UNIT-II

DESIGN OF EXPERIMENTS

ANOVA – Analysis of Variance

2.1. Working Rule (One – Way Classification)

Set the null hypothesis H_0: There is no significance difference between the treatments.

Set the alternative hypothesis H_1: There is a significance difference between the treatments.

Step: 1 Find N = number of observations

Step: 2 Find T = The total value of observations

Step: 3 Find the Correction Factor C . F $= \frac{T^2}{N}$

Step: 4 Calculate the total sum of squares and find the total sum of squares

$$\text{TSS} = \left(\sum X_1{}^2 + \sum X_2{}^2 + \sum X_3{}^2 + \ . . .\right) - C.F$$

Step: 5 Column sum of squares SSC $\left(\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_2} + \frac{(\sum X_3)^2}{N_3} + \ldots\right) - C.F$

Where N_i= Total number of observation in each column $(i = 1, 2, 3, \ldots)$

Step: 6 Prepare the ANOVA to calculate F – ratio

Source of variation	Sum of Degrees	Degrees of Freedom	Mean Square	F - Ratio
Between Samples	SSC	K - 1	MSC = $\frac{SSC}{K-1}$	$F_c = \frac{MSC}{MSE}$ if MSC$>$ MSE
Within Samples	SSE	N - K	MSE = $\frac{SSE}{N-K}$	$F_c = \frac{MSE}{MSC}$ if MSE$>$ MSC

Step: 7 Find the table value (use chi square table)

Step: 8 Conclusion:

Calculated value $<$ Table value, then we accept null hypothesis.

Calculated value $>$Table value, then we reject null hypothesis.

PROBLEMS ON ONE WAY ANOVA

1.A completely randomised design experiment with 10 plots and 3 treatments gave the following results.

Plot No	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Treatment	**A**	**B**	**C**	**A**	**C**	**C**	**A**	**B**	**A**	**B**
Yield	**5**	**4**	**3**	**7**	**5**	**1**	**3**	**4**	**1**	**7**

Analyse the result for treatment effects.

Solution:

Set the null hypothesis H_0: There is no significance difference between the treatments.

Set the alternative hypothesis H_1: There is a significance difference between the treatments.

Treatments	Yields from plots			
A	5	7	3	1
B	4	4	7	-
C	3	5	1	-

TABLE:

Treatment A Treatment B Treatment C

X_1	${X_1}^2$	X_2	${X_2}^2$	X_3	${X_3}^2$
5	25	4	16	3	9
7	49	4	16	5	25
3	9	7	49	7	7
1	1	-	-	-	-
$\sum X_1$ =16	$\sum {X_1}^2$ =84	$\sum X_2$ =5	$\sum {X_2}^2$ =81	$\sum X_3$ =9	$\sum {X_3}^2$ =35

Step: 1 N =10

Step: 2 Sum of all the items (T) $= \sum X_1 + \sum X_2 + \sum X_3 = 16 + 15 + 9 = 40$

Step: 3 Find the Correction Factor C . F $= \frac{T^2}{N} = \frac{(40)^2}{10} = 160$

Step: 4 TSS = Total sum of squares

= sum of squares of all the items – C. F

TSS $=\left(\sum {X_1}^2 + \sum {X_2}^2 + \sum {X_3}^2 + \ . \ . \ .\right) - C.F$

$=(84 + 81 + 35) - 160 = 40$

Step: 5 SSC = Sum of squares between samples

SSC = $\left(\frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_2} + \frac{(\sum X_3)^2}{N_3} + \ldots\right) - C.F$

SSC = $\left(\frac{(16)^2}{4} + \frac{(15)^2}{3} + \frac{(9)^2}{3} + \ldots\right) - 160$

$= 64 + 75 + 27 - 160 = 6$

Step: 6 MSC = Mean squares between samples

$= \frac{Sum\ of\ squares\ between\ samples}{d.f}$

$= \frac{6}{2} = 3$

SSE = Sum of squares within samples

= Total sum of squares – Sum of squares between samples

$= 40 - 6 = 34$

Step:7 MSE = Mean squares within samples

$= \frac{Sum\ of\ squares\ within\ samples}{d.f}$

$= \frac{34}{7} = 4.86$

ANOVA TABLE

Source of variation	Sum of Degrees	Degrees of Freedom	Mean Square	F - Ratio
Between Samples	SSC = 6	K – 1 = 3 – 1 =2	MSC = $\frac{SSC}{K-1} = 3$	
Within Samples	SSE = 34	N – K =10 – 3 = 7	MSE = $\frac{SSE}{N-K} = 4.86$	$F_c = \frac{MSE}{MSC}$= 1.62

d.f for (7, 2) at 5% level of significance is 19.35

Step: 8 Conclusion:

Calculated value < Table value, then we accept null hypothesis.

2. Three different machines are used for a production. On the basis of the outputs, set up one – way ANOVA table and test whether the machines are equally effective.

Outputs		
Machine I	**Machine II**	**Machine III**
10	**9**	**20**
15	**7**	**16**
11	**5**	**10**
10	**6**	**14**

Given that the value of F at 5% level of significance for (2, 9) d. f is 4.26

Solution:

Set the null hypothesis H_0: The machines are equally effective.

TABLE:

Treatment A		Treatment B		Treatment C	
X_1	${X_1}^2$	X_2	${X_2}^2$	X_3	${X_3}^2$
10	100	9	81	20	400
15	225	7	49	16	256
11	121	5	25	10	100
20	400	6	36	14	196
$\sum X_1$ =56	$\sum {X_1}^2$ =846	$\sum X_2$ =27	$\sum {X_2}^2$ =191	$\sum X_3$ =60	$\sum {X_3}^2$ =952

Step: 1 N =12

Step: 2 Sum of all the items (T) $= \sum X_1 + \sum X_2 + \sum X_3 = 56 + 27 + 60 = 143$

Step: 3 Find the Correction Factor C . F $= \frac{T^2}{N} = \frac{(143)^2}{12} = 1704.08$

Step: 4 TSS = Total sum of squares

= sum of squares of all the items – C. F

TSS $=\left(\sum X_1{}^2+\sum X_2{}^2+\sum X_3{}^2+\ ...\right)-C.F$

$=(846+191+952)-1704.08=284.92$

Step: 5 SSC = Sum of squares between samples

SSC = $\left(\frac{(\sum X_1)^2}{N_1}+\frac{(\sum X_2)^2}{N_2}+\frac{(\sum X_3)^2}{N_3}+\dots\right)-C.F$

SSC = $\left(\frac{(56)^2}{4}+\frac{(27)^2}{4}+\frac{(60)^2}{4}+\dots\right)-1704.08$

$=784+182.25+900-1704.08=162.17$

Step: 6 MSC = Mean squares between samples

$=\frac{Sum\ of\ squares\ between\ samples}{d.f}$

$=\frac{162.17}{2}=81.085$

SSE = Sum of squares within samples

= Total sum of squares – Sum of squares between samples

$=284.92-162.17=122.75$

Step: 7 MSE = Mean squares within samples

$=\frac{Sum\ of\ squares\ within\ samples}{d.f}$

$=\frac{122.75}{9}=13.63$

ANOVA TABLE

Source of variation	Sum of Degrees	Degrees of Freedom	Mean Square	F - Ratio
Between Samples	SSC = 162.17	K – 1 = 3 – 1 =2	MSC = $\frac{SSC}{K-1}=$ 81.085	
Within Samples	SSE = 122.75	N – K =12 – 3 = 9	MSE = $\frac{SSE}{N-K}=$ 13.63	$F_c=\frac{MSE}{MSC}$= 5.95

d.f for (2, 9) at 5% level of significance is 4.26.

Step: 8 Conclusion:

Calculated value $>$ Table value, then we reject the null hypothesis.

i.e., the three machines are not equally effective.