## LANGUAGES OF PDA

A language can be accepted by Pushdown automata using two approaches:

1. Acceptance by Final State: The PDA is said to accept its input by the final state if it enters any final state in zero or more moves after reading the entire input.

Let $\mathrm{P}=(\mathrm{Q}, \Sigma, \Gamma, \delta, q 0, \mathrm{Z}, \mathrm{F})$ be a PDA. The language acceptable by the final state can be defined as:

1. $L(P D A)=\left\{w \mid(q 0, w, Z) \vdash^{*}(p, \varepsilon, \varepsilon), q \in F\right\}$
2. Acceptance by Empty Stack: On reading the input string from the initial configuration for some PDA, the stack of PDA gets empty.

Let $\mathrm{P}=(\mathrm{Q}, \Sigma, \Gamma, \delta, \mathrm{q} 0, \mathrm{Z}, \mathrm{F})$ be a PDA. The language acceptable by empty stack can be defined as:

1. $\mathrm{N}(\mathrm{PDA})=\left\{\mathrm{w} \mid(\mathrm{q} 0, \mathrm{w}, \mathrm{Z}) \vdash^{*}(\mathrm{p}, \varepsilon, \varepsilon), \mathrm{q} \in \mathrm{Q}\right\}$

Equivalence of Acceptance by Final State and Empty Stack

- If $L=N(P 1)$ for some PDA P1, then there is a PDA P2 such that $L=L(P 2)$. That means the language accepted by empty stack PDA will also be accepted by final state PDA.
- If there is a language $L=L(P 1)$ for some PDA P1 then there is a PDA P2 such that $L=N(P 2)$. That means language accepted by final state PDA is also acceptable by empty stack PDA.

Example:

Construct a PDA that accepts the language $L$ over $\{0,1\}$ by empty stack which accepts all the string of 0 's and 1 's in which a number of 0 's are twice of number of 1 's.

## Solution:

There are two parts for designing this PDA:

- If 1 comes before any 0 's
- If 0 comes before any 1 's.

We are going to design the first part i.e. 1 comes before 0 's. The logic is that read single 1 and push two 1's onto the stack. Thereafter on reading two 0's, POP two 1's from the stack. The $\delta$ can be

1. $\delta(q 0,1, Z)=(q 0,11, Z) \quad$ Here $Z$ represents that stack is empty
2. $\delta(\mathrm{q} 0,0,1)=(\mathrm{q} 0, \varepsilon)$

Now, consider the second part i.e. if 0 comes before 1's. The logic is that read first 0 , push it onto the stack and change state from q0 to q1. [Note that state q 1 indicates that first 0 is read and still second 0 has yet to read].

Being in q1, if 1 is encountered then POP 0 . Being in q1, if 0 is read then simply read that second 0 and move ahead. The $\delta$ will be:

1. $\delta(\mathrm{q} 0,0, \mathrm{Z})=(\mathrm{q} 1,0 \mathrm{Z})$
2. $\delta(\mathrm{q} 1,0,0)=(\mathrm{q} 1,0)$
3. $\delta(\mathrm{q} 1,0, \mathrm{Z})=(\mathrm{q} 0, \varepsilon) \quad$ (indicate that one 0 and one 1 is already read, so simply read the second 0 )
4. $\delta(\mathrm{q} 1,1,0)=(\mathrm{q} 1, \varepsilon)$

Now, summarize the complete PDA for given L is:

1. $\delta(\mathrm{q} 0,1, \mathrm{Z})=(\mathrm{q} 0,11 \mathrm{Z})$
2. $\delta(\mathrm{q} 0,0,1)=(\mathrm{q} 1, \varepsilon)$
3. $\delta(\mathrm{q} 0,0, \mathrm{Z})=(\mathrm{q} 1,0 \mathrm{Z})$
4. $\delta(\mathrm{q} 1,0,0)=(\mathrm{q} 1,0)$
5. $\delta(\mathrm{q} 1,0, \mathrm{Z})=(\mathrm{q} 0, \varepsilon)$
6. $\delta(\mathrm{q} 0, \varepsilon, \mathrm{Z})=(\mathrm{q} 0, \varepsilon) \quad$ ACCEPT state
