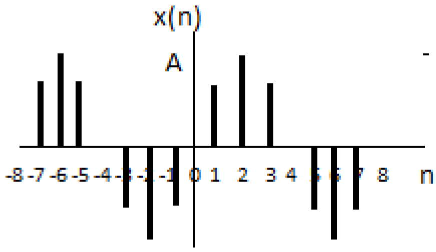
Cosinusoidal signal is defined as

$$x(n) = Acos(\omega n)$$



Sinusoidal signal is defined as

$$x(n) = Asin(\omega n)$$



Sinusoidal signal

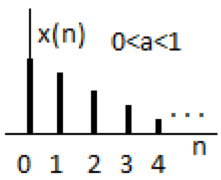
$$where\ \omega = 2\pi f = \frac{2\pi}{N} m\ and\ \omega\ is\ frequency\ in\ radians/sample$$

$m\ is\ the\ smallest\ integer$

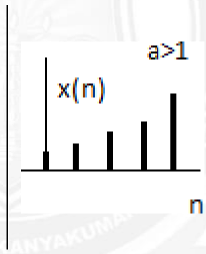
f is frequency in cycles/sample, A is amplitude

Exponential signal

Real Exponential signal is defined as $x(n) = a^n\ for\ n \geq 0$



Decreasing exponential signal

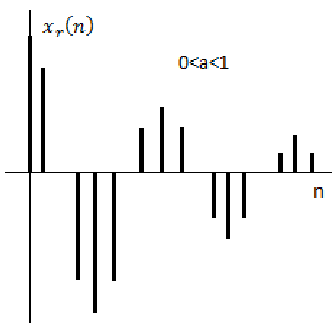


Increasing exponential signal

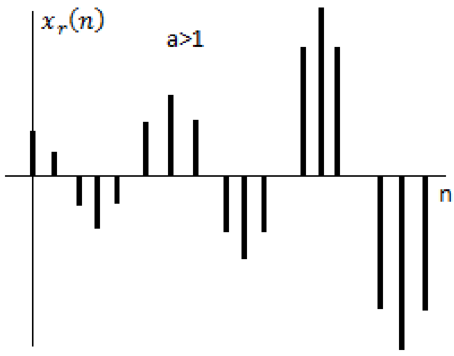
Complex Exponential signal is defined as

$$x(n) = a^n e^{j(\omega 0n)} = a^n[cos\omega_0 n + jsin\omega_0 n]$$

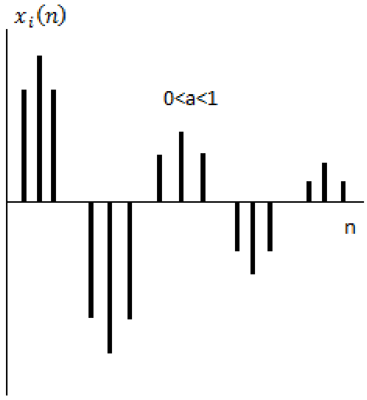
$$where\ x_r(n) = a^n cos\omega_0 n\ and\ x_i(n) = a^n sin\omega_0 n$$



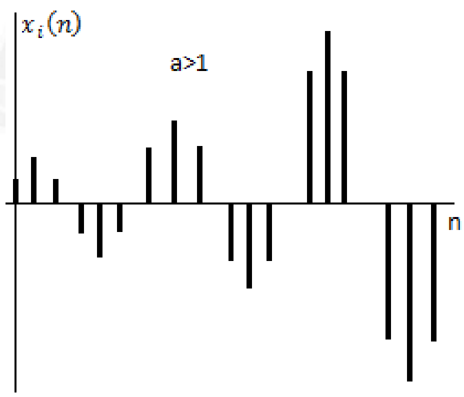
Exponentially decreasing Cosinusoidal signal



Exponentially growing Cosinusoidal signal



Exponentially decreasing sinusoidal signal



Exponentially growing sinusoidal signal

1.2 CLASSIFICATION OF SIGNALS

CONTINUOUS TIME AND DISCRETE TIME SIGNAL

Continuous time signal:

A signal that is defined for every instants of time is known as continuous time signal. Continuous time signals are continuous in amplitude and continuous in time. It is denoted by x(t) and shown in Figure 1.2.1

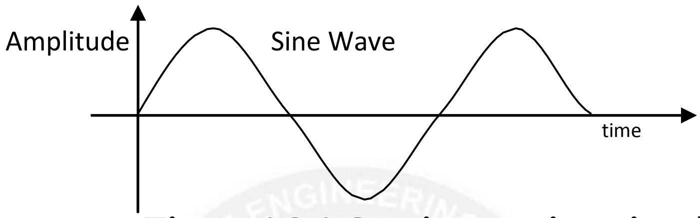


Figure 1.2.1 Continuous time signal

[https://drive.google.com/file/d/1baseARsD-geFLoR-QrEbV5hVsYKKLzGM/view]

Discrete time signal:

A signal that is defined for discrete instants of time is known as discrete time signal. Discrete time signals are continuous in amplitude and discrete in time. It is also obtained by sampling a continuous time signal. It is denoted by x(n) and shown in Figure 1.2.2

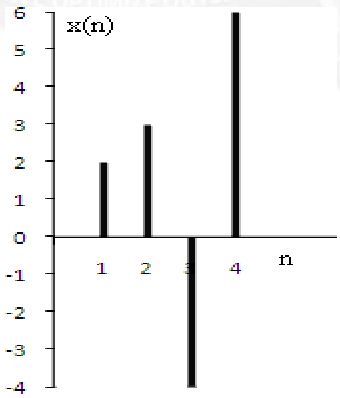


Figure 1.2.2 Discrete time signal

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EVEN (SYMMETRIC) AND ODD (ANTI-SYMMETRIC) SIGNAL

Continuous domain:

Even signal:

A signal that exhibits symmetry with respect to t=0 is called even signal

Even signal satisfies the condition $x(t) = x(-t)$

Odd signal:

A signal that exhibits anti-symmetry with respect to t=0 is called odd signal

Odd signal satisfies the condition $x(t) = -x(-t)$

Even part $\boldsymbol{x_e(t)}$ and Odd part $\boldsymbol{x_0(t)}$ of continuous time signal $\boldsymbol{x(t)}$:

Even component of $x(t)$ *is* $x_e(t) = \frac{1}{2}[x(t) + x(-t)]$

Odd component of $x(t)$ *is* $x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

Discrete domain:

Even signal:

A signal that exhibits symmetry with respect to n=0 is called even signal

Even signal satisfies the condition $x(n) = x(-n)$.

Odd signal:

A signal that exhibits anti-symmetry with respect to n=0 is called odd signal

Odd signalsatisfies the condition $x(n) = -x(-n)$.

Even part $\boldsymbol{x_e(n)}$ and Odd part $\boldsymbol{x_0(n)}$ of discrete time signal $\boldsymbol{x(n)}$:

Even component of $x(n)$ *is* $x_e(n) = \frac{1}{2}[x(n) + x(-n)]$

Odd component of $x(n)$ *is* $x_o(n) = \frac{1}{2}[x(n) - x(-n)]$

PERIODIC AND APERIODIC SIGNAL

Periodic signal:

A signal is said to periodic if it repeats again and again over a certain period of time.

Aperiodic signal:

A signal that does not repeat at a definite interval of time is called aperiodic signal.

Continuous domain:

A Continuous time signal is said to periodic if it satisfies the condition

$$x(t) = x(t + T) \text{ where } T \text{ is fundamental time period}$$

If the above condition is not satisfied, then the signal is said to be aperiodic

Fundamental time period

$$T = 2\pi/\Omega$$

where Ω is fundamental angular frequency in rad/sec

Discrete domain:

A Discrete time signal is said to periodic if it satisfies the condition

$$x(n) = x(n + N) \text{ where } N \text{ is fundamental time period}$$

If the above condition is not satisfied, then the signal is said to be aperiodic

Fundamental time period

$$N = 2\pi m/\omega$$

where ω is fundamental angular frequency in rad/sec,

m is smallest positive integer that makes N as positive integer.

ENERGY AND POWER SIGNAL

Energy signal:

The signal which has finite energy and zero average power is called energy signal. The non-periodic signals like exponential signals will have constant energy and so non periodic signals are energy signals.

i.e., For energy signal, $0 < E < \infty$ and $P = 0$

For Continuous time signals,

$$Energy\ E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt$$

For Discrete time signals,

$$Energy\ E = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2$$

Power signal:

The signal which has finite average power and infinite energy is called power signal. The periodic signals like sinusoidal complex exponential signals will have constant power and so periodic signals are power signals.

i.e., For power signal, $0 < P < \infty\ and\ \ E = \infty$

For Continuous time signals,

$$Average\ power\ P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

For Discrete time signals,

$$Average\ power\ P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

DETERMINISTIC AND RANDOM SIGNALS

Deterministic signal:

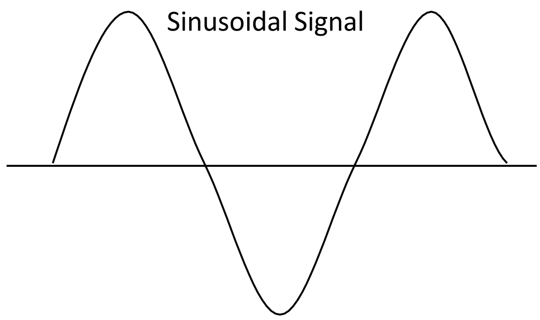
A signal is said to be deterministic if there is no uncertainty over the signal at any instant of time i.e., its instantaneous value can be predicted. It can be represented by mathematical equation.

Example: sinusoidal signal

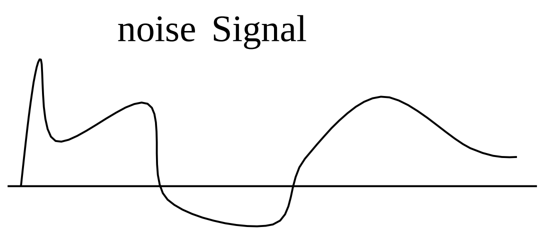
Random signal (Non-Deterministic signal):

A signal is said to be random if there is uncertainty over the signal at any instant of time i.e., its instantaneous value cannot be predicted. It cannot be represented by mathematical equation.

Example: noise signal



Deterministic signal



Random signal

CAUSAL AND NON-CAUSAL SIGNAL

Continuous domain:

Causal signal:

A signal is said to be causal if it is defined for $t \geq 0$.

$$i.e., \quad x(t) = 0 \quad for\ t < 0$$

Non-causal signal:

A signal is said to be non-causal, if it is defined for $t < 0$ or for both $t < 0$ and $t \geq 0$

$$i.e., \quad x(t) \neq 0 \quad for\ t < 0$$

When a non-causal signal is defined only for $t<0$, it is called as anti-causal signal

Discrete domain:

Causal signal:

A signal is said to be causal, if it is defined for $n \geq 0$.

$$i.e., \quad x(n) = 0 \quad for\ n < 0$$

Non-causal signal:

A signal is said to be non-causal, if it is defined, for $n < 0$ or for both $n < 0$ $and\ n \geq 0$

$$i.e., \quad x(n) \neq 0 \quad for\ n < 0$$

When a non-causal signal is defined only for $n<0$, it is called as anti-causal signal.

UNIT I
AMPLITUDE MODULATION

Amplitude modulation or AM as it is often called, is a form of modulation used for radio transmissions for broadcasting and two-way radio communication applications. Although one of the earliest used forms of modulation it is still used today, mainly for long, medium and short wave broadcasting and for some aeronautical point to point communications.

One of the key reasons for the use of amplitude modulation was its ease of use. The system simply required the carrier amplitude to be modulated, but more usefully the detector required in the receiver could be a simple diode based circuit. This meant that AM radios did not need complicated demodulators and costs were reduced - a key requirement for widespread use of radio technology, especially in the early days of radio when ICs were not available.

Amplitude modulation history

The first amplitude modulated signal was transmitted in 1901 by a Canadian engineer named Reginald Fessenden. He took a continuous spark transmission and placed a carbon microphone in the antenna lead. The sound waves impacting on the microphone varied its resistance and in turn this varied the intensity of the transmission. Although very crude, signals were audible over a distance of a few hundred meters, although there was a rasping sound caused by the spark.

With the introduction of continuous sine wave signals, transmissions improved significantly, and AM soon became the standard for voice transmissions. Nowadays, amplitude modulation, AM is used for audio broadcasting on the long medium and short wave bands, and for two way radio communication at VHF for aircraft. However as there now are more efficient and convenient methods of modulating a signal, its use is declining, although it will still be very many years before it is no longer used.

Amplitude modulation applications

Amplitude modulation is used in a variety of applications. Even though it is not as widely used as it was in previous years in its basic format it can nevertheless still be found.

Broadcast transmissions: AM is still widely used for broadcasting on the long, medium and short wave bands. It is simple to demodulate and this means that radio receivers capable of demodulating amplitude modulation are cheap and simple to manufacture. Nevertheless, many people are moving to high quality forms of transmission like frequency modulation, FM or digital transmissions.

Air band radio: VHF transmissions for many airborne applications still use Amplitude Modulation. It is used for ground to air radio communications as well as two-way radio links for ground staff as well.

Single sideband: Amplitude modulation in the form of single sideband is still used for HF radio links. Using a lower bandwidth and providing more effective use of the transmitted power this form of modulation is still used for many point to point HF links.

Quadrature amplitude modulation: AM is widely used for the transmission of data in everything from short range wireless links such as Wi-Fi to cellular telecommunications and much more. Effectively it is formed by having two carriers 90° out of phase. These form some of the main uses of amplitude modulation.

Need for Amplitude modulation

In order that a radio signal can carry audio or other information for broadcasting or for two-way radio communication, it must be modulated or changed in some way. Although there are a number of ways in which a radio signal may be modulated, one of the easiest is to change its amplitude in line with variations of the sound. In this way the amplitude of the radio frequency signal varies in line with the instantaneous value of the intensity of the modulation. This means that the radio frequency signal has a representation of the sound wave superimposed in it.

In view of the way the basic signal "carries" the sound or modulation, the radio frequency signal is often termed the "carrier". A continuous-wave goes on continuously without any intervals and it is the baseband message signal, which contains the information. This wave has to be modulated. According to the standard definition, “The amplitude of the carrier signal varies in accordance with the instantaneous amplitude of the modulating signal.” Which means, the amplitude of the carrier signal containing no information varies as per the amplitude of the signal containing information, at each instant. This can be well explained by the following figures 1.1.1, 1.1.2, 1.1.3 and figure 1.1.4.

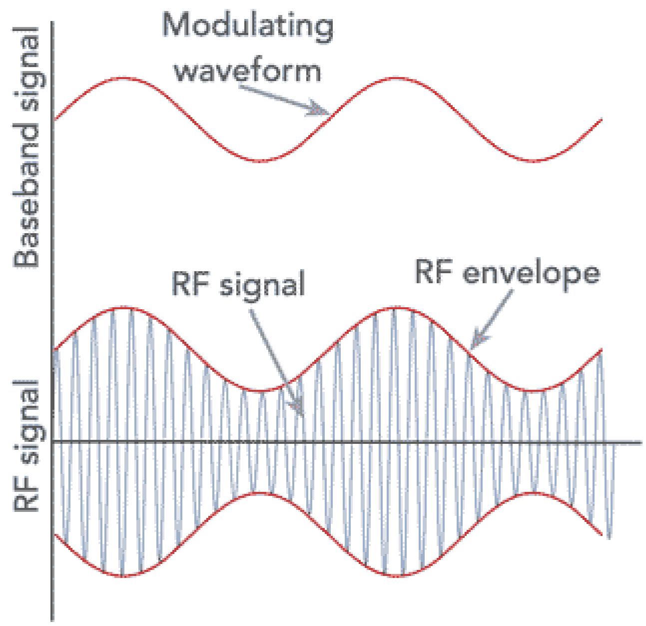


Figure 1.1.1 Modulating wave form and Modulated RF Signal

Diagram Source : Electronic Tutorials

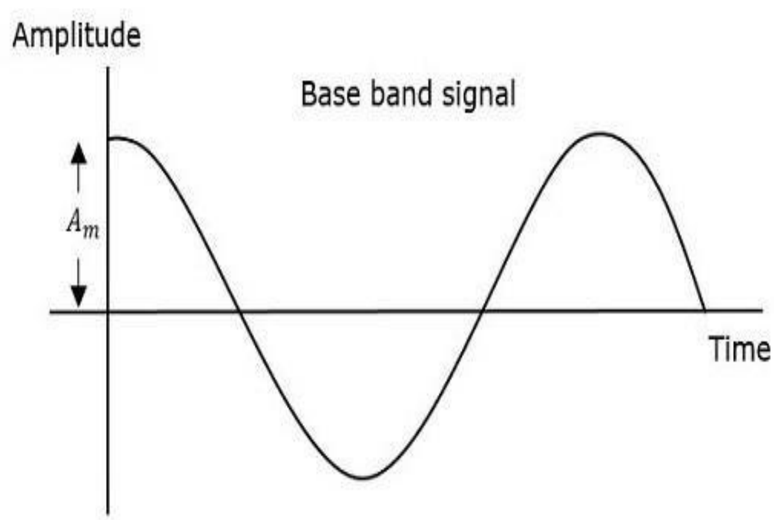


Figure: 1.1.2 Base Band Signal

Diagram Source : Electronic Tutorials

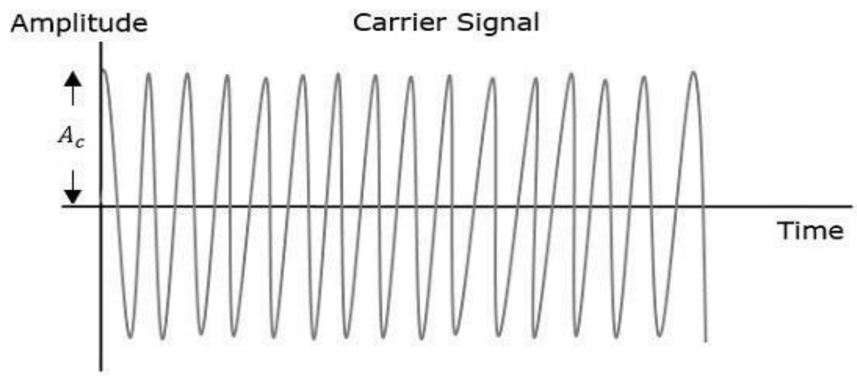


Fig1.1.3 Carrier Signal

Diagram Source : Electronic Tutorial

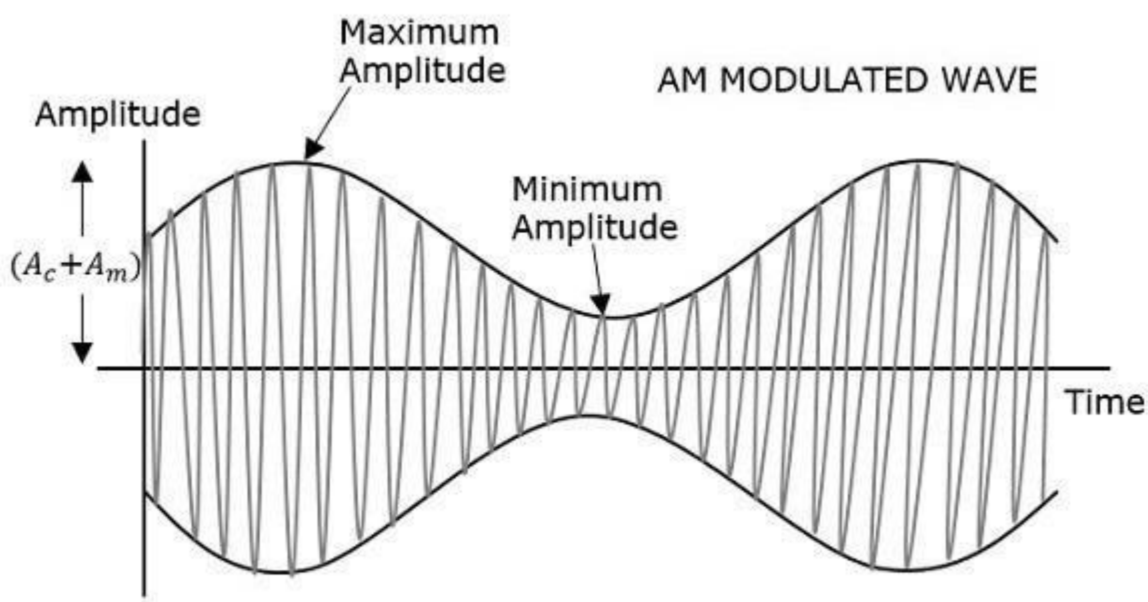


Figure 1.1.4 AM Modulated Signal

Diagram Source : Electronic Tutorial

The figure 1.1.4 shows the modulating wave, which is the message signal. The next one is the carrier wave, which is a high frequency signal and contains no information. While, the last one is the resultant modulated wave.

It can be observed that the positive and negative peaks of the carrier wave, are interconnected with an imaginary line. This line helps recreating the exact shape of the modulating signal. This imaginary line on the carrier wave is called as Envelope. It is the same as that of the message signal.

Mathematical Expressions

Following are the mathematical expressions for these waves. Time-domain Representation of the Waves Let the modulating signal be,

$$m(t)=A_m \mathrm{Cos}(2\pi f_m t) \qquad (1)$$

and the carrier signal be,

$$c(t)=A_c \mathrm{Cos}(2\pi f_c t) \qquad (2)$$

Where,

Am and Ac are the amplitude of the modulating signal and the carrier signal respectively.

fm and fc are the frequency of the modulating signal and the carrier signal respectively.

Then, the equation of Amplitude Modulated wave will be

$$s(t)=[A_c+A_m \cos(2\pi f_m t)]\mathrm{Cos}(2\pi f_c t) \qquad (3)$$

Modulation Index

A carrier wave, after being modulated, if the modulated level is calculated, then such an attempt is called as Modulation Index or Modulation Depth. It states the level of modulation that a carrier wave undergoes.

Rearrange the Equation as below.

$$s(t)=Ac[1+(A_m A_c)\mathrm{Cos}(2\pi f_m t)]\mathrm{Cos}(2\pi f_c t) \qquad (4)$$

$$\Rightarrow s(t)=Ac[1+\mu\cos(2\pi f_m t)Cos(2\pi f_c t) \quad (5)$$

Where, μ is Modulation index and it is equal to the ratio of Am and Ac. Mathematically, we can write it as

$$\mu=Am/Ac \quad (6)$$

Hence, we can calculate the value of modulation index by using the above formula, when the amplitudes of the message and carrier signals are known. Now, let us derive one more formula for Modulation index . We can use this formula for calculating modulation index value, when the maximum and minimum amplitudes of the modulated wave are known.

Let Amax and Amin be the maximum and minimum amplitudes of the modulated wave.

We will get the maximum amplitude of the modulated wave, when cos(2πfmt)

$$\Rightarrow Amax=Ac+Am \quad (7)$$

We will get the minimum amplitude of the modulated wave, when cos(2πfmt)cos(2πfmt) is -1.

$$\Rightarrow Amin=Ac-Am$$

$$\Rightarrow Amin=Ac-Am$$

Adding

$$Amax+Amin=Ac+Am+Ac-Am=2Ac$$

$$Ac=(Amax+Amin)/2\Rightarrow Ac=(Amax+Amin)/2 \quad (8)$$

Subtracting

$$Amax-Amin=Ac+Am-(Ac-Am)$$

$$=2AmAmax-Amin=Ac+Am-(Ac-Am)=2Am$$

$$Am=(Amax-Amin)/2 \Rightarrow Am=(Amax-Amin)/2$$

$$\mu=Amax-Amin\ /\ Amax+Amin$$

$$\mu=Amax-Amin/\ Amax+Amin \qquad (9)$$

Therefore, the above Equations are the two formulas for Modulation index. The modulation index or modulation depth is often denoted in percentage called as Percentage of Modulation. We will get the percentage of modulation, just by multiplying the modulation index value with 100.

For a perfect modulation, the value of modulation index should be 1, which implies the percentage of modulation should be 100%.

For instance, if this value is less than 1, i.e., the modulation index is 0.5, then the modulated output would look like the following figure. It is called as Under-modulation. Such a wave is called as an under-modulated wave. Fig 1.1.5 shows Under Modulated Wave.

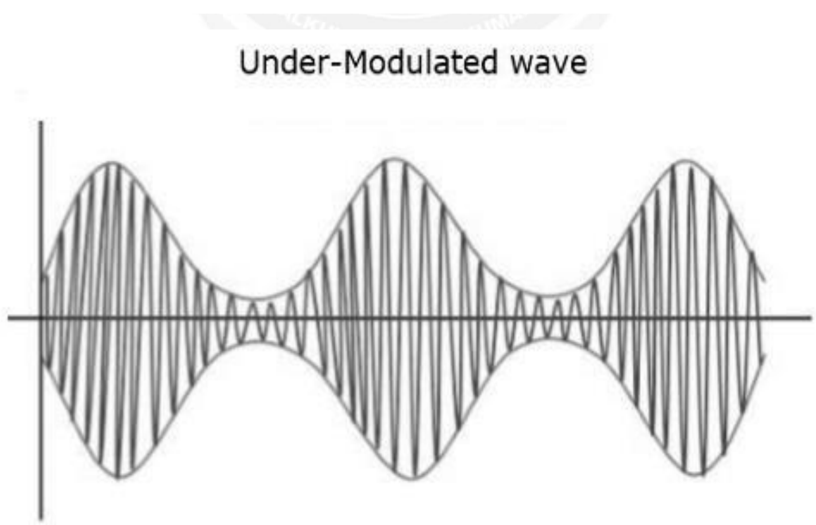


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Figure 1.1.5 Under Modulated Wave

Diagram Source : Brain Kart

If the value of the modulation index is greater than 1, i.e., 1.5 or so, then the wave will be an over-modulated wave. It would look like the following figure 1.1.6.

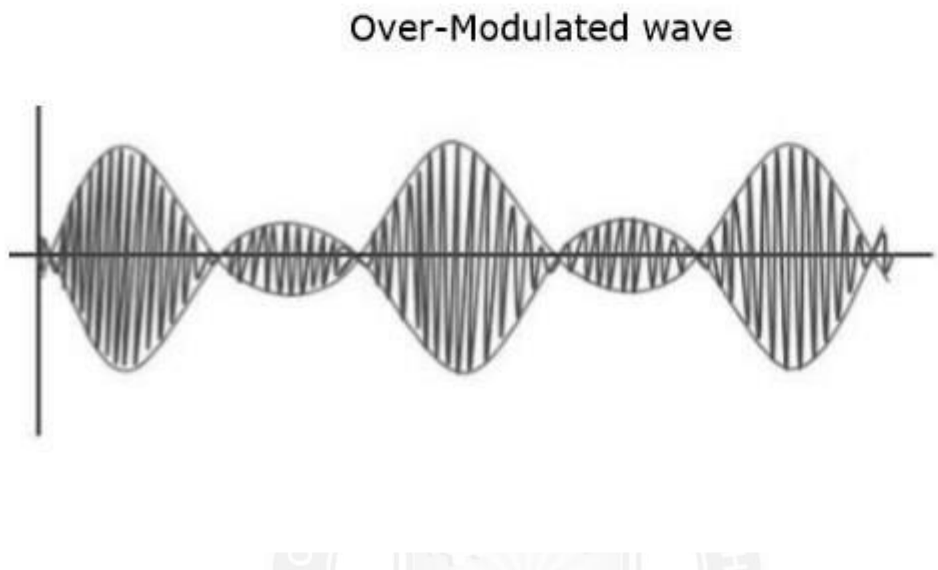

Figure 1.1.6 Over Modulated Signal

Diagram Source : Electronic Tutorials

As the value of the modulation index increases, the carrier experiences a 180^o phase reversal, which causes additional sidebands and hence, the wave gets distorted. Such an over-modulated wave causes interference, which cannot be eliminated.

Let us consider that a carrier signal A cos ωct is amplitude-modulated by a single-tone modulating signal

$$x(t) = Vm\ A \cos \omega_m t. \qquad (9)$$

Then the unmodulated or carrier power

PC = mean square (rms value)

Power Content In Multiple-tone Amplitude Modulation

Mathematical Expression

Let us consider that a carrier signal A cos ωct is modulated by a baseband or modulating signal x(t) which is expressed as :

$$x(t) = V_1 \cos \omega_1 t + V_2 \cos \omega_2 t + V_3 \cos \omega_3 t \quad (10)$$

We know that the general expression for AM wave is

$$s(t) = A \cos \omega_c t + x(t) \cos \omega_c t \quad (11)$$

Putting the value of x(t), we get

$$s(t) = A \cos \omega_c t + [V_1 \cos \omega_1 t + V_2 \cos \omega_2 t + V_3 \cos \omega_3 t\ ] \cos \omega_c t$$

The expression for AM wave can further be expanded as under:

$$s(t) = A \cos \omega_c t + m_1 A \cos \omega_c t \cos \omega_1 t + m_2 A \cos \omega_c t \cos \omega_2 t + m_3 A \cos \omega_c t \cos \omega_3 t$$

Now we know that the total power in AM is given as ,

Pt = carrier power + sideband power

Pt = PC + PS

The carrier power PC is given as considering additional resistance like antenna resistance R.

$$P_{carrier} = [(V_c/\sqrt{2})^2 /R] = V^2C/2R \quad (12)$$

Each side band has a value of mVc /2 and r.m.s value of (maVc/2)/√2. Hence power in LSB and USB can be written as

$PLSB = PUSB = (mVc/2)/\sqrt{2})^2/R = m^2V^2_C/8R = m2/4\ Pcarrier$

$P_{total} = V^2C/2R + [m^2V^2_C/8R] + [m^2V^2_C/8R]$

$= V^2C/2R + [m^2V^2_C/4R\}$

$= V^2_C/2R\ (1 + m^2/2)$

$P_{total} = Pcarrier\ (1 + m^2/2)$ (13)

In some applications, the carrier is simultaneously modulated by several sinusoidal modulating signals. In such a case, the total modulation index is given as

$mt = \sqrt{(}m_1^2 + m_2^2 + m_3^2 + m_4^2 + \ldots..$

If Ic and It are the r.m.s values of unmodulated current and total modulated current and R is the resistance through which these current flow, then

$P_{total}/P_{carrier} = (I_{t.}R/Ic.R)^2 = (It/Ic)^2$

$Pt_{ota}l/P_{carrier} = (1 + m^2/2)$

$It/Ic = 1 + m^2/2$ (14)

Limitations of Amplitude Modulation:

1. Switching modulator Low Efficiency- Since the useful power that lies in the small bands is quite small, so the efficiency of AM system is low.
2. Limited Operating Range – The range of operation is small due to low efficiency. Thus, transmission of signals is difficult.
3. Noise in Reception – As the radio receiver finds it difficult to distinguish between the amplitude variations that represent noise and those with the signals, heavy noise is prone to occur in its reception.

4. Poor Audio Quality – To obtain high fidelity reception, all audio frequencies till 15 Kilo Hertz must be reproduced and this necessitates the bandwidth of 10 Kilo Hertz to minimize the interference from the adjacent broadcasting stations. Therefore in AM broadcasting stations audio quality is known to be poor.

The following two modulators generate AM wave.

- Square law modulator
- Switching Modulator

Square Law Modulator

Following is the block diagram of the square law modulator sown in figure 1.1.7

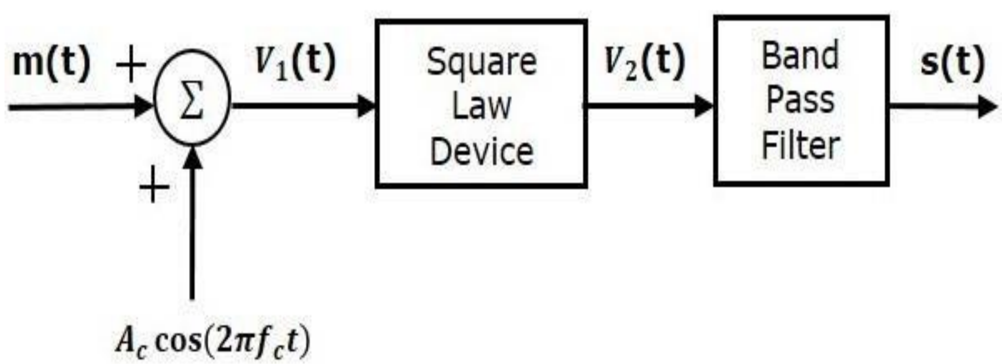

Figure 1.1.7 Block diagram of the square law modulator

Let the modulating and carrier signals be denoted as m(t)m(t) and Acos(2πfct) respectively. These two signals are applied as inputs to the summer (adder) block. This summer block produces an output, which is the addition of the modulating and the carrier signal. Mathematically, we can write it as

$$V_1(t)=m(t)+A_c \cos(2\pi f_c t) \qquad (15)$$

This signal V1tV1t is applied as an input to a nonlinear device like diode. The characteristics of the diode are closely related to square law.

$$V_2(t)=k_1V_1(t)+k_2V_2(t)$$

$$V_2(t)=k_1V_1(t)+k_2V_2(t)$$

Where, k_1 and k_2 are constants.

Substitute V1(t) in $V_2(t)$,

$$V_2(t) = k_1[m(t) + A_c \cos(2\pi f_c t)] + k_2[m(t) + A_c \cos(2\pi f_c t)]^2$$

$$\Rightarrow V_2(t) = k_1 m(t) + k_1 A_c \cos(2\pi f_c t) + k_2 m^2(t) +$$

$$k_2 A_c^2 \cos^2(2\pi f_c t) + 2k_2 m(t) A_c \cos(2\pi f_c t)$$

$$\Rightarrow V_2(t) = k_1 m(t) + k_2 m^2(t) + k_2 A_c^2 \cos^2(2\pi f_c t) +$$

$$k_1 A_c \left[1 + \left(\frac{2k_2}{k_1}\right) m(t)\right] \cos(2\pi f_c t)$$

The last term of the above equation represents the desired AM wave and the first three terms of the above equation are unwanted. So, with the help of band pass filter, we can pass only AM wave and eliminate the first three terms.

Therefore, the output of square law modulator is

$$s(t)=k_1 Ac[1+(2k_2 / k_1)m(t)]\cos(2\pi fct) \quad (16)$$

The standard equation of AM wave is

$$s(t)=Ac[1+kam(t)]\cos(2\pi fct)$$

Where, Ka is the amplitude sensitivity

By comparing the output of the square law modulator with the standard equation of AM wave, we will get the scaling factor as k1k1 and the amplitude sensitivity kaka as 2k /2k1

Switching Modulator

Following is the block diagram of switching modulator shown in Figure 1.1.8.

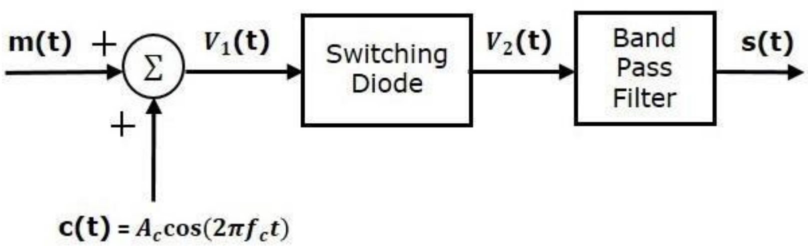

Figure 1.1.8 block diagram of the Switching modulator

Switching modulator is similar to the square law modulator. The only difference is that in the square law modulator, the diode is operated in a non-linear mode, whereas, in the switching modulator, the diode has to operate as an ideal switch.

Let the modulating and carrier signals be denoted as m(t)m(t) and m(t)=Ac cos(2πfmt) and c(t)=Accos(2πfct) respectively. These two signals are applied as inputs to the summer (adder) block. Summer block produces an output, which is the addition of modulating and carrier signals. Mathematically, we can write it as

$$V_1(t)=m(t)+c(t)$$

$$V_1(t)= m(t)+A_c\cos(2\pi f_c t)$$

This signal V1(t) is applied as an input of diode. Assume, the magnitude of the modulating signal is very small when compared to the amplitude of carrier signal Ac. So, the diode's ON and OFF action is controlled by carrier signal c(t). This means, the diode will be forward biased when c(t)>0 and it will be reverse biased when c(t)<0.

Therefore, the output of the diode is

$$V_2(t) = \begin{cases} V_1(t) & if \quad c(t) > 0 \\ 0 & if \quad c(t) < 0 \end{cases}$$

We can approximate this as

$$V_2(t) = V_1(t)\,x(t)$$

Where, $x(t)$ is a periodic pulse train with time period $T = \frac{1}{f_c}$

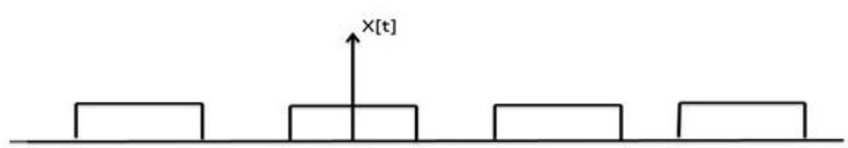

Figure 1.1.9 Periodic pulse train Diagram Source Brain Kart

Diagram Source : Brain Kart

The Fourier series representation of this periodic pulse train is in figure 1.1.9 is ,

$$x(t) = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n - 1}{2n-1}\cos(2\pi(2n-1)f_c t)$$

$$\Rightarrow x(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) - \frac{2}{3\pi}\cos(6\pi f_c t) + \ldots$$

Substitute, $V_1(t)$ and $x(t)$ 2.

$$V_2(t) = [m(t) + A_c\cos(2\pi f_c t)]\left[\frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) - \frac{2}{3\pi}\cos(6\pi f_c t) + \ldots\right]$$

$$V_2(t) = \frac{m(t)}{2} + \frac{A_c}{2}\cos(2\pi f_c t) + \frac{2m(t)}{\pi}\cos(2\pi f_c t) + \frac{2A_c}{\pi}\cos^2(2\pi f_c t) -$$

$$\frac{2m(t)}{3\pi}\cos(6\pi f_c t) - \frac{2A_c}{3\pi}\cos(2\pi f_c t)\cos(6\pi f_c t) + \ldots$$

The 1st term of the above equation represents the desired AM wave and the remaining terms are unwanted terms. Thus, with the help of band pass filter, we can pass only AM wave and eliminate the remaining terms.

Therefore, the output of switching modulator is

$$s(t)=A_c/2(1+(4\pi A_c)m(t))\cos(2\pi f_c t) \qquad (17)$$

We know the standard equation of AM wave is

$$s(t)=Ac[1+k_a m(t)]\cos(2\pi fct)$$

Where, ka is the amplitude sensitivity. By comparing the output of the switching modulator with the standard equation of AM wave, we will get the scaling factor as 0.5 and amplitude sensitivity kaka as 4/πAc.

The switching modulator using a diode has been shown in figures 1.1.10 (a) & (b) .

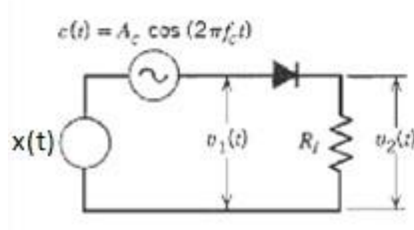

Figure 1.1.10 (a) Switching Modulator

Diagram Source : Brain Kart

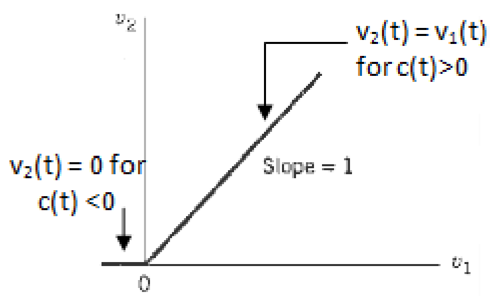

Figure1. 1.10 (b) Input and output voltage Characteristics

Diagram Source : Brain Kart

This diode is assumed to be operating as a switch . The modulating signal x(t) and the sinusoidal carrier signal c(t) are connected in series with each other. Therefore, the input voltage to the diode is given by

$$v_1(t) = c(t) + x(t) = E_c \cos(2\pi f_c t) + x(t) \qquad (18)$$

The amplitude of carrier is much larger than that of x(t) and c(t) decides the status of the diode (ON or OFF) .

Working Operation and Analysis

Let us assume that the diode acts as an ideal switch .Hence, it acts as a closed switch when it is forward biased in the positive half cycle of the carrier and offers zero impedance .Whereas it acts as an open switch when it is reverse biased in the negative half cycle of the carrier and offers an infinite impedance .Therefore, the output voltage v2(t) = v1(t) in the positive half cycle of c(t) and v2(t) = 0 in the negative half cycle of c(t) .

Hence ,

$$v_2(t) = v_1(t) \qquad \text{for } c(t) > 0$$

$$v2(t) = 0 \qquad \text{for } c(t) < 0 \qquad (19)$$

In other words , the load voltage v2(t) varies periodically between the values v1(t) and zero at the rate equal to carrier frequency fc .We can express v2(t) mathematically as under a pulse train. In figure 1.1.11 :

$$v_2(t) = v_1(t) \,.\, g_p(t) = [x(t) + E_c \cos(2\pi f_c t)]\, g_p(t)$$

where, gp(t) is a periodic pulse train of duty cycle equal to one half cycle period i.e. T0 /2 (where T0 = 1/fc) .

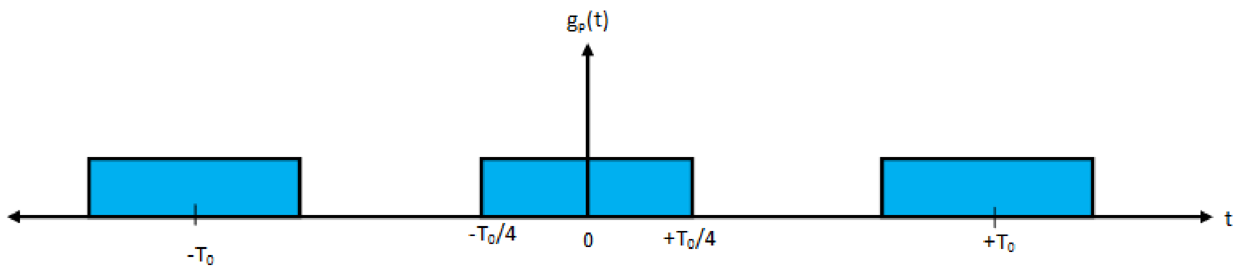

Figure 1.1.11 Periodic pulse train of duty cycle equal to one half cycle period

Diagram Source : Electronic Tutorial

Let us express $g_p(t)$ with the help of Fourier series as under :

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos\,[2\pi f_c t(2n-1)]$$

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) + odd\ harmonic\ components \qquad (20)$$

Substituting $g_p(t)$ into $v_2(t)$ equation , we get

$$v_2(t) = [x(t) + E_c \cos((2\pi f_c t)] \left\{ \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos\,[2\pi f_c t(2n-1)] \right\}$$

Therefore,

$$v_2(t) = [x(t) + E_c \cos((2\pi f_c t)] \left\{ \frac{1}{2} + \frac{2}{\pi} \cos(2\pi f_c t) + odd\ harmonics \right\}$$

The odd harmonics in this expression are unwanted, and therefore, are assumed to be eliminated , Hence,

$$v_2(t) = \underbrace{\frac{1}{2} x(t)}_{\text{Modulating Signal}} + \underbrace{\frac{1}{2} E_c \cos(2\pi f_c t) + \frac{2}{\pi} x(t) \cos(2\pi f_c t)}_{\text{AM Wave}} + \underbrace{\frac{2E_c}{\pi} cos^2(2\pi f_c t)}_{\text{Second harmonic of carrier}} \tag{21}$$

In this expression, the first and the fourth terms are unwanted terms whereas the second and third terms together represents the AM wav .Clubbing the second and third terms together , we obtain

$$v_2(t) = \frac{E_c}{2} \left[1 + \frac{4}{\pi E_c} x(t) \right] \cos(2\pi f_c t) + unwanted\ terms \tag{22}$$

This is the required expression for the AM wave with m=[4/πEc] . The unwanted terms can be eliminated using a band-pass filter (BPF) .

AM Spectra and Band Width

The AM signal has three frequency components, Carrier, Upper Sideband and lower side Band

Bandwidth of AM

The bandwidth of a complex signal like AM is the difference between its highest and lowest frequency components and is expressed in Hertz (Hz). Bandwidth deals with only frequencies.

As shown in the following figure 1.1.12

Bandwidth = **(fc – fm) – (fc + fm) = 2 fm**

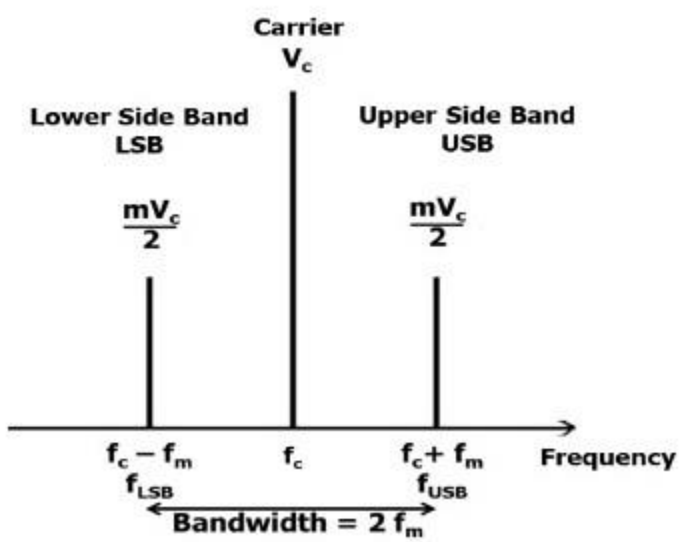

Figure 1.1.12 AM Spectra and Bandwidth

Diagram Source : elprocus.com