5.3 Stability analysis of purely cohesive soil and c-φ soil:

1) Stability analysis of infinite slope

2) Stability analysis of finite slope

5.3.1. Infinite Slopes:

Infinite slopes have dimensions that extended over great distances and the soil mass is inclined to the horizontal. If different strata are present strata boundaries

are assumed to be parallel to the surface. Failure is assumed to occur along a plane parallel to the surface.

Infinite Slopes:

Figure 5.6 shows an infinite slope *AB,* inclined at angle I to the horizontal. For an infinite slope, the soil properties and the soil stresses on any plane parallel to the slope surface are identical, and therefore, the failure of the slope usually involves a sliding of soil mass along a plane parallel to the slope at some depth. Let *CD* represents such a failure plane at depth z below the surface.

Consider a prism of soil, of inclined length b along the slope, and depth z up to the critical surface.

The horizontal length of prism is b cos i, and its volume per unit length of prism is $z.\ b$ .cos i

Weight of prism=W=$\gamma.z.b\cos i$.

Vertical stress σ_z on the surface *CD* is given by

$$\sigma_z = \frac{W}{b} = \gamma . z . cosi - - - - - (1)$$

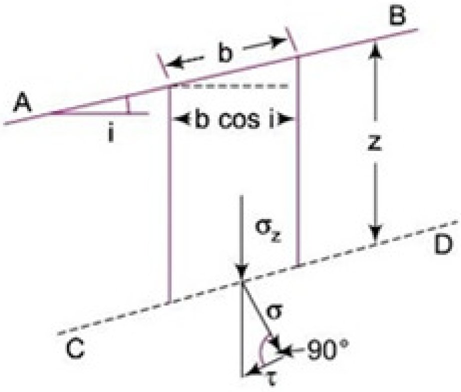

Fig 5.6 infinite slope diagram

If σ and τ are the stress components normal and tangential to the surface *CD*, we have

$$\sigma = \sigma_z cosi = \gamma z cos^2 i - - - - - (2a)$$

$$\tau = \sigma_z sini = \gamma z cosi\ sini - - - - - (2b)$$

The tangential component τ is called the shear stress which induces failure along *CD* and which is resisted by shear strength τ_f of the soil. The factor of safety of the slope, against sliding due to shear is given by

$$F = \frac{\tau_f}{\tau} - - - - - (3)$$

The shear strength in general, consists of both cohesion and internal friction. We shall consider three cases of soil:

*(i)*cohesion less soil,

*(ii)*cohesive, soil.

(iii) Cohesive-frictional soil.

Case(i) Cohesion less soil: In Fig.5.6 OA is the failure envelope for a cohesion less soil, defined by the equation.

$$\tau_f = \sigma \tan\varphi$$

OB represents the locus of the stress components (σ,τ) Acting on the critical surface CD (Fig.5.6)for various values of z.For a given slope i, both σ and τ vary with z, but their ratio

$$\frac{\sigma}{\tau} = \frac{\cos i}{\sin i} = \cot i = \text{constant}$$

The line OB, drawn at inclination I with the σ-axis, therefore, represents the equation

$$\sigma = \tau \cot i$$

$$\tau = \sigma \tan i$$

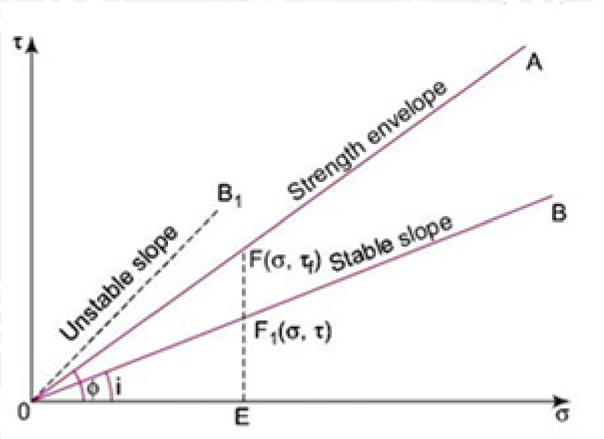

Fig 5.7 stability of slope

For a given value of normal stress σ failure will not occur so long as **τ** is smaller than τ_f ,i.e., so long as i is less than φ.In the limiting caseof stability ,the angle of slope is referred to as the angle of repose.The factor of safety against sliding is given by,

$$F = \frac{\tau_f}{\tau} = \frac{tan\varphi}{\tan i} --- (4)$$

Submerged slope: If the slope is submerged, the bulk unit weight γ should be replaced by submerged unit weight γ' and σ and τ should be calculated using γ'. Also φ should be determined corresponding to submerged condition.

$$\sigma = \gamma' zcos^2 i; \;\; \tau = \gamma' zcosisini$$

$$F = \frac{\tau_f}{\tau} = \frac{\gamma' zcos^2 i \tan\varphi}{\gamma' zcosisini} = \frac{\tan\varphi}{\tan i}$$

This shows that factor safety of a submerged slope is the same as that in dry state.

Steady seepage along the slope:

If the infinite slope is subjected to steady seepage parallel to the slope, the weight W of the prism should be taken corresponding to saturated weight of the soil. Thus

W=W $_{sat}$. z. b cos i

$$\sigma_z = \frac{W}{b} = \gamma_{sat} zcosi$$

$$\sigma = \sigma_z cosi = \gamma_{sat} zcos^2 i ------(i)$$

$$\tau = \sigma_z \sin i = \gamma_{sat} zcosisin\ i ------(ii)$$

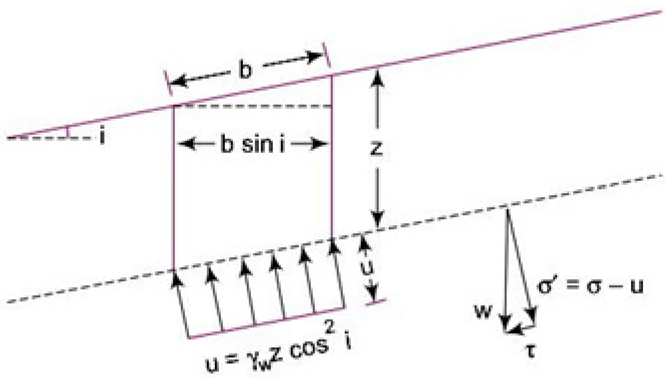

Fig 5.8 steady seepage along infinite slope

In addition to these, there is an upward force u due to seeping water is given by

$$u = \gamma_w zcos^2 i ------(iii)$$

Hence the effective normal stress,

$$\sigma' = \sigma - u = \gamma_{sat} zcos^2 i - \gamma_w zcos^2 i$$

$$= \gamma' zcos^2 i ----- (iv)$$

Hence, $F = \frac{\tau_f}{\tau} = \frac{\sigma' tan\varphi}{\tau} = \frac{\gamma' zcos^2 i tan\varphi}{\gamma_{sat} zcosi \sin i} = \frac{\gamma' tan\varphi}{\gamma_{sat} zcosisin\, i} = \frac{\gamma' tan\varphi}{\gamma_{sat} \tan i} - - - - - - (5)$

Since γ'/γ_{sat} is nearly half, the factor of safety is reduced to nearly one half when there is seepage parallel to the slope.

Case*(ii)*:Cohesivesoil:InFig.5.8,*DA* is strength envelope defined by the equation

$$\tau_f = C + \sigma tan\varphi - - - - - (6)$$

If the slope angle is equal to or less than φ, represented by line *OB* ,no critical state of stress is reached and the slope will be stable. If a line *OF,* at a slope **i>φ** is drawn, it will cut the strength envelope at some point *F,*and a state of incipient failure is reached because the shear stress corresponding to the depth represented by point *F* equals to shear strength τ_f or any depth z less than that represented by point *F,* the shear stress-τ is less than the shear strength τ_f and the slope remains stable. For example ,the

Depth *z* corresponding to point $C_1$is stable for the slope of angle **i>φ**.Hence, if **I** is greater than φ,the slope can be stable only up to a limited depth known as the critical depth *H*,.

The factor of safety against failure, for any depth *z* corresponding to point C_1 of the slope angle **i>φ**, is given by

$$F = \frac{\tau_f}{\tau} = \frac{c + \sigma tan\varphi}{\tau}$$

Putting $\sigma=\gamma z\cos^2 i$ and $\tau=\gamma z\cos i \sin i$

$$F = \frac{c + \gamma z\cos^2 i tan\varphi}{\gamma z\cos i \sin i} - - - - (7)$$

$$F = \frac{c}{\gamma z\cos i \sin i} + \frac{tan\varphi}{tan i} - - - - - (8)$$

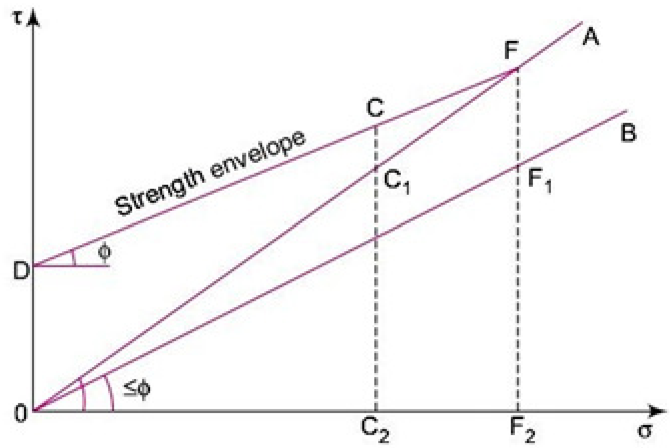

Fig 5.9 failure condition of infinite slope of cohesive soil

For the critical depth z=H_c corresponding to point F,τ_f equals τ(i.e,F=1).Hence we get from eqn(7),

$$\gamma H_c \cos i \sin i = c + \gamma H_c \cos^2 i \tan\varphi$$

$$\gamma H_c \cos i \sin i = c + \gamma H_c \cos^2 i \tan\varphi$$

or

$$H_c = \frac{c}{\gamma} \frac{1}{(tani - tan\varphi)\cos^2 i} ----(9)$$

Eqn (9) indicates for that the given values of I and φ,H_c is proportional to cohesion.

$$(tani - tan\varphi)\cos^2 i = \frac{c}{\gamma}\frac{1}{H_c} ------(10)$$

The dimensional quantity $\frac{c}{\gamma H_c}$ is called the stability S_n

$$S_n = \frac{c}{\gamma H_c} ------(11)$$

Let F_c represents the FOS with respect to cohesion and let C_m be the modified cohesion,at depth H is given by$C_m = \frac{C}{F_c} ------(12)$

The stability number is then written as

$$S_n = \frac{c}{\gamma H_c} = \frac{C_m}{\gamma H} = \frac{C}{F_c \gamma H} = (tani - tan\varphi)\cos^2 i - - - - (13)$$

Eqn (10) &Eqn(13) we get

$$F_c = \frac{H_c}{H} - - - - - - (14)$$

Thus the factor of safety F_c with respect to cohesion, also represents the factor of safety with respect to height. It is based on the assumption that the frictional resistance of the soil is fully developed. However,the true factor of safety is different from F_c and is equal to both cohesion and friction.

Submerged slope:If the slope is submerged ,γ should be replaced by γ';also c and φ should be determined corresponding to submerged condition.

$$F = \frac{C + \gamma' z cos^2 i\, tan\varphi}{\gamma' z cosi \sin i} - - - - (8a)$$

$$H_c = \frac{c}{\gamma'} \frac{1}{(tani - tan\varphi)\cos^2 i} = \frac{c}{\gamma'} \frac{\sec^2 i}{tani - tan\varphi} - - - - (9a)$$

Steady seepage along the slope: As in the case of non cohesive soil ,τ should be taken with respect to saturated weight(γ_{sat})while σ should be completed with respect to the submerged weight .Hence eqn (8) is modified as

$$F = \frac{C + \gamma' z cos^2 i\, tan\varphi}{\gamma_{sat} z cosi \sin i} = \frac{C}{\gamma_{sat} z cosi \sin i} + \frac{\gamma'}{\gamma_{sat}} \frac{tan\varphi}{tani}$$

For the critical height z=H_c corresponding to F=1 $\gamma_{sat} H_c cosi \sin i = C + \gamma' H_c cos^2 i\, tan\varphi$

$$H_c = \frac{C}{(\gamma_{sat} tani - \gamma' tan\varphi) cos^2 i}$$

$$= \frac{c}{\gamma_{sat}\left\{tani - \frac{\gamma'}{\gamma_{sat}} tan\varphi\right\}\cos^2 i} - - - - (9b)$$

Problems:

1. A long natural slope of cohesion less soil is inclined at 12° to the horizontal. Taking φ=30°, determine the factor of safety of the slope. If the slope is completely submerged, what will be change in the factor of safety?

Solution.

$$F = \frac{\tau_c}{\tau} = \frac{\tan\varphi}{\tan i}$$

φ=30°,i=12°

$$F = \frac{\tan 30}{\tan 12} = 2.72$$

When the slope is submerged γ is replaced by γ'

$$F = \frac{\tau_c}{\tau} = \frac{\gamma' z \cos^2 i \tan\varphi}{\gamma' z \cos i \sin i} = \frac{\tan\varphi}{\tan i}$$

$$F = \frac{\tan 30}{\tan 12} = 2.72$$

Thus the factor of safety remains same

2.A long natural slope of sandy soil (φ=25°) is inclined at 100 to the horizontal .The water table is at the surface and the seepage is parallel to the slope. If the saturated unit weight of the soil is19.5kN/m^3 determine the factor of safety of the slope.

$$F = \frac{\gamma' \tan\varphi}{\gamma_{sat} \tan i}$$

$$= \frac{(19.5 - 9.81)\tan 25°}{19.5 \tan 10°}$$

$$= 1.31$$

3.A long natural slope in a c-φ soil is inclined at 12° to the horizontal. The water table is at the surface and the seepage is parallel to the slope. If a plane slip has developed at a depth of 4m.determine the factor of safety.

$$\text{Take } c = \frac{8KN}{m^2}, \varphi = 22° \text{ and } \gamma_{sat} = 19KN/m^3$$

Solution:

$$F = \frac{c + \gamma' z \cos^2 i \tan\varphi}{\gamma_{sat} z \cos i \sin i}$$

$$= \frac{8 + (19 - 9.81)x1 \cos^2 12° \tan 22°}{19x4\cos 12° \sin 12°}$$

$$F = 1.44$$

caseiii)Infinite slope in cohesive frictional soil

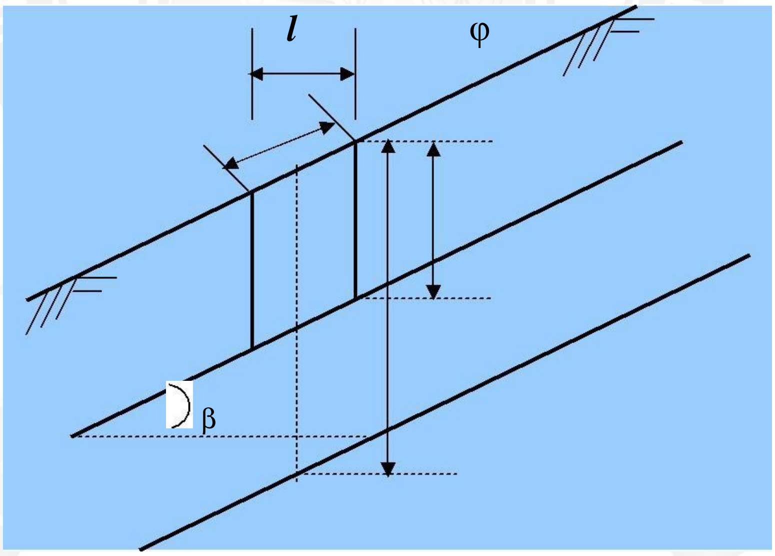

Fig 5.10: Infinite slope in c-φ soil

Consider an infinite slope in c-φ soil as shown with slope angle β

The strength envelope for the c-φ soil is $\tau_f = c + \sigma_{nf} \tan\varphi$. If the slope angle β is less then φ, slope will be stable for any depth.

When the slope angle β>φ the slope will be stable upto a depth Z=Zc corresponding to point P. The point P corresponds to the depth at which the shear stress mobilized will be equal to the available shear strength.

For all depths less than that represented by point P shearing stress will be less than the shear strength and the slope will be stable

$$\tau = c + \sigma \tan \phi$$

$$\tau_f = c + \sigma \tan \phi$$

At P:

$$\sigma_{nf} = \gamma Z_c \operatorname{Cos}^2 \beta$$

$$\tau_f = c + \gamma Z_c \operatorname{Cos}^2\beta \tan\phi \quad \text{----- 1}$$

the developed shear stress is

$$\tau_d = \gamma Z_C \operatorname{Sin}\beta \operatorname{Cos}\beta \quad \text{----- 2}$$

Equating 1 and 2

$$\gamma Z_c \operatorname{Sin}\beta \operatorname{Cos}\beta = c + \gamma Z_c \operatorname{Cos}^2 \beta \tan\phi$$

$$\gamma Z_c (\operatorname{Sin}\beta \operatorname{Cos}\beta - \operatorname{Cos}^2 \beta \tan\phi) = c$$

$$\gamma Z_c = \frac{c}{(\sin\beta\cos\beta - \cos^2 \beta\tan\varphi)}$$

$$\gamma z_c \cos^2 \beta(\tan\beta - \tan\varphi) = c$$

$$Z_c = \frac{C}{\gamma.\cos^2 \beta(\tan\beta - \tan\varphi)}$$

Therefore the critical depth Z_C is proportional to cohesion for a given value ofslope angle (β) and friction angle (β)

Therefore$\frac{c}{\gamma z} = \cos^2 \beta(tan\beta - tan\varphi)$

The term$\frac{c}{\gamma z_c} = S_n$ Stability number (Sn)

For all depth $Z<Z_c$

$$\mathrm{FS} = \frac{\text{shearing strength}}{\text{shearing stress}} = \frac{c + \gamma z \cos^2\beta \tan\varphi}{\gamma z \sin\beta \cos\beta}$$