3.2 IMPEDANCE MATCHING BY STUBS

In microwave and radio-frequency engineering, a stub is a length of transmission line or waveguide that is connected at one end only. The free end of the stub is either left open-circuit or (especially in the case of waveguides) short-circuited. Neglecting transmission line losses, the input impedance of the stub is purely reactive; either capacitive or inductive, depending on the electrical length of the stub, and on whether it is open or short circuit. In Fig 3.2.1 stubs may thus be considered to be frequency-dependent capacitors and frequency-dependent inductors.

Because stubs take on reactive properties as a function of their electrical length, stubs are most common in UHF or microwave circuits where the line lengths are more manageable. Stubs are commonly used in antenna impedance matching circuits and frequency selective filters.

Smith charts can also be used to determine what length line to use to obtain a desired reactance.

Stub matching

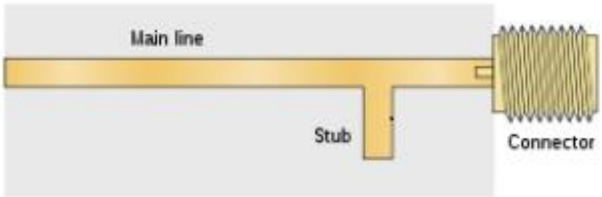

Fig: 3.2.1 Stub matching

Source: John D Ryder, ―Networks, lines and fields‖, 2nd Edition, Prentice Hall India, 2015

In a strip line circuit, a stub may be placed just before an output connector to compensate for small mismatches due to the device's output load or the connector itself.

Stubs can be used to match a load impedance to the transmission line characteristic impedance. The stub is positioned a distance from the load. This distance is chosen so that at that point the resistive part of the load impedance is made equal to the resistive part of the characteristic impedance by impedance transformer action of the length of the main line. The length of the stub is chosen so that it exactly cancels the reactive part of the presented impedance. That is, the stub is made capacitive or inductive according to whether the main line is presenting an inductive or capacitive impedance respectively. This is not the same as the actual impedance of the load since the reactive part of the load impedance will be subject to impedance transformer action as well as the resistive part.

In the method of impendence matching using stub, an open or closed stub line of suitable length is used as a reactance shunted across the transmission line at a designated distance from the load ,to tune the length of the line and the load to resonance with an anti resonant resistance equal to Ro. Matching stubs can be made adjustable so that matching can be corrected on test.

Single stub will only achieve a perfect match at one specific frequency. For wideband matching several stubs may be used spaced along the main transmission line. The resulting structure is filter-like and filter design techniques are applied. For instance, the matching network may be designed as a Chebyshev filter but is optimized for impedance matching instead of passband transmission. The resulting transmission function of the network has a passband ripple like the Chebyshev filter, but the ripples never reach 0dB insertion loss at any point in the passband, as they would do for the standard filter.

SINGLE STUB MATCHING:

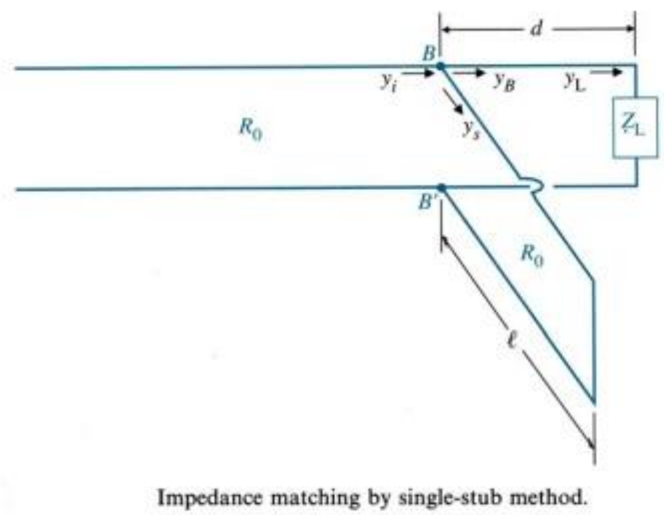

Fig: 3.2.2 Location of single stub for impedance matching
Source: John D Ryder, ―Networks, lines and fields‖, 2nd Edition, Prentice Hall India, 2015

The load should be matched to the characteristic impedance of the line so that as much power as possible is transmitted from the generator to the load for radio-frequency power transmission.

The lines should be matched because reflections from mismatched loads and junctions will result in echoes and will distort the information-carrying signal for information transmission.

Short-circuited (instead of open-circuited) stubs are used for impedance-matching on transmission lines is shown in Fig 3.2.2.

Single-stub method for impedance matching : an arbitrary load impedance can be matched to a transmission line by placing a single short-circuited stub in parallel with the line at a suitable location

When a high frequency line terminated in its characteristic impedance it is operated as a smooth line under such conditions the reflections are absent.

Hence we get maximum power delivered to the load efficiency.

But in practice load antennas does not produce resistance equal to $\boldsymbol{R_O}$.

For example, we use quarter wave line for impedance matching technique.

the input impedance Y is looking towards the load from any point on the transmission line is given by,

$\boldsymbol{Y_S = G_O + j\,B}$ **…….(A)**

$\boldsymbol{G_O}$ - Conductance

$\boldsymbol{Y_S}$ - Susceptance

These may be the admittance at point A before the stub connected.

The point A located such that at which $\boldsymbol{G_O = \frac{1}{R_O}}$

Then at point A a short stub line is connected and this line is selected such that its input susceptance is $\mp$ **jB.**

This stub is connected across the transmission line. The total admittance and and i/p can be written as,

$\boldsymbol{Y_S = G_O \pm j\,B \mp jB}$

$\boldsymbol{Y_S = G_O} = \frac{1}{R_O}$

$\boldsymbol{Z_S} = \frac{1}{Y_S}$

$\boldsymbol{R_O} = \frac{1}{G_O}$

The input impedance is given by,

$$z_{in} = z_s = R_O \left[\frac{1+|K| \angle\emptyset-2\beta s}{1-|K| \angle\emptyset-2\beta s}\right] \qquad \ldots\ldots(1)$$

The i/p admittance is given by,

$$Y_s = \frac{1}{z_s} = \frac{1}{R_O}\left[\frac{1-|K| \angle\emptyset-2\beta s}{1+|K| \angle\emptyset-2\beta s}\right] \qquad \ldots\ldots(2)$$

$$Y_s = G_O\left[\frac{1-K\cos(\emptyset-2\beta s)-\mathrm{j\,k}\sin(\emptyset-2\beta s)}{1+K\cos(\emptyset-2\beta s)+\mathrm{j\,k}\sin(\emptyset-2\beta s)}\right]$$

$$Y_s = G_O\left[\frac{[1-K\cos(\emptyset-2\beta s)]-\mathrm{j\,k}\sin(\emptyset-2\beta s)}{[1+K\cos(\emptyset-2\beta s)]+\mathrm{j\,k}\sin(\emptyset-2\beta s)}\right]$$

$$Y_s = G_O\left[\frac{[1-K\cos(\emptyset-2\beta s)]-\mathrm{j\,k}\sin(\emptyset-2\beta s)}{[1+K\cos(\emptyset-2\beta s)]+\mathrm{j\,k}\sin(\emptyset-2\beta s)}\right] \mathrm{x} \left[\frac{[1+K\cos(\emptyset-2\beta s)]-\mathrm{j\,k}\sin(\emptyset-2\beta s)}{[1+K\cos(\emptyset-2\beta s)]-\mathrm{j\,k}\sin(\emptyset-2\beta s)}\right]$$

$$Y_s = G_O\left[\frac{\ldots.}{[1+K\cos(\emptyset-2\beta s)]^2+[\mathrm{k}\sin(\emptyset-2\beta s)]^2}\right]$$

$\ldots.. \quad = \quad [1+ K\cos(\emptyset-2\beta s) - \mathrm{j\,k}\sin(\emptyset-2\beta s) - K\cos(\emptyset-2\beta s) - K^2cos^2(\emptyset-2\beta s) + jK^2\cos(\emptyset-2\beta s)\sin(\emptyset-2\beta s)$- $\mathrm{j\,k}\sin(\emptyset-2\beta s) - \mathrm{j}\,K^2\cos(\emptyset-2\beta s)\sin(\emptyset-2\beta s)$- $K^2sin^2(\emptyset-2\beta s)$]

$$Y_s = G_O\left[\frac{1-\mathrm{j2\,k}\sin(\emptyset-2\beta s)-K^2}{1+2K\cos(\emptyset-2\beta s)+K^2cos^2(\emptyset-2\beta s)+K^2sin^2(\emptyset-2\beta s)}\right]$$

$$Y_s = G_O\left[\frac{1-K^2-\mathrm{j2\,k}\sin(\emptyset-2\beta s)}{1+2K\cos(\emptyset-2\beta s)+K^2}\right]$$

We know that,

$$\boldsymbol{Y_S} = \boldsymbol{G_S} + \mathbf{j}\,\boldsymbol{B_S}$$

$$\boldsymbol{G_S} + \mathbf{j}\,\boldsymbol{B_S} = G_O\left[\frac{1-K^2}{1+2K\cos(\emptyset-2\beta s)+K^2} - \frac{\mathrm{j2\,k}\sin(\emptyset-2\beta s)}{1+2K\cos(\emptyset-2\beta s)+K^2}\right]$$

$$\frac{\boldsymbol{G_S}+\mathbf{j}\,\boldsymbol{B_S}}{G_O} = \left[\frac{1-K^2}{1+2K\cos(\emptyset-2\beta s)+K^2} - \frac{\mathrm{j2\,k}\sin(\emptyset-2\beta s)}{1+2K\cos(\emptyset-2\beta s)+K^2}\right]$$

$$\frac{\boldsymbol{G_S}}{G_O} + \mathrm{j}\,\frac{\boldsymbol{B_S}}{G_O} = \left[\frac{1-K^2}{1+2K\cos(\emptyset-2\beta s)+K^2} - \frac{\mathrm{j2\,k}\sin(\emptyset-2\beta s)}{1+2K\cos(\emptyset-2\beta s)+K^2}\right]$$

$$\frac{\boldsymbol{G_S}}{G_O} = \frac{1-K^2}{1+2K\cos(\emptyset-2\beta s)+K^2} \qquad \ldots\ldots(3)$$

$$\frac{\boldsymbol{B_S}}{G_O} = \frac{-2\,\mathrm{k}\sin(\emptyset-2\beta s)}{1+2K\cos(\emptyset-2\beta s)+K^2} \qquad \ldots\ldots(4)$$

From the values of $\frac{\boldsymbol{G_S}}{G_O}$ and $\frac{\boldsymbol{B_S}}{G_O}$ we can relate this value to the stub length and the point at which the stub is to be connected.

From the plot shown in figure it is absorbed that the value of $\frac{\boldsymbol{G_S}}{G_O}$ is maximum if the value cosine term in the expression in negative i.e,

$\emptyset - 2\beta s$ = -π

$2\beta s_2$ = ∅ + π

$s_2 = \frac{\emptyset + \pi}{2\beta}$

At distance s_2 the maximum value of $\frac{\boldsymbol{G_S}}{G_O}$ is,

$$\left(\frac{\boldsymbol{G_S}}{G_O}\right)_{max} = \frac{1 - K^2}{1 + 2K\cos(\emptyset - 2\beta s) + K^2}$$

$$\left(\frac{\boldsymbol{G_S}}{G_O}\right)_{max} = \frac{1 - K^2}{1 + 2K\cos(-\pi) + K^2}$$

$$\left(\frac{\boldsymbol{G_S}}{G_O}\right)_{max} = \frac{1 - K^2}{1 - 2K + K^2}$$

$$\left(\frac{\boldsymbol{G_S}}{G_O}\right)_{max} = \frac{1 - K^2}{(1 - K)^2}$$

$$\left(\frac{\boldsymbol{G_S}}{G_O}\right)_{max} = \frac{(1 - K)(1 + K)}{(1 - K)^2}$$

$$\left(\frac{\boldsymbol{G_S}}{G_O}\right)_{max} = \frac{1 + K}{1 - K}$$

$$\left(\frac{\boldsymbol{G_S}}{G_O}\right)_{max} = S$$

$\boldsymbol{G_S} = G_O$ S

$\frac{1}{R_s} = \frac{1}{R_O}$. S

$R_s = \frac{R_O}{S}$

Thus at the point s_2 the point impedance R_s is resistive and its value is minimum voltage at distance s_2 from the load.

At distance s_1 from the load ,

$\boldsymbol{G_S} = G_O$

This is the point at which the stub is to be connected where the value of $\frac{G_S}{G_O}$ is unity.

$\frac{G_S}{G_O} = 1$

From equ (3),

$1 = \frac{1-K^2}{1+2K\cos(\emptyset-2\beta s_1)+K^2}$

$1 + 2K\cos(\emptyset - 2\beta s_1) + K^2 = 1 - K^2$

$2K\cos(\emptyset - 2\beta s_1) = 1 - K^2 - 1 - K^2$

$2K\cos(\emptyset - 2\beta s_1) = -2K^2$

$\cos(\emptyset - 2\beta s_1) = \frac{-2K^2}{2K}$

$\cos(\emptyset - 2\beta s_1) = -k$

$\emptyset - 2\beta s_1 = cos^{-1}(-K)$

$\emptyset - 2\beta s_1 = -\pi \pm cos^{-1}(K)$

$\emptyset + \pi \pm cos^{-1}(K) = 2\beta s_1$

$s_1 = \frac{1}{2\beta}\left(\emptyset + \pi \pm cos^{-1}(K)\right)$

$\beta = \frac{2\pi}{\lambda}$

Sub β value in s_1

$s_1 = \frac{\lambda}{4\pi}\left(\emptyset + \pi \pm cos^{-1}(K)\right)$

The distance ‘d’ from the voltage minimum to the point of stub connection is given by,

$d = s_2 - s_1$

$d = \frac{\emptyset+\pi}{2\beta} - \frac{\emptyset+\pi \pm cos^{-1}(K)}{2\beta}$

$d = \frac{\emptyset+\pi-\emptyset-\pi \pm cos^{-1}(K)}{2\beta}$

$d = \frac{\pm cos^{-1}(K)}{2\beta}$

we know that,

K = $\frac{S-1}{S+1}$, $\beta = \frac{2\pi}{\lambda}$

Sub these values in above equ,

d = $\frac{\pm cos^{-1}\left(\frac{S-1}{S+1}\right)}{\frac{4\pi}{\lambda}}$

d = $\left[\frac{\pm cos^{-1}\left(\frac{S-1}{S+1}\right)}{\pi}\left(\frac{\lambda}{4}\right)\right]$

hence the stub mat be located at distance 'd' measured in either direction from voltage minimum but for better performance the stub is placed on the load side of the voltage minimum which is nearest to the load.

To calculate the input susceptance of the line at s distance 's_1' ,

From equ (4),

$\frac{\boldsymbol{B_S}}{G_O} = \frac{-2\,\mathrm{k}\sin(\emptyset-2\beta s_1)}{1+2K\cos(\emptyset-2\beta s_1)+K^2}$

We know that,

$s_1 = \frac{1}{2\beta}\left(\emptyset+\pi \pm cos^{-1}(K)\right)$

$\cos(\emptyset-2\beta s_1) = \cos\left(\emptyset-2\beta\ X\ \frac{\emptyset+\pi \pm cos^{-1}(K)}{2\beta}\right)$

$= \cos(\emptyset-\emptyset-\pi \pm cos^{-1}(K))$

$= \cos(-\pi \pm cos^{-1}(K))$

$= \cos(cos^{-1}(-K))$

$\cos(\emptyset-2\beta s_1) = -K$

$\sin(\emptyset-2\beta s_1) = \sin\left(\emptyset-2\beta\ X\ \frac{\emptyset+\pi \pm cos^{-1}(K)}{2\beta}\right)$

$= \sin(\emptyset-\emptyset-\pi \pm cos^{-1}(K))$

$= \sin(-\pi \pm cos^{-1}(K))$

$= \sin(cos^{-1}(-K))$

Let $cos^{-1}(-K) = \theta$

$-K = \cos\theta$

$\sin(\emptyset-2\beta s_1) = \sin(cos^{-1}(\cos\theta))$

$\sin(\emptyset - 2\beta s_1) = \sin\theta$

[$sin^2\theta + cos^2\theta = 1$

$sin^2\theta = 1 - cos^2\theta$

$\sin\theta = \pm\sqrt{1 - cos^2\theta}$]

$$= \pm\sqrt{1 - cos^2\theta}$$

$\sin(\emptyset - 2\beta s_1) = \pm\sqrt{1 - K^2}$

$\frac{\boldsymbol{B_S}}{G_O} = \pm\frac{2k\sqrt{1-K^2}}{1+2K(-K)+K^2}$

$\frac{\boldsymbol{B_S}}{G_O} = \pm\frac{2k\sqrt{1-K^2}}{1-2K^2+K^2}$

$\frac{\boldsymbol{B_S}}{G_O} = \pm\frac{2k\sqrt{1-K^2}}{1-K^2}$

$\frac{\boldsymbol{B_S}}{G_O} = \pm\frac{2k}{\sqrt{1-K^2}}$

$\boldsymbol{B_S} = \pm G_O\left(\frac{2k}{\sqrt{1-K^2}}\right)$

$\boldsymbol{B_S} = \pm\frac{2k\,G_O}{\sqrt{1-K^2}}$

The above equation gives susceptance of the line with a distance ‘s’ where the stub is to be connected.

So, this is given as,

$\boldsymbol{B_{Stub}} = \pm\frac{2k\,G_O}{\sqrt{1-K^2}}$

In general, the input impedance of a short circuited line is given by,

$z_{sc} = jR_O\tan\beta s$

Y_{sc} = G + jB

$\frac{1}{Z}$ = Y

$Y_{sc} = \frac{1}{jR_O\tan\beta s}$

G + jB = $\frac{1}{jR_O\tan\beta s}$

B = $\frac{1}{R_O\tan\beta s}$

The stub connected to the transmission line is also a short circuited line with a total length L.

$$\boldsymbol{B_{Stub}} = \frac{1}{R_O \tan\beta L} = \pm \frac{2k\, G_O}{\sqrt{1-K^2}}$$

$$2k\, G_O\, R_O \tan\beta L = \pm\sqrt{1-K^2}$$

$$2k\, G_O \frac{\mathbf{1}}{G_O} \tan\beta L = \pm\sqrt{1-K^2}$$

$$\tan\beta L = \pm\frac{\sqrt{1-K^2}}{2k}$$

$$\beta L = tan^{-1}\left(\pm\frac{\sqrt{1-K^2}}{2k}\right)$$

$$L = \frac{1}{\beta}\, tan^{-1}\left(\pm\frac{\sqrt{1-K^2}}{2k}\right)$$

$$\beta = \frac{2\pi}{\lambda}$$

Sub β value in L,

$$L = \frac{\lambda}{2\pi}\, tan^{-1}\left(\pm\frac{\sqrt{1-K^2}}{2k}\right)$$

We know that,

$$K = \frac{S-1}{S+1}$$

$$\frac{\sqrt{1-K^2}}{2k} = \frac{\sqrt{1-\left(\frac{S-1}{S+1}\right)^2}}{2\left(\frac{S-1}{S+1}\right)}$$

$$= \frac{\sqrt{\frac{(S+1)^2-(S+1)^2}{(S+1)^2}}}{2\left(\frac{S-1}{S+1}\right)}$$

$$= \frac{\sqrt{\frac{S^2+1+2S-S^2-1-2S}{(S+1)^2}}}{2\left(\frac{S-1}{S+1}\right)}$$

$$= \frac{\frac{\sqrt{4S}}{S+1}}{2\left(\frac{S-1}{S+1}\right)}$$

$$= \frac{\sqrt{4S}}{2(S-1)}$$

$$= \frac{2\sqrt{S}}{2(S-1)}$$

$$\frac{\sqrt{1-K^2}}{2k} = \frac{\sqrt{S}}{(S-1)}$$

$$L = \frac{\lambda}{2\pi} \tan^{-1}\left(\pm \frac{\sqrt{S}}{(S-1)}\right)$$

This is the length of the stub which is short circuited.

DOUBLE STUB MATCHING:

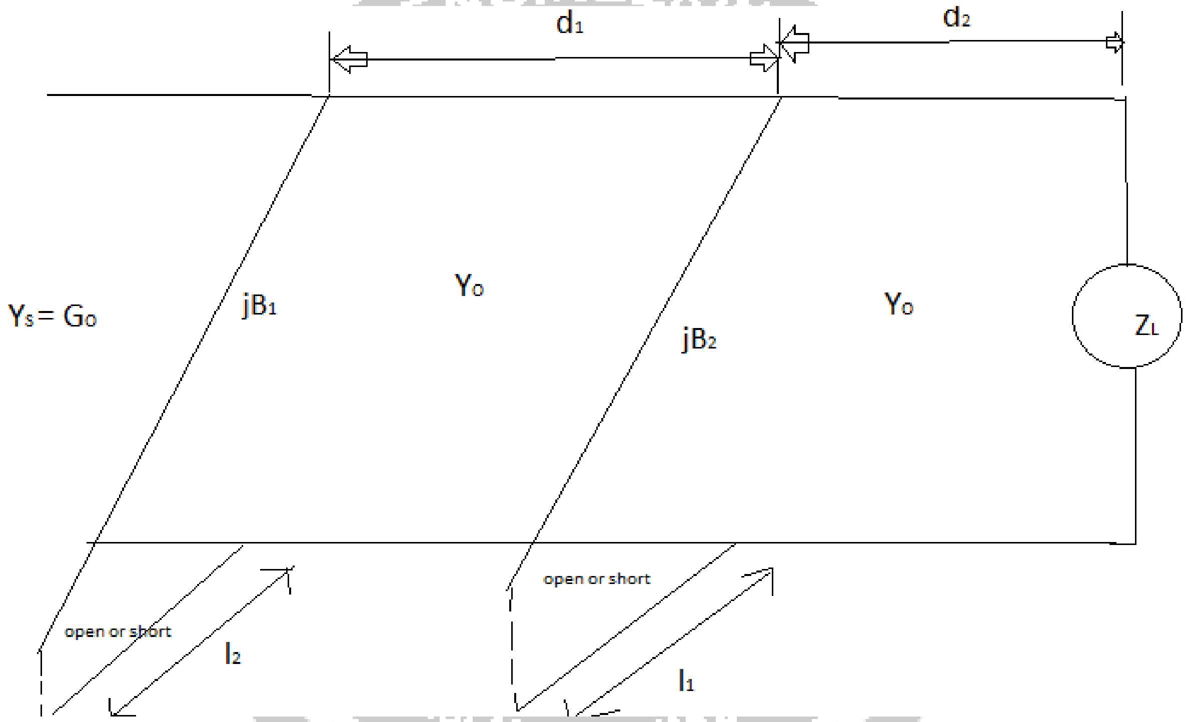

Fig: 3.2.3 Illustrating double stub matching

The Fig 3.2.3 shows that the another possible method of impedance matching is to use two stubs in which the locations of the stub are arbitrary, the two stub lengths furnishing the required adjustments. The spacing is frequently made l/4.This is called double stub matching.

Limitations of single stub matching:

It provides matching at single frequency

The stub must be located at a fixed position on the line.

Not easy to tune.

$Y_S = G_O$

$\frac{Y_S}{G_O}$ = 1 + jb

Y_S = 1 + jb – jb

The input impedance is given by,

$$z_s = R_O \left[\frac{Z_R + jR_O\tan\beta s}{R_O + jZ_R\tan\beta s}\right]$$

$R_O = Z_O$, $z_R = Z_L$ sub in above equ,

$$z_s = Z_O \left[\frac{Z_L + jZ_O\tan\beta s}{Z_O + jZ_L\tan\beta s}\right]$$

$z_s = \frac{1}{Y_S}$, $Z_O = \frac{1}{Y_O}$, $Z_L = \frac{1}{Y_L}$ sub in above equ,

$$\frac{1}{Y_S} = \frac{1}{Y_O}\left[\frac{\frac{1}{Y_L} + j\frac{1}{Y_O}\tan\beta s}{\frac{1}{Y_O} + j\frac{1}{Y_L}\tan\beta s}\right]$$

$$\frac{Y_O}{Y_S} = \frac{\frac{\frac{Y_O}{Y_L} + j\tan\beta s}{Y_O}}{\frac{1 + j\frac{Y_O}{Y_L}\tan\beta s}{Y_O}}$$

$$\frac{Y_O}{Y_S} = \frac{\frac{Y_O}{Y_L} + j\tan\beta s}{1 + j\frac{Y_O}{Y_L}\tan\beta s}$$

$$\frac{Y_O}{Y_S} = y_S$$

$$\frac{Y_O}{Y_L} = y_L$$

$$y_S = \frac{y_L + j\tan\beta s}{1 + jy_L\tan\beta s}$$

$$y_S = \frac{y_L + j\tan\beta s}{1 + jy_L\tan\beta s} \text{ x } \frac{1 - jy_L\tan\beta s}{1 - jy_L\tan\beta s}$$

$$y_S = \frac{y_L + j\tan\beta s - jy_L{}^2\tan\beta s + y_L\tan^2\beta s}{1 + y_L{}^2\tan^2\beta s}$$

$$y_S = \frac{y_L + y_L\tan^2\beta s + j\tan\beta s - jy_L{}^2\tan\beta s}{1 + y_L{}^2\tan^2\beta s}$$

$$y_S = \frac{y_L(1 + \tan^2\beta s) + j\tan\beta s(1 - y_L{}^2)}{1 + y_L{}^2\tan^2\beta s}$$

$$y_S = \frac{y_L(1 + \tan^2\beta s)}{1 + y_L{}^2\tan^2\beta s} + \frac{j\tan\beta s(1 - y_L{}^2)}{1 + y_L{}^2\tan^2\beta s}$$

$$y_S = g_1 + jb_1$$

$y_S{}' = g_1 + jb_1{}'$

$y_S{}'' = y_S = 1 \pm j\, b_2$