ROHINI COLLEGE OF ENGINEERING & TECHNOLOGY

DEPARTMENT OF MATHEMATICS

UNIT II – FOURIER SERIES

2.4 PERIODIC FUNCTION

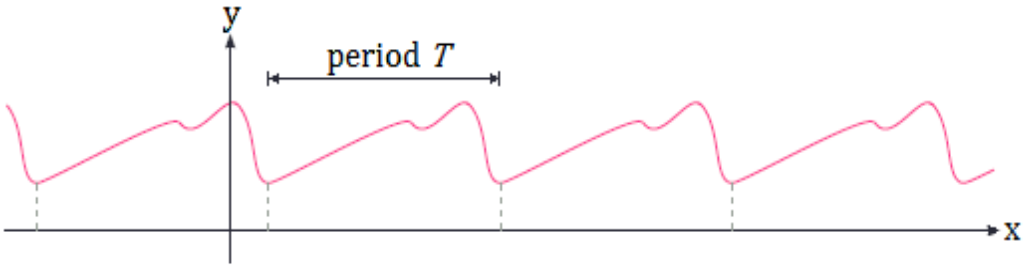
Periodic Functions

- A function $f(x)$ is periodic if there is a positive number T such that for every value of x:

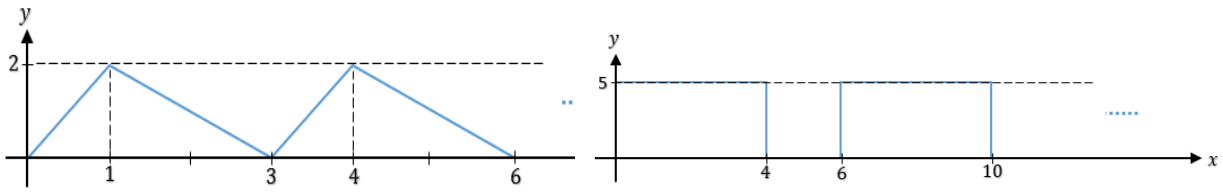
$$\boxed{f(x+T) = f(x)}$$

The smallest such value of $T > 0$ is called the fundamental period or simply the period.

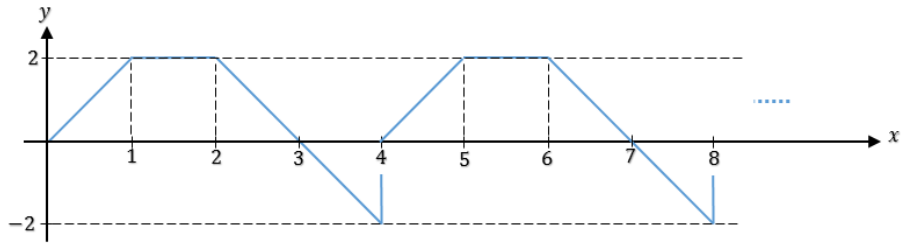
- If we know what the graph looks like in an interval of length T, then we can use replication to sketch the entire graph.



Problem 1: What is the fundamental period of the following function?



(a) (b)



(c)

Solution:

a) The period is 3

b) The period is 6

c) The period is 4

Period of Trigonometric Functions

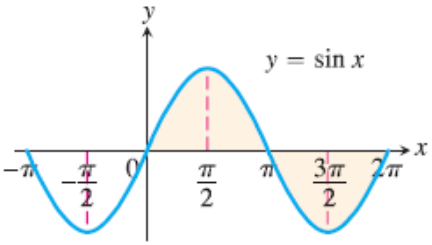
Period π:

$$\tan(x+\pi) = \tan x$$
$$\cot(x+\pi) = \cot x$$

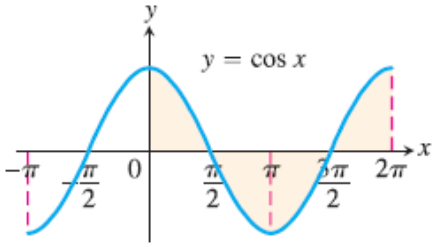
$$\tan x = \frac{\sin x}{\cos x} \quad , \quad \cot x = \frac{1}{\tan x}$$
$$\sec x = \frac{1}{\cos x} \quad , \quad \csc x = \frac{1}{\sin x}$$

Period 2π:

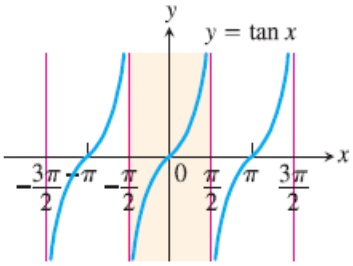
$$\sin(x+2\pi) = \sin x$$
$$\cos(x+2\pi) = \cos x$$
$$\sec(x+2\pi) = \sec x$$
$$\csc(x+2\pi) = \csc x$$



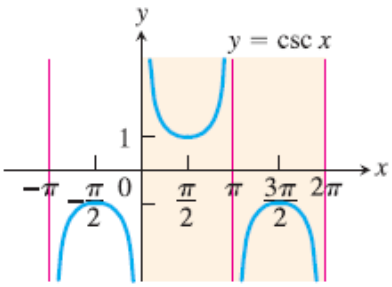
Domain: $-\infty < x < \infty$
Range: $-1 \le y \le 1$
Period: 2π



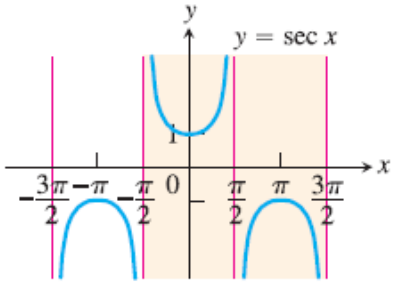
Domain: $-\infty < x < \infty$
Range: $-1 \le y \le 1$
Period: 2π



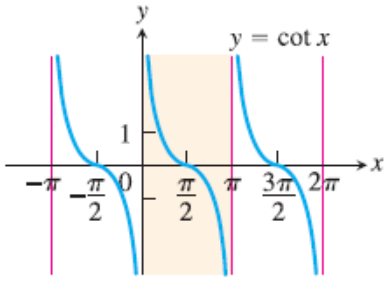
Domain: $x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$
Range: $-\infty < y < \infty$
Period: π



Domain: $x \neq 0, \pm\pi, \pm 2\pi, \ldots$
Range: $y \leq -1$ or $y \geq 1$
Period: 2π



Domain: $x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$
Range: $y \leq -1$ or $y \geq 1$
Period: 2π

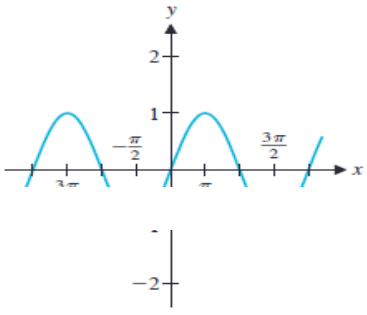


Domain: $x \neq 0, \pm\pi, \pm 2\pi, \ldots$
Range: $-\infty < y < \infty$
Period: π

<u>Problem 2:</u> Sketch a) $y = \sin x$, b) $y = 2\sin x$,and c) $y = \sin(2x)$, What is the period of each function?

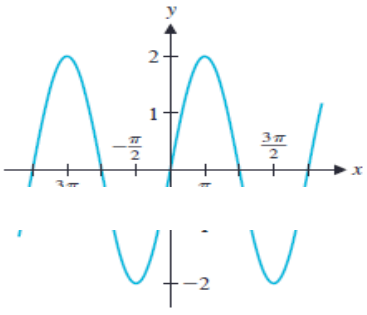
<u>Solution:</u>

a) The period is 2π



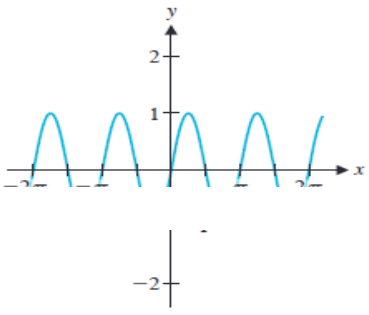
$y = \sin x$

b) The period is 2π



$y = 2\sin x$

c) The period is π



$y = \sin(2x)$