## LANGUAGES OF PDA

A language can be accepted by Pushdown automata using two approaches:

**1.** Acceptance by Final State: The PDA is said to accept its input by the final state if it enters any final state in zero or more moves after reading the entire input.

Let  $P = (Q, \Sigma, \Gamma, \delta, q0, Z, F)$  be a PDA. The language acceptable by the final state can be defined as:

1. L(PDA) = {w |  $(q0, w, Z) \vdash^* (p, \varepsilon, \varepsilon), q \in F$ }

**2.** Acceptance by Empty Stack: On reading the input string from the initial configuration for some PDA, the stack of PDA gets empty.

Let  $P = (Q, \Sigma, \Gamma, \delta, q0, Z, F)$  be a PDA. The language acceptable by empty stack can be defined as:

1. N(PDA) = {w |  $(q0, w, Z) \vdash^* (p, \varepsilon, \varepsilon), q \in Q$ }

Equivalence of Acceptance by Final State and Empty Stack

- If L = N(P1) for some PDA P1, then there is a PDA P2 such that L = L(P2). That means the language accepted by empty stack PDA will also be accepted by final state PDA.
- If there is a language L = L (P1) for some PDA P1 then there is a PDA P2 such that L = N(P2). That means language accepted by final state PDA is also acceptable by empty stack PDA.

## Example:

Construct a PDA that accepts the language L over  $\{0, 1\}$  by empty stack which accepts all the string of 0's and 1's in which a number of 0's are twice of number of 1's.

## Solution:

There are two parts for designing this PDA:

- If 1 comes before any 0's
- If 0 comes before any 1's.

We are going to design the first part i.e. 1 comes before 0's. The logic is that read single 1 and push two 1's onto the stack. Thereafter on reading two 0's, POP two 1's from the stack. The  $\delta$  can be

- 1.  $\delta(q0, 1, Z) = (q0, 11, Z)$  Here Z represents that stack is empty
- 2.  $\delta(q0, 0, 1) = (q0, \varepsilon)$

Now, consider the second part i.e. if 0 comes before 1's. The logic is that read first 0, push it onto the stack and change state from q0 to q1. [Note that state q1 indicates that first 0 is read and still second 0 has yet to read].

Being in q1, if 1 is encountered then POP 0. Being in q1, if 0 is read then simply read that second 0 and move ahead. The  $\delta$  will be:

- 1.  $\delta(q0, 0, Z) = (q1, 0Z)$
- 2.  $\delta(q1, 0, 0) = (q1, 0)$
- 3.  $\delta(q1, 0, Z) = (q0, \varepsilon)$  (indicate that one 0 and one 1 is already read, so simply read the second 0)
- 4.  $\delta(q1, 1, 0) = (q1, \varepsilon)$

Now, summarize the complete PDA for given L is:

- 1.  $\delta(q0, 1, Z) = (q0, 11Z)$
- 2.  $\delta(q0, 0, 1) = (q1, \epsilon)$
- 3.  $\delta(q0, 0, Z) = (q1, 0Z)$
- 4.  $\delta(q1, 0, 0) = (q1, 0)$
- 5.  $\delta(q1, 0, Z) = (q0, \varepsilon)$
- 6.  $\delta(q0, \varepsilon, Z) = (q0, \varepsilon)$  ACCEPT state