Equilibrium of Rigidbodies – support Reactions

Beam:

A beam is horizontal structural member which carries a load transverse (perpendicular) to its axis and transfers the load through support reactions to supporting columns or walks

Frame:

A structure made up of up of several members riveted or welded together is known as frame.

Support Reactions of Beam:-

The force of resistance exerted by the support on the beam is called support reaction.

Types of support

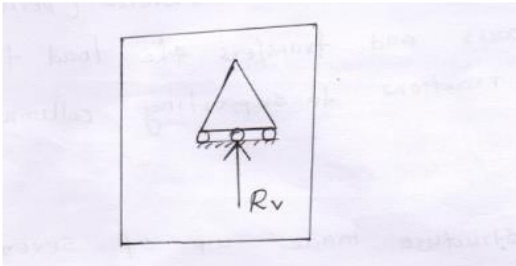
1. Roller support

2. Hinged support

3. Fixed support

1. Roller support:

It consist of the rollers as the bottom. It has only one vertical reaction.

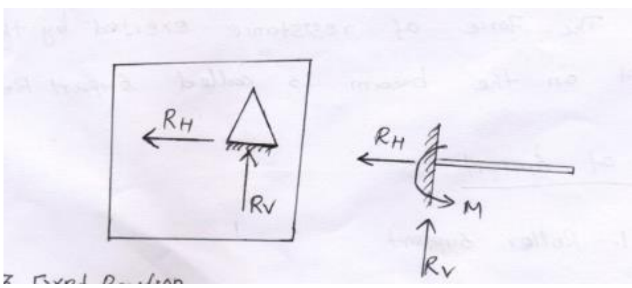


2. Hinged support:

It resists the horizontal and vertical moment. It has two Reaction

(i) Horizontal reaction

(ii) Vertical reaction



3. Fixed reaction:

It is the Stronged support. This support has following reaction

(i) Vertical reaction

(ii) Horizontal reaction

(iii) Rotational reaction (moment)

Types of load:

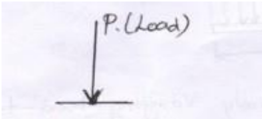
1. Point load

2. Uniformly distributed load (UDL)

3. Uniformly varying load (UVL)

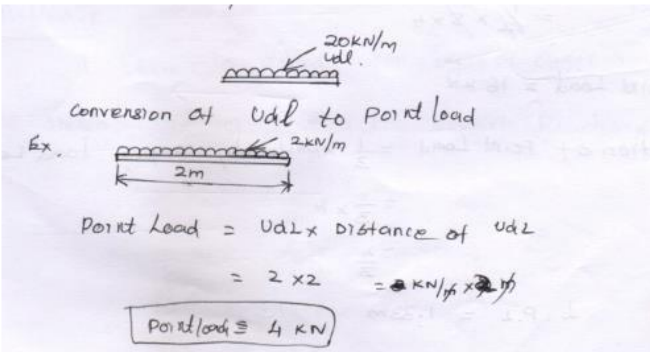
1. Point load:

Load which is acting at a particular point (i.e.) point load;



2. Uniformly distributed Load

The load which is spread over a beam in such a manner that each unit length at the beam carries same intensity of the load is called uniformly distributed load.



$$Point\ load\ = udl \times distance of\ udl$$

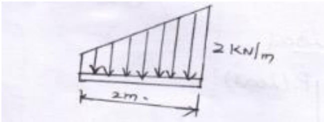
$$= 2 \times 2$$

$$Point\ = 4KN$$

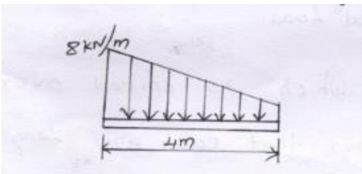
$$Location\ of\ point\ = \frac{uniformly\ disbuted\ load\ length}{2}$$

3. Uniformly varying load

A load which is varying from the particular point along particular length is called uniformly varying load



Conversion of uniformly varying load to point load



$Point\ load\ = 1/2 \times\ uniformly\ varying\ load\ \times\ \ length\ of\ uniformly\ load$

$= 1/2 \times 8 \times 4$

$Point\ load\ = 16KN$

$Location\ of\ point\ load\ = \frac{1}{3} \times\ uniformly\ varying\ load\ length$

$= \frac{1}{3} \times 4$

$= 4/3$

$L.P.L = \ 1.33m$

Procedure for solving the support reaction problem

1. Sum of all the horizontal force is zero $\sum F_H = 0$

To find R_H (R_{HB})

2. Sum of all the vertical force is zero $\sum FV$=0

To find R_{VA}+R_{VB}

3. Take moment of force about A ($\sum MA$=0 To R_{FV}=0 to find R_{VA}

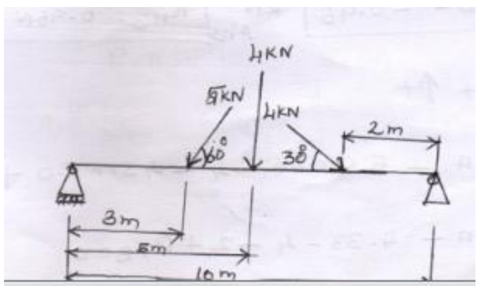
(OR)

4. Substitute R_{VB} in$\sum FV$=0 Eqn

To find R_{VA}

Problem-I

A beam is acted upon by a system of forces shown in fig. Find the support Reactions

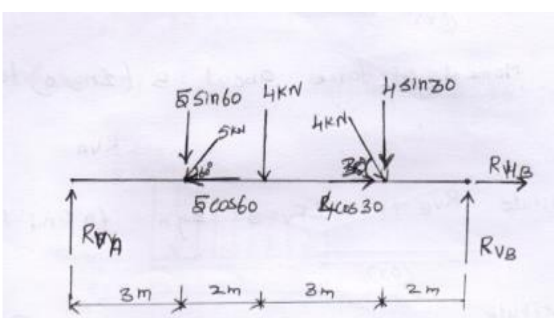


To find

Reaction at the support. R_V&R_{VB},R_{HB}

Soln:

Free body diagram



$\sum F_H$ =0 $\quad \overset{+}{\rightarrow}\overset{-}{\leftarrow}$

$$\sum F_H = -5\cos 60 + 4cos30 + R_{HB}$$

$\sum F_H = 0.96 + R_{HB}$

$R_{HB} = -0.96KN$

$R_{HB} = 0.96N(\rightarrow)$

$\sum F_V = 0$

$\sum F_V = -R_{VA} - 5sin\,60 - 4 - 4sin\,30 + R_{VB} = 0$

$RV_A - 4.33 - 4 - 2 + RV_B = 0$

$RV_A + RV_B - 10.33 = 0$

$RV_A + RV_B = 10.33$ ------> (1)

Take moment of force about A

$\sum M_A = 0$

$\sum M_A = (5sin\,60 \times 3) + (4 \times 2) + (4sin\,30 \times 8) + (Rvb \times 10) = 0$

$12.99 + 8 + 16 - 10\,R_{VB} = 0$

$36.99 - 10\,R_{VB} = 0$

$-10\,R_{VB} = (-36.99)/(-10)$

$R_{VB} = \frac{-36.99}{-10}$

$Ans: R_{VB} = 3.69\,N$

$R_{VB\ value}$ sub in Eqn ----- > (1)

$R_{VA} + R_{VB} = 10.33$

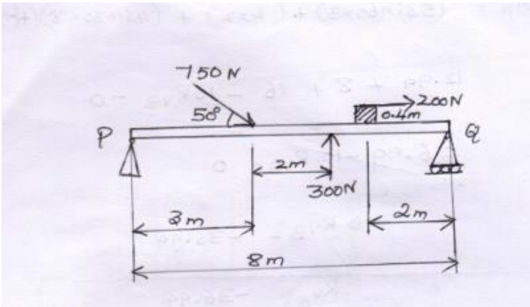
$R_{VA} + 3.69 = 10.33$

$R_{VA} = 10.33 - 3.69$

$R_{VA} = 6.64N$

Problem 2

A beam is loaded as shown in fig find the magnitude direction and the location of the resultant of the system of forces.



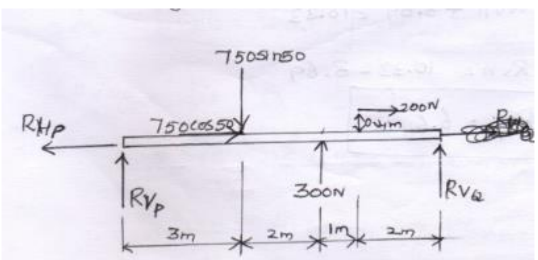
To find

1. Resultant force &direction

2. location of resultant force

Soln

Free body diagram



$\sum F_V = 0$ ↑ + ↓ -

$-R_{HP} + 750\,cos\,50 + 200 = 0$

$-\,R_{HP} + 482 + 200 = 0$

$-\,R_{HP} + 682 = 0$

$-R_{HP} = -682N$

↑ ↓

$\sum F_V = 0$ +   -

$R_{VP} - 750 \sin 50 + 300 + R_{VQ} = 0$

$R_{VP} + R_{VQ} - 574.53 + 300 = 0$

$R_{VP} + R_{vQ} = -300 + 574.53$

$R_{VP} + R_{VQ} = 274.53\ N$______(1)

Take moment About 'p'

$\sum M_P = 0$

$(750 \sin 50 \times 3) + (200 \times 0.4)(-R_{VQ} \times 8) = 0$

$1753.59 - 1500 + 80 - 8\ R_{VQ} = 0$

$-8\ R_{VQ} = -1753.59 + 1500 - 80$

$-8\ R_{VQ} = 333.59$

$R_{VQ} = (-333.59)/(-8)$

$R_{VQ} = 41.9N$

$R_{VQ} = 41.69N$

R_{VQ} value sub in Eqn(1)

$R_{VP} + R_{VQ} = 274.53$

$R_{VP} + 41.69 = 274.53$

$R_{VP} = 274.53 + 41.69$

$R_{VP} = 232.83N$

Resultant $R = \sqrt{(\sum F_H)^2 + (\sum(F_V)^2}$

Resultant force consider $R = \sqrt{(\sum H_P)^2 + RV_P{}^2}$

Only hinged support $R = \sqrt{[682]^2 + [232.83]^2}$

$R = 720N$

Direction $\theta = tan^{-1}\left[\frac{\sum V_P}{\sum V_H}\right]$

$\theta = tan^{-1}\left(\frac{232.83}{682}\right)$

$\theta = 18°50'$

Location

By varignon's theorem

$\sum M_P = R \times x$

$\sum M_p = 0$ $+\downarrow$ $\uparrow-$

$\sum M_P = (750 \times sin\,50 \times 3) + (300 \times 5) + (200 \times 0.4) + (-R_{VQ} \times 8)$

$\sum M_P = 1753.59 - 1500 - 800 - 333.52$

$\sum M_P = 0.07\,N.m$

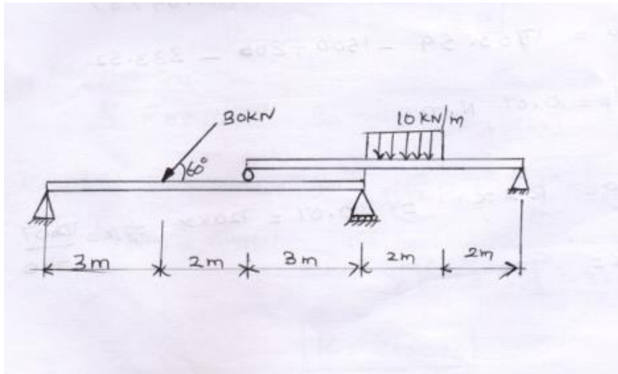
$\sum M_P = R \times x$ => $0.07 = 720 \times x$ => $x = 0.07/720$

Problem: 3

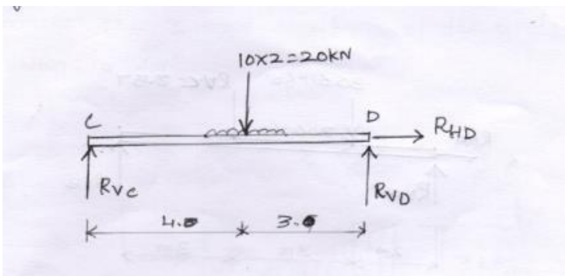
Two beams AB and CD are shown in fig Aand D are hinged supports Band C are rollers supports.

(i) Sketch the free body diagram of the beam AB and deter mine the reaction at the support A and B.

(ii) Sketch the free body diagram of the beam CD and determine the reactions at the supports C and D_0



Free diagram of beam CD



$\sum F_H = 0 \underset{+}{\rightarrow} \underset{-}{\leftarrow}$

$R_{HD} = 0$

$R_{VC} - 20 + R_{VD} = 0$

$R_{VC} + R_{VD} = 20$_______(2)

Take moment abut c=0

$\sum M_C = (20 \times 4)(-R_{VD} \times 7) = 0$

$80 - 7\ R_{VD} = 0$

$R_{VD} = \dfrac{-80}{-7}$

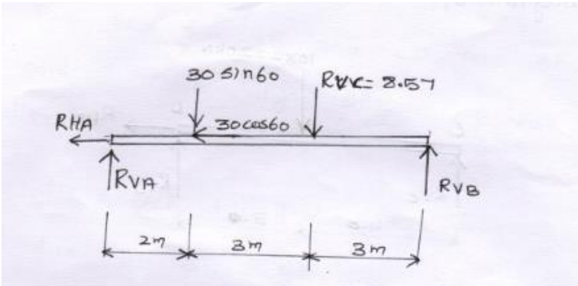
$R_{VD} = 11.42KN$

R_{VD} value sub in Eqn (1)

$R_{VC}\ +\ 11.42 = 20\ \ => R_{VC} = 20\ -\ 11.42$

$R_{VC} = 8.57KN$

Free body diagram of beam AB



$\sum F_H = 0 \quad \underset{+}{\rightarrow} \underset{-}{\leftarrow}$

$\sum F_H = R_{HA} - 30cos\,60 = 0$

$RH_A - 15 = 0$

$RH_A = 15KN$

$\sum F_V = 0 \quad \uparrow + \downarrow -$

$R_{VA} - 30sin\,60 - 8.57 + R_{VB} = 0$

$R_{VA} + R_{VB} - 25.98 - 8.57 = 0$

$R_{VA} + R_{VB} = 34.55$_______(1)

Take moment about A

$\sum M_A = 0$

$\sum M_A = (30sin\,60 \times 2) + (8.57 \times 5) + (R_{VB} \times 8) = 0$

$51.96 + 42.85 - 8\,R_{VB} = 0$

$-8\,R_{VB} - \times 94.81/\times 8$

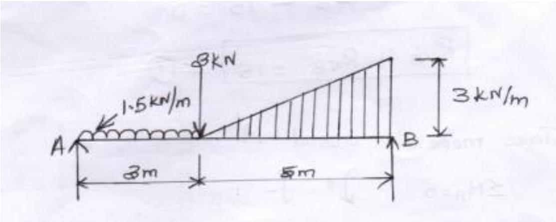
$R_{VB} = 11.85KN$

$R_{VB} + 11.85 = 34.85$

$R_{VA} = 22.69KN$

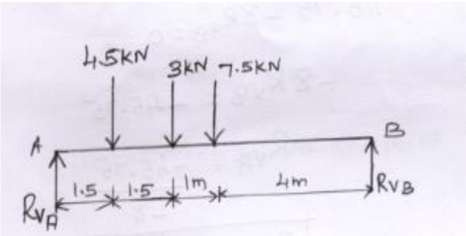
Problem:4

Determine the support reaction of the simply supported beam shown in fig



Soln:

Free body diagram



$UDL\ to\ PL = UDL\ \times dis\ of\ VDL$

$= 1.5 \times 3 = 4.5$

$Length\ of\ UDL = \frac{UDL}{2} = \frac{3}{2} = 1.5$

$UVL\ to\ PL\ = \frac{1}{2} \times UVL \times\ length\ of\ UVL$

$= \frac{1}{2} \times 3 \times 5$

$PL = 7.5\ KN$

$Length\ of\ PL = \frac{1}{3} \times 3 = 1m$

$\sum F_H = 0 \overset{+}{\rightarrow}\overset{-}{\leftarrow}$

$\sum F_V = 0 \quad \downarrow + \quad \uparrow +$

$R_{VA} \; - \; 4.5 \; - 3 \; - 7.5 + R_{VB} = \; 0$

$+ \; R_{VA} \; - \; 15 \; = \; 0$

$R_{VA} \; + \; R_{VB} \; = \; 15$ ______(1)

Take moment about A

$\sum M_A = 0 \downarrow + \uparrow -$

$(4.5 \times 1.5) + (3 \times 3) + (7.5 \times 4) + (R_{VB}) = 0$

$45.75 - 8\,R_{VB} = 0$

$-8\,R_{VB} = -45.75$

$R_{VB} = \frac{-45.75}{-8}$

$R_{VB} = 5.71KN$

$R_{VB} \; + R \quad = 15 - - - - - (1)$

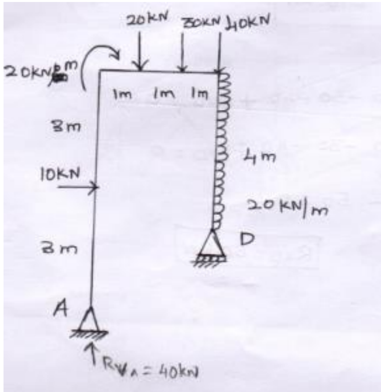
$R_{VA} \quad + \; 5.71 = 15$

$R_{VA} = 15 - 5.71$

$R_{VA} = 9.28KN$

Problem 5

Find the reactions for the frame shown in fig. the line of action of 40k passes through the point A.

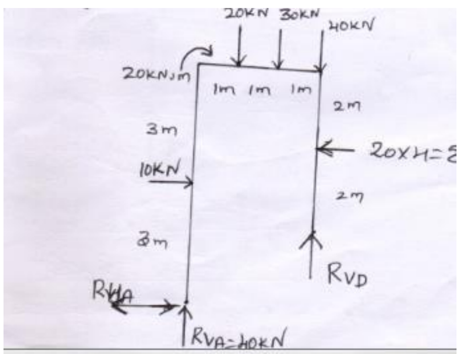


Given

Line of action at point A R_{VA}=40KN

Soln:

Free body diagram



$\sum FH$=0 $\xrightarrow[+]{}\xleftarrow[-]{}$

$-R_{HA} + 10 - 80 = 0$

$-R_{HA} - 70 = 0$

$-R_{HA} = 70$

$R_{HA} = -70KN$

$R_{HA} = -70KN$

$\sum FV$=0

$$R_{VA} - 20 - 30 - 40 + R_{VD} = 0$$

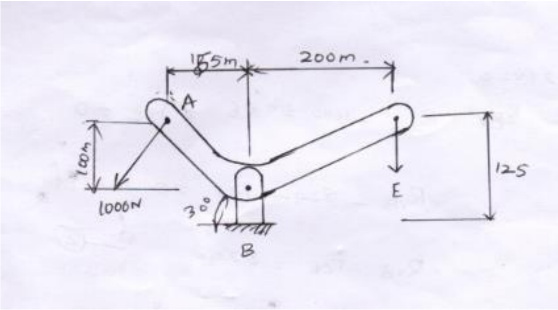
$$40 - 20 - 30 - 40 + R_{VD} = 0$$

$$-50 + R_{VD} = 0$$

$$R_{VD} = 50KN$$

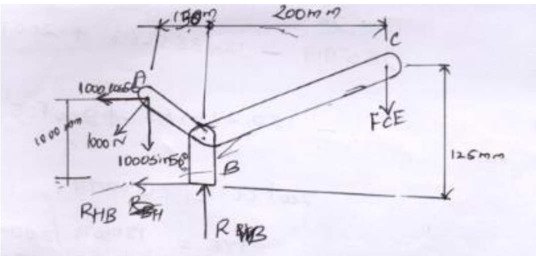
Problem:

Lever ABC of machine component is subjected to a force 1000n at point A as shown in fig. compute the reaction act B and the force at CE.



Soln:

Free body diagram



θ=tan^{-1}($\frac{100}{500}$)

θ=33^{0} 41^{1}

$\sum F_H$=0

$-R_{HB} - 1000\ cos\ 560 = 0$

$-\ R_{HB} = 1000\ cos\ 560$

$R_{HB} = -554.7\ N$_____(1)

$\sum F_V$=0

$R_{VB} - 1000\ Sin\ 56 - FCE = 0$

$R_{VB} = -829 -\ \ FCE = 829 - - - - - (2)$

$M_B =\ [-1000\ cos\ 56\ \times 100] + [-1000 sin\ 560 \times 150] + [F_{CE}] = 0$

$-55919 - 124355.63 + 200\ F_{CE} = 0$

$-180274.63200\ F_{CE} = 0$

$200\ F_{CE} = 180274$

$F_{CE} = 180274/200$

$F_{CE} = 901.373N$

Resultant $R_B = \sqrt{(RV_B)}^2 + (RH_B)^2$

$= \sqrt{(1730)}2 + (-554)^2$

R_B=1817N

$\theta = \tan^{-1}(\frac{RV_B}{RH_B}) = \tan^{-1}(\frac{1730}{554})$

$\theta = 72^0 14^1$

To find

Reaction of support

Soln

Sum of horizontal forces $\sum F_H$

$\sum F_H$=0

$-RH_A - 10\ cos\ 60\ = 0$

$-\ RH_A = -10 cos\ 60 = -5$

$RH_A\ = 5N$

Sum of vertical forces $\sum F_V$

∑FV=0

$R_{VA} = 10 sin\ 60\ \ - 8 = 0$

$R_{VA} = -8.66 - 8 = 0$

$R_{VA} - 16.66 = 0$

$R_{VA} = 16.66N$

Moment About 'o'

$Mo = 0$

$Mo = M = [10 sin\ 60 \times 4] + 20 + [8 \times 8] = 0$

$M + 34.64 + 20 + 64 = 0$

$M +\ 118.64 = 0$

$M\ = 118.64 KN.m$

$M\ = 118.64\ m$