5.4 PRINCIPLE OF IMPULSE AND MOMENTUM

The **momentum**, p, of a body of mass mm which is moving with a velocity v is p=m×v= mv. The units are Ns.

The **impulse** of a force is I=Ft - when a constant force F acts for a time t. The units are Ns.

The **Impulse-Momentum Principle** says I= mv−mu which is **final momentum - initial momentum** so Impulse is the change in momentum.

The principle of states that total momentum before impact is equal to total momentum after impact, $m_1u_1+m_2u_2=m_1v_1+m_2v_2$.

Solved Examples

1.A body weighing 196.2 N slides up a 30° inclined plane under the action of an applied force of 300 N acting parallel to the plane. The coefficient of friction is 0.2. The body moves from rest. Determine at the end of the 4 s, the acceleration, distance travelled, velocity, kinetic energy, work done, momentum and impulse applied on the body.

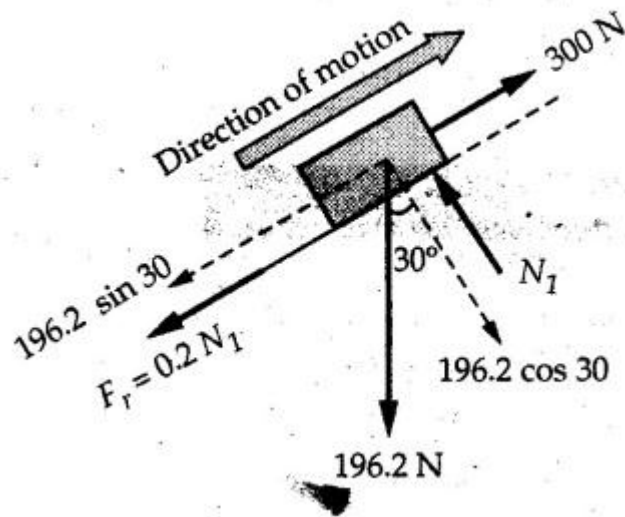**Solution:**

Let a = acceleration of the body

$$u = 0, \quad t = 4\,s$$

$$s = ut + \frac{1}{2}at^2$$

$$\therefore \quad s = 0 + \frac{1}{2}a \times 4^2 = 8a$$

$$v = u + at = 0 + a \times 4$$

$$\therefore \quad v = 4a$$

To use work-energy principle, draw F.B.D. and resolve forces parallel and perpendicular to the direction of motion as shown in Fig.

By work-energy principle,

$$0 + [300 \times s - 196.2 \sin 30 \times 5 - 0.2\, N_1 \times s] = \frac{1}{2}\left(\frac{196.2}{9.81}\right) v^2$$

$$N_1 = 196.2 \cos 30$$

Substituting for N, s and v,

$$300 \times 8a - 196.2 \sin 30 \times 8a - 0.2 \times 196.2 \cos 30 \times 8a = \frac{1}{2}\left(\frac{196.2}{9.81}\right)(4a)^2$$

∴
$$\boxed{a = 8.396 \text{ m/s}^2}$$

$$s = 8a = 8 \times 8.396$$

∴
$$\boxed{s = 67.168 \text{ m}}$$

$$v = 4a = 4 \times 8.396$$

∴
$$\boxed{v = 33.584 \text{ m/s}}$$

$$K.E. \;=\; \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{196.2}{9.81}\right) \times 33.584^2$$

∴ $\boxed{K.E. \;=\; 11278.85 \text{ J}}$

Work done $=$ Change in K.E.

∴ $\boxed{\text{Work done} \;=\; 11278.85 \text{ J}}$

$$\text{Momentum} \;=\; mv = \frac{196.2}{9.81} \times 33.584$$

∴ $\boxed{\text{Momentum} \;=\; 671.68 \text{ N-s}}$

Impulse $=$ Change in momentum

∴ $\boxed{\text{Impulse} \;=\; 671.68 \text{ N-s}}$

2.A 40 kg block A is connected to a 60 kg block B by a spring of constant k = 180 N/m. The blocks are placed on a smooth horizontal surface and are at rest when the spring is stretched 2 m. If they are released from rest, determine the speeds of blocks A and B, at the instant the spring becomes unstretched.
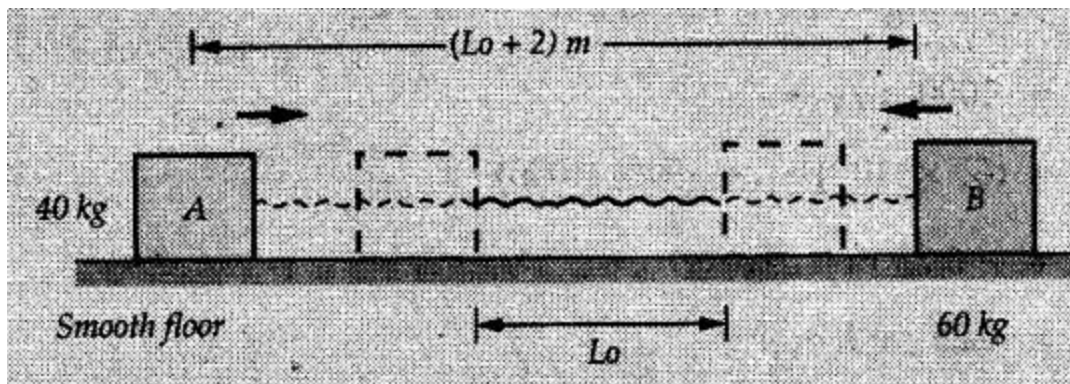
Solution:

When spring is released, A moves towards right and B moves towards left.

By conservation of momentum,

$$0 = m_A v_A + m_B(-v_B)$$

$$\therefore 40 v_A - 60 v_B = 0$$

$$v_A = 1.5 v_B \qquad \qquad \dots (1)$$

By conservation of energy,

$$\frac{1}{2} k x^2 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

$$180 \times 2^2 = 40 v_A^2 + 60 v_B^2$$

$$180 \times 2^2 = 40 (1.5 v_B)^2 + 60 v_B^2$$

$$\therefore \boxed{v_B = 2.19 \text{ m/s} \leftarrow}$$

$$v_A = 1.5 \times 2.19$$

$$\therefore \boxed{v_A = 3.285 \text{ m/s} \rightarrow}$$

3.A 900 kg car travelling at 48 km/h couples to a 680 kg car travelling at 24 km/h in the same direction. Determine their common velocity after coupling. What is the amount of energy lost?

Solution:

As collision is plastic, $v_1 = v_2 = v$

By conservation of momentum,

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$(900)\left(48 \times \frac{5}{18}\right) + (680)\left(24 \times \frac{5}{18}\right) = 900\, v + 680\, v$$

$$\therefore \boxed{v = 10.464 \text{ m/s} = 37.67 \text{ km/h}}$$

$$\text{Energy lost} = \left(\frac{1}{2}m_1 u_1^2 + \frac{1}{2}m_2 u_2^2\right) - \frac{1}{2}(m_1 + m_2)v^2$$

$$= \frac{1}{2}(900)\left(48 \times \frac{5}{18}\right)^2 + \frac{1}{2}(680)\left(24 \times \frac{5}{18}\right)^2 - \frac{1}{2}(900 + 680)(10.464^2)$$

$$= 8609.8 \text{ J}$$

$$\therefore \quad \boxed{\text{Energy lost} = 8.61 \text{ kJ}}$$

4.A 45 Mg rail car moving at 3 km/h is to be coupled, to a 25 Mg car which is at rest. Determine average impulsive force acting on each car during coupling process which lasts for 0.3 s. Also find final velocity of coupled cars.

Solution:

By conservation of momentum,

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

As the collision is plastic, $v_1 = v_2 = v$

$$m_1 = 45 \times 10^6 \text{g} = 45 \times 10^3 \text{ kg}$$

$$m_2 = 25 \times 10^3 \text{kg}$$

$$u_1 = 3 \times \frac{5}{18} \text{ m/s} = \frac{5}{6} \text{ m/s}$$

$$u_2 = 0$$

$$\therefore \quad (45 \times 10^3)\left(\frac{5}{6}\right) + 0 = (45 \times 10^3 + 25 \times 10^3)v$$

$$\therefore \quad \boxed{v = 0.5357 \text{ m/s}}$$

By impulse-momentum theorem for 25 Mg car,

$$m\,v_1 + F_{av}\,\Delta t \;=\; m\,v_2$$

$$v_1 = 0 \;,\; v_2 \;=\; 0.5357 \text{ m/s} \;,\; \Delta t \;=\; 0.3 \text{ s}$$

$$\therefore\quad 0 + F_{av} \times 0.3 \;=\; 25 \times 10^3 \times 0.5357$$

$$\therefore\qquad\qquad \boxed{F_{av} \;=\; 44.64 \times 10^3 \text{N} = 44.64 \text{ kN}}$$

The same result can be obtained using impulse momentum theorem for the 45 kg car with a negative sign as the force on the two cars will have same magnitude but opposite direction.