SERIES RESONANCE

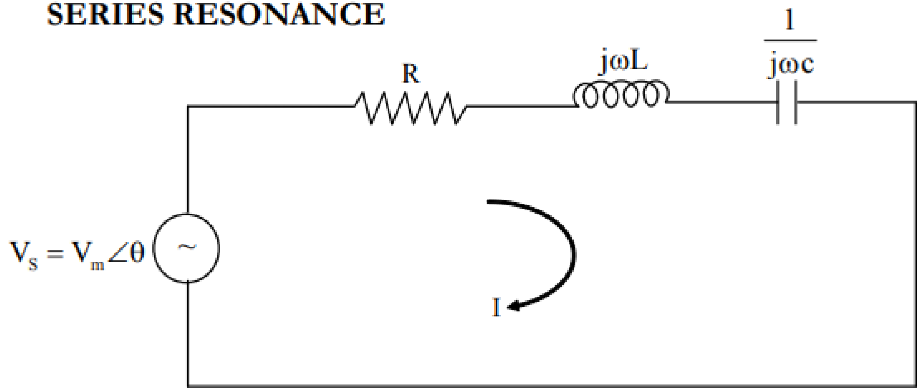


Fig. 5.1 Series RLC Circuit

Resonance occuring in a series circuit is known as series resonance. Consider a series RLC circuit as shown in figure 3.1.

The impedance of the given circuit is,

$$Z = \frac{V_s}{I} = R + j\omega L + \frac{1}{j\omega C}$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

Resonance results when imaginary part is zero.

$$\therefore \quad \omega L - \frac{1}{\omega C} = 0$$

Value of ω that satisfies this condition is called resonant frequency, "ω_0".

$$\therefore \omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec}$$

Since, $\omega_0 = 2\pi f_0$, we can write

$$2\pi f_0 = \frac{1}{\sqrt{LC}} \Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz.}$$

where, $f_0 \rightarrow$ resonant frequency in Hz (or) rad/sec.

Variance of Impedance with Frequency for RLC Series Circuit :

The circuit impedance is, $Z = R + j\,(X_L - X_C) = R + jX$

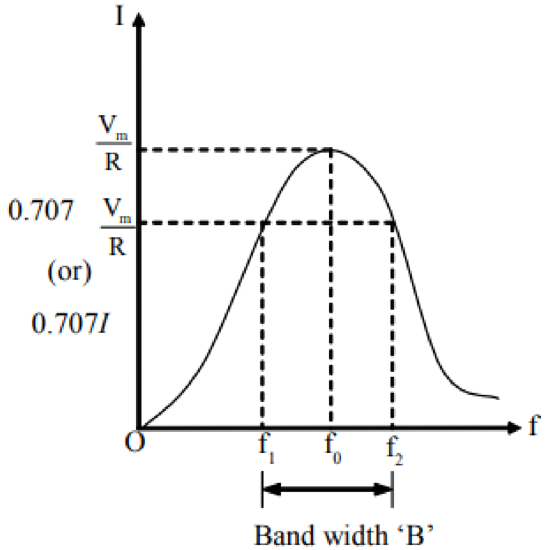
$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

At resonant frequency as $X = 0$, the impedance $|Z|$ is equal to R. At all other frequencies, reactance value is not equal to zero. Hence $|Z|$ is greater than R.

i.e., At $f = f_0$, $|Z| = R$

BAND WIDTH - HALF POWER FREQUENCIES :

The current Vs frequency curve of a RLC series circuit is symmetrical about the resonant frequency. At f_0, the current is maximum and is given by $\frac{V_m}{R}$. There will be two frequencies f_1, f_2 on either side of the resonant frequency f_0 at which the power is half the power at resonance. (Refer fig.5.7). Hence they are called half power frequencies.



$$\therefore\ f_1 = f_0 - \frac{R}{4\pi L}$$

$$\therefore\ f_2 = f_0 + \frac{R}{4\pi L}$$

Fig. 5.7 Current Vs frequency curve of a RLC series circuit

$f_1 \rightarrow$ Lower half power frequency

$f_2 \rightarrow$ Upper half power frequency

Bandwidth :

It is defined as the band of frequency between the two half power frequencies f_2 & f_1.

$$\text{Bandwidth} = f_2 - f_1 = f_0 + \frac{R}{4\pi L} - \left(f_0 - \frac{R}{4\pi L}\right)$$

$$B\,W = \frac{2R}{4\pi L} = \frac{R}{2\pi L}$$

The Q-factor relates the maximum or peak energy stored to the energy dissipated in the circuit per cycle of oscillation.

$$Q = 2\pi \frac{\text{Peak energy stored in the circuit}}{\text{Energy dissipated by the circuit in one period at resonance}}$$

(or)

It is the measure of energy storage property in relation to its energy dissipation property.

$$\therefore \quad Q = 2\pi \frac{\frac{1}{2} LI^2}{\frac{1}{2} I^2 R \left(\frac{1}{f_0}\right)} = \frac{2\pi f_0 L}{R}$$

$$\text{(or)} \qquad Q = \frac{\omega_0 L}{R} \qquad \text{----------(6)}$$

The quality factor is also given by, $Q = \frac{f_0}{f_2 - f_1} = \frac{f_0}{BW}$

$$= \frac{f_0}{R/2\pi L} = \frac{2\pi f_0 L}{R} = \frac{\omega_0 L}{R} \qquad \text{Selectivity} = \frac{f_0}{BW} = \frac{f_0}{f_2 - f_1}$$