Boundary Condition for Magnetic Fields:

Similar to the boundary conditions in the electro static fields, here we will consider the behavior of $\vec{B}$ and $\vec{H}$ at the interface of two different media. In particular, we determine how the tangential and normal components of magnetic fields behave at the boundary of two regions having different permeabilities.

The figure 4.9 shows the interface between two media having permeabities μ_1 and μ_2, $\hat{a}_n$ being the normal vector from medium 2 to medium 1.

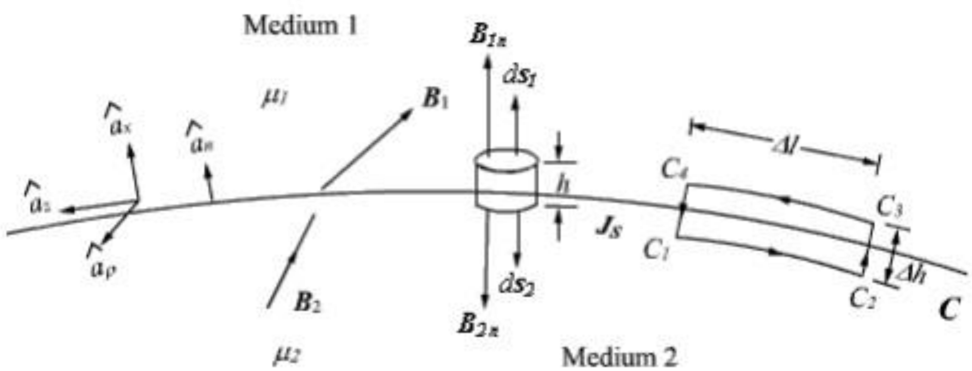

Figure 4.9: Interface between two magnetic media

To determine the condition for the normal component of the flux density vector $\vec{B}$, we consider a small pill box P with vanishingly small thickness *h* and having an elementary area ΔS for the faces. Over the pill box, we can write

$$\oint_s \vec{B}.d\vec{s} = 0 \quad \text{...(4.36)}$$

Since h --> 0, we can neglect the flux through the sidewall of the pill box.

$$\therefore \int_{\Delta S} \vec{B}_1 . d\vec{S}_1 + \int_{\Delta S} \vec{B}_2 . d\vec{S}_2 = 0 \qquad \text{.............................(4.37)}$$

$$d\vec{S}_1 = dS\hat{a}_n \quad \text{and} \quad d\vec{S}_2 = dS\left(-\hat{a}_n\right) \qquad \text{..................(4.38)}$$

$$\therefore \int_{\Delta S} B_{1n} dS - \int_{\Delta S} B_{2n} dS = 0$$

where

$$B_{1n} = \vec{B}_1 . \hat{a}_n \text{ and } B_{2n} = \vec{B}_2 . \hat{a}_n \qquad \text{..........................(4.39)}$$

Since ΔS is small, we can write

$$\left(B_{1n} - B_{2n}\right)\Delta S = 0$$

or,

$$B_{1n} = B_{2n} \qquad \text{...................................(4.40)}$$

That is, the normal component of the magnetic flux density vector is continuous across the interface.

In vector form,

$$\hat{a}_n \cdot \left(\vec{B}_1 - \vec{B}_2\right) = 0 \quad \text{.............................(4.41)}$$

To determine the condition for the tangential component for the magnetic field, we consider a closed path C as shown in figure 4.8. By applying Ampere's law we can write

$$\oint \vec{H} \cdot d\vec{l} = I \quad \text{.....................................(4.42)}$$

Since h -->0,

$$\int_{c_1 - c_2} \vec{H} \cdot d\vec{l} + \int_{c_3 - c_4} \vec{H} \cdot d\vec{l} = I \quad \text{...................................(4.43)}$$

We have shown in figure 4.8, a set of three unit vectors $\hat{a}_n$, $\hat{a}_t$ and $\hat{a}_\rho$ such that they satisfy $\hat{a}_t = \hat{a}_\rho \times \hat{a}_n$ (R.H. rule). Here $\hat{a}_t$ is tangential to the interface and $\hat{a}_\rho$ is the vector perpendicular to the surface enclosed by C at the interface

The above equation can be written as

$$\vec{H}_1 \cdot \Delta l \, \hat{a}_t - \vec{H}_2 \cdot \Delta l \, \hat{a}_t = I = J_{sn} \Delta l$$

or,

$$H_{1t} - H_{2t} = J_{sn} \quad \text{...................................(4.44)}$$

i.e., tangential component of magnetic field component is discontinuous across the interface where a free surface current exists.

If $J_S = 0$, the tangential magnetic field is also continuous. If one of the medium is a perfect conductor J_S exists on the surface of the perfect conductor.

In vector form we can write,

$$\left(\vec{H}_1 - \vec{H}_2\right).\hat{a}_t \Delta l$$

$$= \left(\vec{H}_1 - \vec{H}_2\right).\left(\hat{a}_\rho \times \hat{a}_n\right)\Delta l$$

$$= J_{Sn\Delta l} = \vec{J}_S.\hat{a}_\rho \Delta l \quad \text{.....................................(4.45)}$$

Therefore,

$$\hat{a}_n \times \left(\vec{H}_1 - \vec{H}_2\right) = \vec{J}_S \quad \text{.....................................(4.46)}$$