## BA4101 - STATISTICS FOR MANAGEMENT

## UNIT I - Introduction to Probability

## Random Experiment:

An experiment whose output is uncertain even though all the outcomes are known.

Example: Tossing a coin, Throwing a fair die, Birth of a baby.

## Sample Space:

The set of all possible outcomes in a random experiment. It is denoted by S .

## Example:

For tossing a fair coin, $S=\{H, T\}$

For throwing a fair die, $S=\{1,2,3,4,5,6$,

For birth of a baby, $S=\{M, F\}$

## Event:

A subset of sample space is event. It is denoted by A .

## Mutually Exclusive Events:

Two events A and B are said to be mutually exclusive events if they do not occur simultaneously. If A and B are mutually exclusive, then $A \cap B=\Phi$

## Example:

Tossing two unbiased coins

$$
S=\{H H, H T, T H, T T\}
$$

(i) Let $A=\{H H\}, B=\{H T\}$

$$
A \cap B=\{H\} \neq \Phi
$$

Then A and B are not mutually exclusive.
(i) Let $A=\{H H\}, B=\{T T\}$

$$
A \cap B=\Phi
$$

Then A and B are mutually exclusive.

### 1.1 Probability:

Probability of an event A is $P(A)=\frac{n(A)}{n(S)}$
i.e. , $P(A)=\frac{\text { number of cases favourable to } A}{\text { Total number of cases }}$

## Axioms of Probability:

(i) $0 \leq P(A) \leq 1$
(ii) $P(S)=1$
(iii) $P(A \cup B)=P(A)+P(B)$, if A and B are mutually exclusive.

Note:
(i) $P(\phi)=0$
(ii) $P(\overline{A)}=1-P(A)$, for any event A
(iii) $P(A \cup B)=P(A)+P(B)-P(A \cap B)$, for any two events $A$ and $B$.

## Independent events:

Two events $A$ and $B$ are said to be independent if occurrence of $A$ does not affect the occurrence of B.

Condition for two events and B are independent:

$$
P(A \cap B)=P(A) P(B)
$$

## Conditional Probability:

If the probability of the event A provided the event B has already occurred is called the conditional probability and is defined as

$$
P(A \mid B)=\frac{P(A \cap B)}{P(B)}, \text { provided } P(B) \neq 0
$$

The probability of an event B provided A has occurred already is given by

$$
P(B \mid A)=\frac{P(A \cap B)}{P(A)}, \text { provided } P(A) \neq 0
$$

## RANDOM VARIABLE

## Define Random Variables:

A random variable is a function that assign a real number for all the outcomes in the sample space of a random experiment.

## Example:

Toss two coins then the sample space $S=\{H H, H T, T H, T T\}$

Now we define a random variable X to denote the number of heads in 2 tosses.
$\mathrm{X}(\mathrm{HH})=2$
$\mathrm{X}(\mathrm{HT})=1$
$X(T H)=1$
$X(T T)=0$

## Types of Random Variables:

(i) Discrete Random Variables
(ii) Continuous Random Variables

## DISCRETE RANDOM VARIABLE

Definition : A discrete random variable is a R.V.X whose possible values consitute finite set of values or countably infinite set of values.

## Probability mass function (PMF):

Let X be discrete random variable. Then $P\left(X=x_{i}\right)=p\left(x_{i}\right)=p_{i}$ is said to be a Probability mass function of $X$, if
(i) $0 \leq p\left(x_{i}\right) \leq 1$
(ii) $\sum_{i} p\left(x_{i}\right)=1$

The collection of pairs $\left\{x_{i}, p_{i}\right\}, i=1,2,3, \ldots$ is called the probability distribution of the random variable X , which is sometimes in the form of a table as given below:

| $X=x_{i}$ | $x_{1}$ | $x_{2}$ | $\cdots$ | $x_{r}$ | $\cdots$ |
| :---: | :---: | :---: | :--- | :---: | :--- |
| $\mathrm{P}\left(X=x_{i}\right)$ | $p_{1}$ | $p_{2}$ | $\cdots$ | $p_{r}$ | $\cdots$ |

## CUMULATIVE DISTRIBUTION FUNCTION

Let X be a R.V. The CDF of X is $F(x)=P(X \leq x)=\sum_{X \leq x} p(x)$
Note : $p\left(x_{i}\right)=P\left(X=x_{i}\right)=F\left(x_{i}\right)-F\left(x_{i-1}\right)$, Where F is the distribution function of the random variable $X$.

## Problems on Discrete Random Variables

## 1. A Discrete Random Variable $X$ has the following probability

distribution

| X | $\mathbf{0}$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| $\mathbf{P}(\mathbf{x})$ | $\mathbf{a}$ | 3a | 5a | 7a | 9a | 11a | 13a | 15a | 17a |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

(i) Find the value of "a".
(ii) Find $P[X<3], P[0<X<3], P[X \geq 3]$
(iii) Find the distribution of $X$.

## Solution:

(i) We know that $\sum P(x)=1$

$$
\begin{aligned}
& \Rightarrow a+3 a+5 a+7 a+9 a+11 a+13 a+15 a+17 a=1 \\
& \quad \Rightarrow 81 a=1 \\
& \quad \Rightarrow a=\frac{1}{81}
\end{aligned}
$$

(ii) $\quad P[X<3]=P[X=0]+P[X=1]+P[X=2]$

$$
\begin{aligned}
& =a+3 a+5 a \\
& =9 a \\
& =\frac{9}{81}
\end{aligned}
$$

$$
\begin{aligned}
P[0<X<3] & =P[X=1]+P[X=2] \\
& =3 a+5 a \\
& =8 a \\
& =\frac{8}{81} \\
P[X \geq 3]= & 1-P[X<3] \\
& =1-\frac{9}{81}=\frac{72}{81}
\end{aligned}
$$

(iii) Distribution of X :

| X | $\mathbf{P}(\mathbf{x})$ | $\mathbf{F}(\mathbf{X})=\mathbf{P}[\mathbf{X} \leq x]$ |
| :---: | :---: | :---: |
| 0 | a | $\mathrm{F}(0)=\mathrm{P}[\mathrm{X} \leq 0]=\frac{1}{81}$ |
| 1 | 3 a | $\mathrm{F}(1)=\mathrm{P}[\mathrm{X} \leq 1]=F(0)+P(1)=\frac{1}{81}+\frac{3}{81}=\frac{4}{81}$ |
| 2 | 5a | $\mathrm{F}(2)=\mathrm{P}[\mathrm{X} \leq 2]=F(1)+P(2)=\frac{4}{81}+\frac{5}{81}=\frac{9}{81}$ |
| 3 | 7 a | $\mathrm{F}(3)=\mathrm{P}[\mathrm{X} \leq 3]=F(2)+P(3)=\frac{9}{81}+\frac{7}{81}=\frac{16}{81}$ |
| 4 | 9a | $\mathrm{F}(4)=\mathrm{P}[\mathrm{X} \leq 4]=F(3)+P(4)=\frac{16}{81}+\frac{9}{81}=\frac{25}{81}$ |
| 5 | 11a | $\mathrm{F}(5)=\mathrm{P}[\mathrm{X} \leq 5]=F(4)+P(5)=\frac{25}{81}+\frac{11}{81}=\frac{36}{81}$ |
| 6 | 13a | $\mathrm{F}(6)=\mathrm{P}[\mathrm{X} \leq 6]=F(5)+P(6)=\frac{36}{81}+\frac{13}{81}=\frac{49}{81}$ |
| 7 | 15a | $\mathrm{F}(7)=\mathrm{P}[\mathrm{X} \leq 7]=F(6)+P(7)=\frac{49}{81}+\frac{15}{81}=\frac{64}{81}$ |
| 8 | 17a | $\mathrm{F}(8)=\mathrm{P}[\mathrm{X} \leq 8]=F(7)+P(8)=\frac{64}{81}+\frac{17}{81}=\frac{81}{81}$ |

2. A Discrete Random Variable $X$ has the following probability distribution

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{P}(\mathrm{x})$ | 0 | k | 2 k | 2 k | 3 k | $\boldsymbol{k}^{2}$ | $2 k^{2}$ | $7 k^{2}+k$ |

(i) Find the value of " $k$ ".
(ii) Find $P[X<6], P[1<X<5], P[X \geq 6], P[X>2]$
(iii) Find $P[1.5<X<4.5 / X>2]$
(iv) Find the distribution of $X$ and find the value of $k$ if $P[X<k]>\frac{1}{2}$

## Solution:

(i) We know that $\sum P(x)=1$

$$
\begin{aligned}
& \Rightarrow 0+k+2 k+2 k+3 k+k^{2}+2 k^{2}+7 k^{2}+k=1 \\
& \Rightarrow 10 k^{2}+9 k=1 \\
& \Rightarrow 10 k^{2}+9 k-1=0 \\
& \Rightarrow(k+1)(10 k-1)=0 \\
& \Rightarrow k=-1(\text { or }) k=\frac{1}{10}
\end{aligned}
$$

(ii) $\quad P[X \geq 6]=P[X=6]+P[X=7]$

$$
\begin{aligned}
& =2 k^{2}+7 k^{2}+k \\
& =9 k^{2}+k \\
& =\frac{9}{100}+\frac{1}{10} \\
& =\frac{19}{100}
\end{aligned}
$$

(iii) $\quad P[X<6]=1-P[X \geq 6]$

$$
\begin{gathered}
=1-\frac{19}{100} \\
=\frac{81}{100}
\end{gathered}
$$

(iv) $\quad P[1<X<5]=P[X=2]+P[X=3]+P[X=4]$

$$
=2 k+2 k+3 k
$$

$$
\begin{aligned}
& =7 k \\
& =\frac{7}{10}
\end{aligned}
$$

(v) $\quad P[1.5<X<4.5 / X>2]=\frac{P[1 \cdot 5<X<4 \cdot 5 \cap X>2]}{P[X>2]}$

$$
=\frac{P[2<X<4 \cdot 5]}{P[X>2]}
$$

$$
\begin{aligned}
& =\frac{P[X=3]+P[X=4]}{P[X>2]} \\
& =\frac{\frac{5}{\frac{10}{7}}}{10} \\
& =\frac{5}{7}
\end{aligned}
$$

## Distribution of X :

| $\mathbf{X}$ | $\mathbf{P}(\mathbf{x})$ | $\mathbf{F}(\mathbf{X})=\mathbf{P}[\mathbf{X} \leq \boldsymbol{x}]$ |
| :---: | :---: | :--- |
| 0 | 0 | $\mathrm{~F}(0)=\mathrm{P}[\mathrm{X} \leq 0]=0$ |
| 1 | k | $\mathrm{F}(1)=\mathrm{P}[\mathrm{X} \leq 1]=F(0)+P(1)=0+\frac{1}{10}=\frac{1}{10}$ |
| 2 | 2 k | $\mathrm{F}(2)=\mathrm{P}[\mathrm{X} \leq 2]=F(1)+P(2)=\frac{1}{10}+\frac{2}{10}=\frac{3}{10}$ |
| 3 | 2 k | $\mathrm{F}(3)=\mathrm{P}[\mathrm{X} \leq 3]=F(2)+P(3)=\frac{3}{10}+\frac{2}{10}=\frac{5}{10}$ |
| 4 | 3 k | $\mathrm{F}(4)=\mathrm{P}[\mathrm{X} \leq 4]=F(3)+P(4)=\frac{5}{10}+\frac{3}{10}=\frac{8}{10}$ |
|  |  |  |


| 5 | $k^{2}$ | $\mathrm{~F}(5)=\mathrm{P}[\mathrm{X} \leq 5]=F(4)+P(5)=\frac{8}{10}+\frac{1}{100}=\frac{81}{100}$ |
| :---: | :---: | :--- |
| 6 | $2 k^{2}$ | $\mathrm{~F}(6)=\mathrm{P}[\mathrm{X} \leq 6]=F(5)+P(6)=\frac{81}{100}+\frac{2}{100}=\frac{83}{100}$ |
| 7 | $7 k^{2}+k$ | $\mathrm{~F}(7)=\mathrm{P}[\mathrm{X} \leq 7]=F(6)+P(7)=\frac{83}{100}+\frac{7}{100}+\frac{1}{10}=\frac{100}{100}$ |

The value of $\mathrm{k}=4$ when $P[X<k]>\frac{1}{2}$
3. If the random variable $X$ takes the values $1,2,3$ and 4 such that $2 P(X=$

1) $=3 P(X=2)=P(X=3)=5 P(X=4)$. Find the probability distribution.

## Solution:

Let $2 P(X=1)=3 P(X=2)=P(X=3)=5 P(X=4)=k$

$$
\begin{aligned}
& \Rightarrow 2 P(X=1)=k \\
& \Rightarrow P(X=1)=\frac{k}{2} \\
& \Rightarrow 3 P(X=2)=k \\
& \Rightarrow P(X=2)=\frac{k}{3} \\
& \Rightarrow P(X=3)=k \\
& \Rightarrow 5 P(X=3)=k \\
& \Rightarrow P(X=3)=\frac{k}{5}
\end{aligned}
$$

We know that $\sum P(x)=1$

$$
\begin{aligned}
& \Rightarrow P(1)+P(2)+P(3)+P(4)=1 \\
& \Rightarrow \frac{k}{2}+\frac{k}{3}+k+\frac{k}{5}=1 \\
& \Rightarrow \frac{15 k+10 k+30 k+6 k}{30}=1 \\
& \Rightarrow \frac{61 k}{30}=1 \Rightarrow k=\frac{30}{61}
\end{aligned}
$$

The Probability Distribution is

| X | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{P}(\mathrm{x})$ | $\frac{k}{2}=\frac{1}{2} \times \frac{30}{61}=\frac{15}{61}$ | $\frac{k}{3}=\frac{1}{3} \times \frac{30}{61}=\frac{10}{61}$ | $k=\frac{30}{61}$ | $\frac{k}{5}=\frac{1}{5} \times \frac{30}{61}=\frac{6}{61}$ |

4. Suppose that the random variable $X$ assumes three values 0,1 and 2 with probabilities $1 / 3,1 / 6$ and $1 / 2$ respectively. Obtain the distribution function of $X$.

## Solution:

| Values of X = x | 0 | 1 | 2 |
| :--- | :--- | :--- | :--- |
| $\mathrm{P}(\mathrm{x})$ | $1 / 3$ | $1 / 6$ | $1 / 2$ |
|  | $\mathrm{P}(0)$ | $\mathrm{P}(1)$ | $\mathrm{P}(2)$ |

The distribution of $\mathbf{X}$

| $\mathbf{X}$ | $\mathbf{P}(\mathbf{x})$ | $\mathbf{F}(\mathbf{X})=\mathbf{P}[\mathbf{X} \leq \boldsymbol{x}]$ |
| :---: | :---: | :--- |
| 0 | 0 | $\mathrm{~F}(0)=\mathrm{P}[\mathrm{X} \leq 0]=\frac{1}{3}$ |
| 1 | k | $\mathrm{F}(1)=\mathrm{P}[\mathrm{X} \leq 1]=F(0)+P(1)=\frac{1}{3}+\frac{1}{6}=\frac{1}{2}$ |
| 2 | 2 k | $\mathrm{F}(2)=\mathrm{P}[\mathrm{X} \leq 2]=F(1)+P(2)=\frac{1}{2}+\frac{1}{2}=1$ |

5. The Probability function of an infinite distribution is given by $\boldsymbol{P}(X=$ $j)=\left(\frac{1}{2}\right)^{j}, j=1,2, \ldots ., \infty$. Verify that the total probability is 1 and find also mean, variance, $P(X$ is even $), P(X$ is divisible by 3$), P(X \geq 5)$

## Solution:

| $x$ | 1 | 2 | 3 | 4 | 5 | $\cdots$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(\mathrm{X}=\mathrm{x})$ | $\frac{1}{2}$ | $\left(\frac{1}{2}\right)^{2}$ | $\left(\frac{1}{2}\right)^{3}$ | $\left(\frac{1}{2}\right)^{4}$ | $\left(\frac{1}{2}\right)^{5}$ | $\ldots .$. |

$$
\begin{array}{r}
\sum p(x)=\frac{1}{2}+\left(\frac{1}{2}\right)^{2}+\left(\frac{1}{2}\right)^{3}+\left(\frac{1}{2}\right)^{4}+\cdots \ldots \\
=\frac{1}{2}\left[1+\frac{1}{2}+\left(\frac{1}{2}\right)^{2}+\cdots \cdot\right]=\frac{1}{2}\left[\left(1-\frac{1}{2}\right)^{-1}\right]=\frac{1 / 2}{1 / 2}=1
\end{array}
$$

$\therefore$ Total Probability is 1

Mean, $E(X)=\sum x p(x)$

$$
\begin{aligned}
& =1 \times \frac{1}{2}+2 \times\left(\frac{1}{2}\right)^{2}+3 \times\left(\frac{1}{2}\right)^{3}+4 \times\left(\frac{1}{2}\right)^{4}+\cdots \cdots \\
& =\frac{1}{2}\left[1+2 \times \frac{1}{2}+3\left(\frac{1}{2}\right)^{2}+\cdots \cdot\right] \\
& =\frac{1}{2}\left[\left(1-\frac{1}{2}\right)^{-2}\right]\left(\because 1+2 x+3 x^{2}+\cdots \cdot=(1-x)^{-2}\right) \\
& =\frac{1 / 2}{1 / 4}=2
\end{aligned}
$$

$$
\therefore E(X)=2
$$

$$
E\left(X^{2}\right)=\sum x^{2} p(x)
$$

$$
=1^{2} \times \frac{1}{2}+2^{2} \times\left(\frac{1}{2}\right)^{2}+3^{2} \times\left(\frac{1}{2}\right)^{3}+4^{2} \times\left(\frac{1}{2}\right)^{4}+\cdots \ldots
$$

$$
=\frac{1}{2}\left[1+4 \times \frac{1}{2}+9\left(\frac{1}{2}\right)^{2}+\cdots .\right]
$$

$$
=\frac{1}{2}\left[\left(1+\frac{1}{2}\right)\left(1-\frac{1}{2}\right)^{-3}\right]\left(\because 1+4 x+9 x^{2}+\cdots=(1+x)(1-x)^{-3}\right)
$$

$$
=\frac{1}{2} \times \frac{3}{2} \times \frac{8}{1}=6
$$

$$
\operatorname{Var}(X)=E\left(X^{2}\right)-[E(X)]^{2}
$$

$=6-4=2$
$P(X$ is even $)=P(X=2)+P(X=4)+P(X=6)+\ldots$

$$
\begin{aligned}
& =\left(\frac{1}{2}\right)^{2}+\left(\frac{1}{2}\right)^{4}+\left(\frac{1}{2}\right)^{6}+\cdots \ldots \\
& =\left(\frac{1}{2}\right)^{2}\left[1+\left(\frac{1}{2}\right)^{2}+\left(\frac{1}{2}\right)^{4}+\cdots \ldots\right] \\
& =\frac{1}{4}\left[\left(1-\frac{1}{4}\right)^{-1}\right]=\frac{1}{4} \times \frac{4}{3} \\
& \mathrm{P}(\mathrm{X} \text { is even })=\frac{1}{3}
\end{aligned}
$$

$P(X$ is divisible by 3$)=P(X=3)+P(X=6)+P(X=9)+\ldots$.

$$
\begin{aligned}
&=\left(\frac{1}{2}\right)^{3}+\left(\frac{1}{2}\right)^{6}+\left(\frac{1}{2}\right)^{9}+\cdots \ldots \\
&=\left(\frac{1}{2}\right)^{3}\left[1+\left(\frac{1}{2}\right)^{3}+\left(\frac{1}{2}\right)^{6}+\cdots \ldots\right]
\end{aligned}
$$

$$
=\frac{1}{8}\left[\left(1-\frac{1}{8}\right)^{-1}\right]=\frac{1}{8} \times \frac{8}{7}
$$

$\mathrm{P}(\mathrm{X}$ is divisible by 3$)=\frac{1}{7}$

$$
\mathrm{P}(\mathrm{X} \geq 5)=\mathrm{P}(\mathrm{X}=5)+\mathrm{P}(\mathrm{X}=6)+\mathrm{P}(\mathrm{X}=7)+\ldots
$$

$$
=\left(\frac{1}{2}\right)^{5}+\left(\frac{1}{2}\right)^{6}+\left(\frac{1}{2}\right)^{7}+\cdots \ldots
$$

$$
=\left(\frac{1}{2}\right)^{5}\left[1+\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^{2}+\cdots \ldots\right]
$$

$$
=\frac{1}{32}\left[\left(1-\frac{1}{2}\right)^{-1}\right]=\frac{1}{32} \times 2=\frac{1}{16}
$$

## MATHEMATICAL EXPECTATION FOR DISCRETE RANDOM

## VARIABLE

Note:
(i) $E(c)=c$
(ii) $\operatorname{Var}(c)=0$
(iii) $E(a X)=a E(X)$
(iv) $E(a X+b)=a E(X)+b$
(v) $\operatorname{Var}(a X)=a^{2} \operatorname{Var}(X)$
(vi) $\operatorname{Var}(a X \pm b)=a^{2} \operatorname{Var}(X)$

## Problems:

1. If $\operatorname{Var}(X)=4$, find $\operatorname{Var}(4 X+5)$, where $X$ is a random variable.

## Solution:

We know that $\operatorname{Var}(a X+b)=a^{2} \operatorname{Var}(X)$

Here $a=4, \operatorname{Var}(X)=4$
$\operatorname{Var}(4 X+5)=4^{2} \operatorname{Var}(X)=16 \times 4=64$
2. Let $X$ be the number on a die when a die is thrown. Find the mean and variance of $X$.

## Solution:

The PMF is given by

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{P}(\mathrm{X}=\mathrm{x})$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ |

Mean,$E(X)=\sum x p(x)$

$$
\begin{aligned}
& =1 \times \frac{1}{6}+2 \times \frac{1}{6}+3 \times \frac{1}{6}+4 \times \frac{1}{6}+5 \times \frac{1}{6}+6 \times \frac{1}{6} \\
& =\frac{21}{6}=3.5 \\
& =E\left(X^{2}\right)=\sum x^{2} p(x) \\
& =1^{2} \times \frac{1}{6}+2^{2} \times \frac{1}{6}+3^{2} \times \frac{1}{6}+4^{2} \times \frac{1}{6}+5^{2} \times \frac{1}{6}+6^{2} \times \frac{1}{6} \\
& \\
& \quad \operatorname{Var}(X)=E\left(X^{2}\right)-[E(X)]^{2} \\
& =15.16-(3.5)^{2}=2.91
\end{aligned}
$$

3. A Fair coin is tossed three times. Let $X$ be the number if tails appearing, Find the probability distribution of $\mathbf{X}$. And also calculate $\mathbf{E}(\mathbf{X})$

Solution:

S = \{ HHH, HTH,HHT,THH,TTH,THT,HTT,TTT $\}$
X denotes the number of tail
$\therefore \mathrm{X}(\mathrm{HHH})=0, \mathrm{X}(\mathrm{HTH})=1, \mathrm{X}(\mathrm{HHT})=1, \mathrm{X}(\mathrm{THH})=1, \mathrm{X}(\mathrm{TTH})=2$,
$\mathrm{X}(\mathrm{HTT})=2, \mathrm{X}(\mathrm{TTT})=3$
$\therefore$ Random variable X takes the value $0,1,2,3$

$$
\begin{gathered}
P(X=0)=P(H H H)=\frac{1}{8} \\
P(X=1)=P(H T H, H H T, T H H)=\frac{3}{8} \\
P(X=2)=P(T T H, T H T, T T H)=\frac{3}{8} \\
P(X=3)=P(T T T)=\frac{1}{8}
\end{gathered}
$$

The probability function of X is

| x | 0 | 1 | 2 | 3 |
| :--- | :---: | :---: | :---: | :---: |
| $\mathrm{P}(\mathrm{X}=\mathrm{x})$ | $\frac{1}{8}$ | $\overline{3}$ | $\overline{3}$ | $\frac{3}{8}$ |

Mean,$E(X)=\sum x p(x)$

$$
\begin{aligned}
& =0 \times \frac{1}{8}+1 \times \frac{3}{8}+2 \times \frac{3}{8}+3 \times \frac{1}{8} \\
& =\frac{12}{8}=1.5
\end{aligned}
$$

## Continuous Random Variable:

If X is a random variable which can take all the values in an interval then X is called continuous random variable.

## Properties of Probability Density Function:

The Probability density function of the random variable X denoted by $f(x)$ has the following properties.
(i) $f(x) \geq 0$
(ii) $\int_{-\infty}^{\infty} f(x) d x=1$

## Cumulative Distribution Function (CDF):

## Properties of CDF:

(i) $F(-\infty)=0$
(ii) $F(\infty)=1$
(iii) $\frac{d}{d x}[F(x)]=f(x)$
(iv) $P(X \leq a)=F(a)$
(v) $P(X>a)=1-F(a)$
(vi) $P(a \leq X \leq b)=F(b)-F(a)$

## Problems on Continuous Random Variables:

1. A continuous random variable $X$ has a density function $f(x)=$ $\frac{K}{1+x^{2}},-\infty \leq X \leq \infty$. i) Find the values of $K$ ii) Cumulative distribution function iii) $\mathbf{P}(\mathbf{X}>0)$ iv) Mean of $\mathbf{X}$.

Solution: Given X is a continuous RV defined in $(-\infty, \infty)$
i) To find of $k$

We know that $\int_{-\infty}^{\infty} f(x) d x=1$

$$
\begin{aligned}
& \Rightarrow \int_{-\infty}^{\infty} \frac{K}{1+x^{2}} d x=1 \\
& \Rightarrow K \int_{-\infty}^{\infty} \frac{d x}{1+x^{2}}=1 \\
& \Rightarrow K \int_{-\infty}^{\infty} \frac{d x}{1+x^{2}}=1
\end{aligned}
$$

$$
\Rightarrow K\left[\tan ^{-1} x\right]_{-\infty}^{\infty}=1
$$

$$
\Rightarrow K\left[\tan ^{-1} \infty-\tan ^{-1}(-\infty)\right]=1
$$

$$
\Rightarrow K\left[\frac{\pi}{2}+\frac{\pi}{2}\right]=1
$$

$$
\Rightarrow K\left[\frac{2 \pi}{2}\right]=1
$$

$$
\Rightarrow K=\frac{1}{\pi}
$$

ii) $\quad \mathrm{CDF}$ of X

$$
\begin{gathered}
F(x)=P(X \leq x)=k \int_{-\infty}^{x} \frac{d x}{1+x^{2}} \\
=\frac{1}{\pi}\left[\tan ^{-1} x\right]_{-\infty}^{x} \\
=\frac{1}{\pi}\left(\tan ^{-1} x-\tan ^{-1}(-\infty)\right) \\
F(x)=\frac{1}{\pi}\left(\tan ^{-1} x+\frac{\pi}{2}\right) ;-\infty<x<\infty
\end{gathered}
$$

iii)

$$
\begin{aligned}
& P(X>0)=\int_{0}^{\infty} f(x) d x=k \int_{0}^{\infty} \frac{d x}{1+x^{2}} \\
& =\frac{1}{\pi}\left[\tan ^{-1} x\right]_{0}^{\infty} \\
& =\frac{1}{\pi}\left(\tan ^{-1} \infty-\tan ^{-1} 0\right)=\frac{1}{2} \\
& \text { iv) Mean of } X, E(X)=\int_{-\infty}^{x} x f(x) d x \\
& E(X)=0 \quad \because \frac{x}{1+x^{2}} \text { is an odd function }
\end{aligned}
$$

2. A Continuous random variable $X$ can assume any value between $X=2$ and $X=5$ has the density function given by $f(x)=$ $k(1+x)$. Find i) $k$ ii) $p[X>4] \quad$ iii) $P[3>X>4]$

## Solution:

Solution: Given X is a continuous RV defined in $(2,5)$
i) To find of k

We know that $\int_{-\infty}^{\infty} f(x) d x=1$

$$
\begin{aligned}
& \Rightarrow \int_{2}^{5} K(1+x) d x=1 \\
& \Rightarrow K \int_{2}^{5}(1+x) d x=1 \\
& \Rightarrow K\left[x+\frac{x^{2}}{2}\right]_{2}^{5}=1 \\
& \Rightarrow K\left[5+\frac{25}{2}-2-2\right]=1 \\
& \Rightarrow K\left[\frac{27}{2}\right]=1 \\
& \Rightarrow K=\frac{2}{27} \\
& \text { ii) } \boldsymbol{p}[\boldsymbol{X}>4]=\int_{4}^{5} f(x) d x \\
& =\boldsymbol{K} \int_{4}^{5}(1+x) d x \\
& =K\left[x+\frac{x^{2}}{2}\right]_{4}^{5} \\
& \Rightarrow K\left[5+\frac{25}{2}-4-8\right]
\end{aligned}
$$

$$
\Rightarrow K\left[\frac{25}{2}-7\right]
$$

$$
=\frac{2}{27}\left[\frac{11}{2}\right]=\frac{11}{27}
$$

$$
\text { iii) } \boldsymbol{P}[3>\boldsymbol{X}>4]=\int_{3}^{4} f(x) d x
$$

$$
\begin{aligned}
=K[x & \left.+\frac{x^{2}}{2}\right]_{3}^{4} \\
& =K\left[4+8-3-\frac{9}{2}\right]
\end{aligned}
$$

$$
=\frac{2}{27}\left[\frac{9}{2}\right]=\frac{1}{3}
$$

$$
P[3>X>4]=\frac{1}{3}
$$

3. If a random variable $X$ has $\operatorname{PDF} f(x)=\left\{\begin{array}{l}\frac{1}{4},|x|<2 \\ 0,|x|>2\end{array}\right.$

$$
\text { Find (i) } P[X<1] \text { ii) } P[|X|>1](i i i) P[2 X+3>5]
$$

## Solution:

(i) $\quad P[X<1]=\int_{-2}^{1} f(x) d x$

$$
\begin{aligned}
& =\int_{-2}^{1} \frac{1}{4} d x \\
& =\frac{1}{4}[x]_{-2}^{1} \\
& =\frac{1}{4}[1-(-2)] \\
& =\frac{3}{4}
\end{aligned}
$$

(ii) $\quad P[|X|>1]=1-P[-1<X<1]$

$$
\begin{aligned}
& =1-\int_{-1}^{1} f(x) d x \\
& =1-\int_{-1}^{1} \frac{1}{4} d x \\
& =1-\frac{1}{4}[x]_{-1}^{1} \\
& =1-\frac{1}{4}[1-(-1)] \\
& =1-\frac{2}{4} \\
& =\frac{2}{4}
\end{aligned}
$$

(iii) $\quad P[2 X+3>5]=P[2 X>5-3]$

$$
\begin{aligned}
& =P\left[X>\frac{5-3}{2}\right] \\
& =P\left[X>\frac{2}{2}\right] \\
& =P[X>1] \\
& =\int_{1}^{2} f(x) d x \\
& =\int_{1}^{2} \frac{1}{4} d x \\
& =\frac{1}{4}[x]_{1}^{2}
\end{aligned}
$$

$$
=\frac{1}{4}[2-(1)]=\frac{1}{4}
$$

## MATHEMATICAL EXPECTATION OF CONTINUOUS RANDOM

## VARIABLES

(i) $E(X)=\int_{-\infty}^{\infty} x f(x) d x$
(ii) $E\left(X^{2}\right)=\int_{-\infty}^{\infty} x^{2} f(x) d x$
(iii) $\operatorname{Var}(X)=E\left(X^{2}\right)-E[(X)]^{2}$

## Problems:

1. Let $X$ be a continuous random variable with probability density function $f(x)=k x(2-x), 0<x<2$. Find (i) $k$ (ii) mean (iii) variance (iv) cumulative distribution function of $\mathbf{X}(\mathbf{v})$ rth moment.

Solution:
(i) To find $k$,

$$
\begin{aligned}
\int_{0}^{2} f(x) d x=1 & \Rightarrow k \int_{0}^{2}\left(2 x-x^{2}\right) d x=1 \\
& \Rightarrow k\left[\frac{2 x^{2}}{2}-\frac{x^{3}}{3}\right]_{0}^{2}=1 \\
& \Rightarrow k\left[4-\frac{8}{3}\right]=1 \\
& \Rightarrow k\left(\frac{4}{3}\right)=1
\end{aligned}
$$

$$
\Rightarrow k=\frac{3}{4}
$$

## (ii) To calculate mean of X

$$
\begin{aligned}
E(X) & =\int_{0}^{2} x f(x) d x \\
& =\int_{0}^{2} x^{2} \frac{3}{4}(2-x) d x \\
& =\frac{3}{4} \int_{0}^{2}\left(2 x^{2}-x^{3}\right) d x \\
& =\frac{3}{4}\left[\frac{2 x^{3}}{3}-\frac{x^{4}}{4}\right]_{0}^{2} \\
& =\frac{3}{4}\left(\frac{16}{3}-4\right) \\
& =\frac{3}{4} \times \frac{4}{3}=1
\end{aligned}
$$

(iii) To calculate variance of $X$

$$
\begin{aligned}
E\left(X^{2}\right)= & \int_{0}^{2} x^{2} f(x) d x \\
& =\int_{0}^{2} x^{3} \frac{3}{4}(2-x) d x \\
& =\frac{3}{4} \int_{0}^{2}\left(2 x^{3}-x^{4}\right) d x \\
& =\frac{3}{4}\left[\frac{2 x^{4}}{4}-\frac{x^{5}}{5}\right]_{0}^{2} \\
& =\frac{3}{4}\left(8-\frac{32}{5}\right)
\end{aligned}
$$

$$
=\frac{3}{4} \times \frac{8}{5}=\frac{6}{5}
$$

$$
\begin{aligned}
\operatorname{Var}(X) & =E\left(X^{2}\right)-E[(X)]^{2} \\
& =\frac{6}{5}-1=\frac{1}{5}
\end{aligned}
$$

## (iv) To calculate CDF of $\mathbf{X}$

$$
\begin{aligned}
F_{X}(x)= & P(X \leq x)=\int_{-\infty}^{x} f(x) d x \\
& =\int_{0}^{x} f(x) d x \\
& =\int_{0}^{x} \frac{3}{4} x(2-x) d x \\
& =\frac{3}{4} \int_{0}^{x}\left(2 x-x^{2}\right) d x \\
& =\frac{3}{4}\left[\frac{2 x^{2}}{2}-\frac{x^{3}}{3}\right]_{0}^{x} \\
& =\frac{3}{4}\left(x^{2}-\frac{x^{3}}{2}\right) \\
= & \frac{1}{4}\left(3 x^{2}-x^{3}\right)
\end{aligned}
$$

$$
F(x)=\left\{\begin{array}{cl}
0 ; & x<0 \\
\frac{1}{4}\left(3 x^{2}-x^{3}\right) ; & 0 \leq x<2 \\
1 ; & x \geq 2
\end{array}\right.
$$

(v) To find the rth moment:

$$
E\left(x^{r}\right)=\int_{-\infty}^{\infty} x^{r} f(x) d x
$$

$$
\begin{aligned}
& =\int_{0}^{2} x^{r} \frac{3}{4} x(2-x) d x \\
& =\frac{3}{4} \int_{0}^{2}\left(2 x^{r+1}-x^{r+2}\right) d x \\
& =\frac{3}{4}\left[\frac{2 x^{r+2}}{r+2}-\frac{x^{r+3}}{r+3}\right]_{0}^{2} \\
& =\frac{3}{4}\left[\left(2 \frac{2^{r+2}}{r+2}-\frac{2^{r+3}}{r+3}\right)-(0-0)\right] \\
& =\frac{3}{4} \times 2^{r} 2^{2}\left[\frac{1}{r+2}-\frac{1}{r+3}\right]
\end{aligned}
$$

$$
=6 \cdot \frac{2^{r}}{(r+2)(r+3)}
$$

2. The probability distribution function of a random variable $X$ is

$$
f(x)=\left\{\begin{array}{cc}
x ; & 0<x<1 \\
2-x ; & 1<x<2 \text { Find the cumulative distribution function } \\
0 ; & x>2
\end{array}\right.
$$

of $\mathbf{X}$.

## Solution:

We know that c.d.f $F(x)=\int_{-\infty}^{x} f(x) d x$
(i) When $0<x<1$

$$
\begin{aligned}
F(x) & =\int_{-\infty}^{0} f(x) d x+\int_{0}^{x} f(x) d x \\
& =0+\int_{0}^{x} x d x \\
& =\left[\frac{x^{2}}{2}\right]_{0}^{x}
\end{aligned}
$$

$$
\begin{aligned}
& =\frac{x^{2}}{2}-0 \\
& =\frac{x^{2}}{2}
\end{aligned}
$$

(ii) When $1<x<2$

$$
\begin{aligned}
F(x) & =\int_{-\infty}^{0} f(x) d x+\int_{0}^{1} f(x) d x+\int_{1}^{x} f(x) d x \\
& =0+\int_{0}^{1} f(x) d x+\int_{1}^{x} x d x \\
& =\left[\frac{x^{2}}{2}\right]_{0}^{1}+\left[2 x-\frac{x^{2}}{2}\right]_{1}^{x} \\
& =\left(\frac{1}{2}-0\right)+\left[\left(2 x-\frac{x^{2}}{2}\right)-\left(2-\frac{1}{2}\right)\right] \\
& =\frac{1}{2}+2 x-\frac{x^{2}}{2}-\frac{3}{2} \\
=2 x & -\frac{x^{2}}{2}-1
\end{aligned}
$$

(iii) When $x>2$

$$
\begin{aligned}
F(x) & =\int_{-\infty}^{0} f(x) d x+\int_{0}^{1} f(x) d x+\int_{1}^{2} f(x) d x+\int_{2}^{x} f(x) d x \\
& =0+\int_{0}^{1} f(x) d x+\int_{1}^{2} f(x) d x+\int_{2}^{x} x d x \\
& =0+\int_{0}^{1} x d x+\int_{1}^{2}(2-x) d x+0 \\
& =\left[\frac{x^{2}}{2}\right]_{0}^{1}+\left[2 x-\frac{x^{2}}{2}\right]_{1}^{2}
\end{aligned}
$$

$$
\begin{aligned}
& =\left(\frac{1}{2}-0\right)+\left[(4-2)-\left(2-\frac{1}{2}\right)\right] \\
& =\frac{1}{2}+2-\frac{3}{2}=1 \\
F(x) & =\left\{\begin{array}{cc}
\frac{x^{2}}{2} ; & 0<x<1 \\
2 x-\frac{x^{2}}{2}-1 ; & 1<x<2 \\
1 ; & x>2
\end{array}\right.
\end{aligned}
$$

## 3. The Cumulative distribution function of a random variable $X$ is given by

$$
F(x)=\left\{\begin{array}{c}
0 ; \quad x<0 \\
x^{2} ; \quad 0 \leq x<\frac{1}{2} \\
1-\frac{3}{25}(3-x)^{2} ; \frac{1}{2} \leq x<3 \\
1, \quad x \geq 3
\end{array}\right.
$$

Find the pdf of $X$ and evaluate $P(|X| \leq 1)$ using both pdf and cdf.

Solution:

Given

$$
F(x)=\left\{\begin{array}{cc}
0 ; & x<0 \\
x^{2} ; & 0 \leq x<\frac{1}{2} \\
1-\frac{3}{25}(3-x)^{2} ; \frac{1}{2} \leq x<3 \\
1, & x \geq 3
\end{array}\right.
$$

$\operatorname{Pdf} \operatorname{id} f(x)=\frac{d}{d x}[F(x)]$

$$
f(x)=\left\{\begin{array}{ccc}
0 ; & & x<0 \\
2 x ; & & 0 \leq x<\frac{1}{2} \\
\frac{6}{25}(3-x) ; & \frac{1}{2} \leq x<3 \\
0, & & x \geq 3
\end{array}\right.
$$

To find $P(|X| \leq 1)$ using cdf:

$$
\begin{aligned}
P(|X| \leq 1) & =P(-1 \leq X \leq 1) \\
& =F(1)-F(-1) \\
& =\left[1-\frac{3}{25}(3-1)^{2}\right]-0 \\
& =1-\frac{12}{25} \\
& =\frac{25-12}{25}=\frac{13}{25}
\end{aligned}
$$

To find $P(|X| \leq 1)$ using pdf:

$$
\begin{aligned}
P(|X| \leq 1) & =P(-1 \leq X \leq 1) \\
& =\int_{-1}^{1} f(x) d x \\
& =\int_{-1}^{0} f(x) d x+\int_{0}^{\frac{1}{2}} f(x) d x+\int_{\frac{1}{2}}^{1} f(x) d x \\
& =0+\int_{0}^{\frac{1}{2}} 2 x d x+\int_{\frac{1}{2}}^{1} \frac{6}{25}(3-x) d x
\end{aligned}
$$

$$
\begin{aligned}
& =2\left(\frac{x^{2}}{2}\right)_{0}^{\frac{1}{2}}+\frac{6}{25}\left[3 x-\frac{x^{2}}{2}\right]_{\frac{1}{2}}^{1} \\
& =\frac{1}{4}+\frac{6}{25}\left[3-\frac{1}{2}-\frac{3}{2}+\frac{1}{8}\right] \\
& =\frac{13}{25}
\end{aligned}
$$

### 1.2 Baye's Theorem

If $B_{1}, B_{2}, \ldots, B_{n}$ be a set of exhaustive and mutually exclusive events associated with a random experiment and D is another event associated with $B_{i}$, then $P\left(D / B_{i}\right)=\frac{P\left(B_{i}\right) \cdot P\left(D / B_{i}\right)}{\sum_{i=1}^{n} P\left(B_{i}\right) P\left(D / B_{i}\right)}$

## State and Prove Bayes Theorem

## (OR)

## State and Prove Theorem of Probability of Causes.

Soln :
Statement

If $B_{1}, B_{2}, \ldots B_{n}$ be a set of exhaustive and mutually exclusive events associated with random experiment and $D$ is another event associated with (or caused )by Bi. Then

$$
P\left(D \mid B_{i}\right)=\frac{P\left(B_{i}\right) \cdot P\left(D / B_{i}\right)}{\sum_{i=1}^{n} P\left(B_{i}\right) P\left(D / B_{i}\right)}
$$

Proof :

$$
P\left(B_{i} \cap D\right)=P\left(B_{i}\right) \cdot P\left(\frac{D}{B_{i}}\right)
$$

$$
\begin{array}{r}
P\left(D \cap B_{i}\right)=P(D) \cdot P\left(\frac{B_{i}}{D}\right) \\
P\left(B_{i} \mid D\right)=\frac{P\left(B_{i}\right) \cdot P\left(D / B_{i}\right)}{P(D)} \ldots \ldots \tag{1}
\end{array}
$$

The inner circle reprosents the events D.D can occur along with $B_{1}, B_{2}, \ldots B_{n}$ that are exhaustive and mutually exclusive
$\therefore D B_{1}, D B_{1}, \ldots . D B_{n}$ are also mutually exclusive such that

$$
D=D B_{1}+D B_{2}+\cdots+D B_{n}
$$

$$
\therefore D=\sum D B_{i}
$$

$$
P[D]=P\left[\Sigma D B_{i}\right]
$$

$$
=\sum P\left[D B_{i}\right]
$$

$$
=\sum P\left[D \cap B_{\mathrm{i}}\right]
$$

$$
P[D]=\sum_{i=1}^{n} P\left(B_{i}\right) \cdot P\left(D / B_{i}\right)
$$

Substitute $P[D]$ in eqn (1)

$$
\text { (1) } \Rightarrow P\left[B_{i} / D\right]=\frac{P\left(B_{i}\right) \cdot P\left(D / B_{i}\right)}{\sum_{i=1}^{n} P\left(B_{\mathrm{i}}\right) P\left(D / B_{i}\right)}
$$

Hence the proof,

## Problem based on Baye's Theorem

1. Four boxes $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ contain fuses. The boxes contain $5000, \mathbf{3 0 0 0}$, 2000 and 1000 fuses respectively. The percentages of fuses in boxes which are defective are $\mathbf{3 \%}, \mathbf{2 \%}, \mathbf{1 \%}$ and $5 \%$ respectively.one fuse in selected at random arbitrarily from one of the boxes. It is found to be defective fuse. Find the probability that it has come from box $D$.
(OR)

Four boxes A,B,C,D contain fuses. Box A contain 5000 fuses , box B contain 3000 fuses, box $C$ contain 2000 fuses and box $D$ contain 1000 fuses. The percentage of fuses in boxes which are defective are $\mathbf{3 \%}, \mathbf{2 \%}, \mathbf{1 \%}$ and $\mathbf{0 . 5 \%}$ respectively. One fuse is select at random from one of the boxes. It is found to be defective fuse. What is the probability that it has come from box $D$.

Soln:

Since selection ratio is not given
Assume selection ratio is $1: 1: 1: 1$

$$
\text { Total }=1+1+1+1=4
$$

$$
P(A)=\frac{1}{4}
$$

$P(B)=\frac{1}{4}$
$P(C)=\frac{1}{4}$
$P(D)=1 / 4$
Let E be the event selecting a defective fuse from any one of the machine

$$
\begin{aligned}
& P(E / A)=3 \%=0.03 \\
& P(E / B)=2 \%=0.02 \\
& P(E / C)=1 \%=0.01 \\
& \begin{aligned}
& P(E / D)=5 \%=0.05 \\
& P(E)=P(A) P(E / A)+P(B) P(E / B)+P(C) P(F / C)+P(D)(F / D) \\
& \quad= \frac{1}{4} \times 0.03+1 / 4 \times 0.02+1 / 4 \times 0.01+1 / 4 \times 0.05 \\
& \quad= 0.0275 \\
& P(D / E)=\frac{P(D) P(E / D)}{P(E)} \\
&=\frac{\frac{1}{4} \times 0.05}{0.0275}=0.4545 \\
&=0.4545
\end{aligned}
\end{aligned}
$$

2. In a bolt Factory, Machines $A, B$ and $C$ manufacture respectively $\mathbf{2 5 \%}, \mathbf{3 5 \%}$ and $\mathbf{4 0 \%}$ of total output . also out of these output of $A, B, C$ are $5,4,2$ percent respectively are defective. A bolt is drawn at random from the total output and it is found to be defective. What is the probability that it was manufactured by the machine $B$ ?

In a company machine $A, B$ and $C$ manufactured bolts, $\mathbf{2 5 \%}, \mathbf{3 5 \%}$ and $\mathbf{4 0 \%}$ of total output . also out of these output of $A, B, C$ are $5,4,2$ percent respectively are defective. A bolt is taken random from the total output and
it is found to be defective. Find the probability that it was manufactured by the machine $\mathbf{B}$ ?

Soln:

Given, $\mathrm{P}\left(E_{1}\right)=\mathrm{P}(\mathrm{A})=25 \%=0.25$

$$
\begin{aligned}
& \mathrm{P}\left(E_{2}\right)=\mathrm{P}(\mathrm{~B})=35 \%=0.35 \\
& \mathrm{P}\left(E_{3}\right)=\mathrm{P}(\mathrm{C})=40 \%=0.40
\end{aligned}
$$

Let $D$ be the event of drawing defective bolt
$P\left(D / E_{1}\right)=5 \%=\frac{5}{100}=0.05$
$P\left(D / E_{2}\right)=4 \%=0.04$
$\left.P\left(D / E_{3}\right)=2 \%\right)=0 \cdot 02$

To find $P\left(E_{2} / D\right)$
By Bayes theorem
$P\left(E_{2} / D\right)=\frac{P\left(E_{2}\right) P\left(D / E_{2}\right)}{P\left(E_{1}\right) P\left(D / E_{1}\right)+P\left(E_{2}\right) P\left(D / E_{2}\right)+P\left(E_{3}\right) P\left(D / E_{3}\right)}$
$=\frac{(0.35)(0.04)}{(0.25)(0.05)+(0.35)(0.04)+(0.4)(0.02)}$
$=\frac{0.014}{0.0345}$
$=0.406$
3. A bag A contains 2 white and 3 red balls and a bag $B$ contains 4 white and 5 red balls. One ball is drawn at random from one of the
bag and is found to be red. Find the Probability that it was drawn from bag B

## (OR)

A box A contains 2 white and 3 red balls and a box $B$ contains 4 white and 5 red balls at random one ball is taking and is found to be red. What is the probability that it was drawn from bag $B$ ?

Soln:
Let $B_{1}$ be the event that the ball is drawn from the bag $A$.

Let $B_{2}$ be the event that the ball is drawn from the bag $B$.
Lat $A$ be the event that the drawn ball is red

$$
\begin{aligned}
& P\left(B_{1}\right)=P\left(B_{2}\right)=\frac{1}{2} \\
& P\left(A / B_{1}\right)=\frac{3 C_{1}}{5 C_{1}}=\frac{3}{5} \\
& P\left(A / B_{2}\right)=\frac{5 C_{1}}{9 C_{1}}=\frac{5}{9}
\end{aligned}
$$

$$
P\left(B_{2} / A\right)=\frac{P\left(B_{2}\right) P\left(A / B_{2}\right)}{P\left(B_{1}\right) P\left(A / B_{1}\right)+P\left(B_{2}\right) P\left(A / B_{2}\right)}
$$

$$
=\frac{\left(\frac{1}{2}\right)\left(\frac{5}{9}\right)}{\left(\frac{1}{2}\right)\left(\frac{3}{5}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{9}\right)}
$$

$$
=\frac{\frac{5}{18}}{\frac{52}{90}}
$$

$$
P\left(B_{2} / A\right)=\frac{25}{52}
$$

