## 1.6. MOHR'S CIRCLE

Mohr's circle is a graphical method of finding normal, tangential and resultant stresses on an oblique plane. Mohr's circle will be drawn for the following cases:

(i) A body subjected to two mutually perpendicular principal tensile stresses of unequal intensities

(ii) A body subjected to two mutually perpendicular principal stresses which are unequal and unlike (i.e., one is tensile and other is compressive).

(iii) A body subjected to two mutually perpendicular principal tensile stresses accompanied by a simple shear stress.

### 1.26.1 Mohr's Circle When A Body Is Subjected To Two Mutually Perpendicular Principal Tensile Stresses Of Unequal Intensities.

Consider a rectangular body subjected to two mutually perpendicular principal tensile stresses of unequal intensities. It is required to find the resultant stress on an oblique plane.

Let $\sigma_1$ = Major tensile stress

$\sigma_2$ = Minor tensile stress and

$\theta$ = Angle made by the oblique plane with the axis of minor tensile stress.

Mohr's Circle is drawn as follows:

Take any point A and draw a horizontal line through A. Take AB = $\sigma_1$

and AC = $\sigma_2$ towards right from A to some suitable scale. With BC as diameter describe a circle. Let O is the centre of the circle. Now through O, draw a line OE making an angle 2θ with OB.

From E, draw ED perpendicular on AB. Join AE. Then the normal and tangential stresses on the oblique plane are given by AD and ED respectively. The resultant stress on the oblique plane is given by AE

From Fig.

Length AD = Normal stress on oblique plane

Length ED = Tangential stress on oblique plane.

Length AE = Resultant stress on oblique plane.

Radius of Mohr's circle = $\frac{\sigma 1-\sigma 2}{2}$

Angle Ø = obliquity.

**Problem1.25.**The tensile stresses at a point across two mutually perpendicular planes are 120N/mm$^2$ and 60N/mm$^2$. Determine the normal, tangential and resultant stresses on a plane inclined at 30$^o$ to the axis of minor stress by Mohr's circle method

**Given Data**

Major principal stress, $\sigma_1$ = 120N/mm$^2$(tensile)

Minor principal stress, $\sigma_2$ = 60N/mm$^2$(tensile)

Angle of oblique plane with the axis of minor principal stress,

$\theta = 30^o$

**To find**

The normal, tangential and resultant stresses

**Solution**

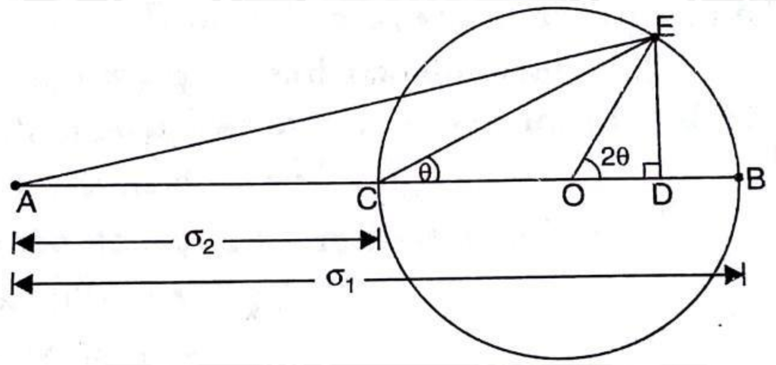
Scale. Let 1cm = 10N/mm$^2$

Then $\sigma_1 = \frac{120}{10}$ = 12cm and

$\sigma_2 = \frac{60}{10}$ = 6cm

Mohr's circle is drawn as:

Take any point A and draw a horizontal line through A. Take AB = $\sigma_1$=12cm and AC




= $\sigma_2$=6cm. With BC as diameter (i.e.,BC=12-6=6cm) describe a circle. Let O is the centre of the circle. Through O, draw a line OE making an angle 2θ (i.e., 2×30 =60$^o$) with OB. From E, draw ED perpendicular to CB. Join AE. Measure the length AD, ED and AE.

By measurements :

Length AD = 10.50cm

Length ED = 2.60cm

Length AE = 10.82cm

Then normal stress = Length AD × Scale

= 10.50× 10 = **105N/mm$^2$**

Tangential or shear stress = Length ED × Scale

= 2.60×10 = **26 N/mm$^2$**.

Resultant stress = Length AE × Scale.

$=10.82\times10=$ **108.2N/mm²**.

**1.26.2. Mohr's Circle when a Body is subjected to two Mutually perpendicular Principal stresses which are Unequal and Unlike (i.e., one is Tensile and other is Compressive).**

Consider a rectangular body subjected to two mutually perpendicular principal stresses which are unequal and one of them is tensile and the other is compressive. It is required to find the resultant stress on an oblique plane.

Let $\sigma_1$ = Major principal tensile stress

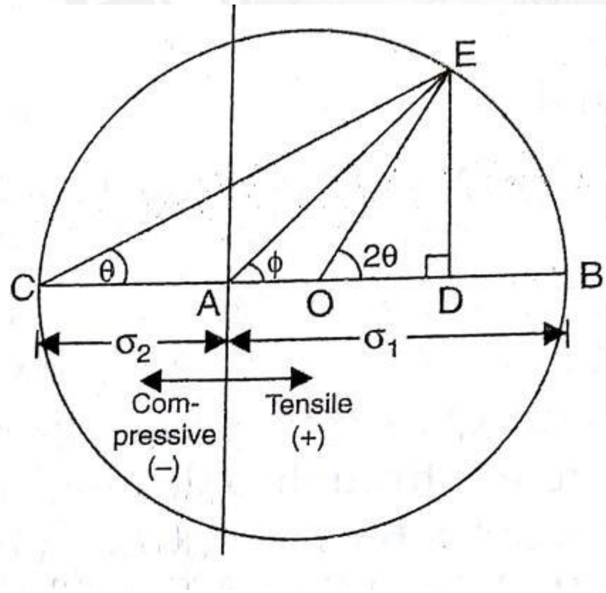
$\sigma_2$ = Minor principal compressive stress and

$\theta$ = Angle made by the oblique plane with the a×is of minor tensile stress.

Mohr's Circle is drawn as follows:

Take any point A and draw a horizontal line through A on both sides of A as shown in fig. Take AB = $\sigma_1$ (+) towards right of A and AC = $\sigma_2$ (-)towards left of A to some suitable scale. Bisect BC at O. With O as centre and radius equal to CO or OB, draw a circle. Through O draw a line OE making an angle 2θ with OB.

From E, draw ED perpendicular to AB.Join AE and CE. Then the normal and tangential stresses on the oblique plane are given by AD and ED respectively. The resultant stress on the oblique plane is given by AE




From Fig.

Length AD = Normal stress on oblique plane

Length ED = Tangential stress on oblique plane.

Length AE = Resultant stress on oblique plane.

Radius of Mohr's circle = $\frac{\sigma_1+\sigma_2}{2}$

Angle Ø = obliquity.

**Problem1.26.** The stresses at a point in a bar are 200N/mm$^2$ (tensile) and 100N/mm$^2$ (compressive). Determine the resultant stress in magnitude and direction on a plane inclined at 60$^o$ to the axis of major stress. Also determine the maximum intensity of shear stress in the material at the point.

**Given Data**

Major principal stress, $\sigma_1$ = 200N/mm$^2$

Minor principal stress, $\sigma_2$ = -100N/mm$^2$

(-ve sign is due to compressive stress)

Angle of oblique plane with the axis of minor principal stress,

$$\theta = 90^o - 60^o = 30^o$$

**To find**

The Magnitude and direction Resultant stress and maximum intensity of shear stress

**Solution**

Scale. Let 1cm =20N/mm$^2$

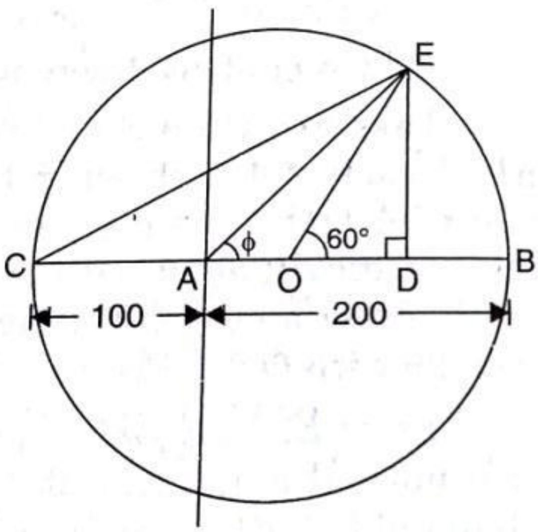
Then $\sigma_1 = \frac{200}{20}$ = 10cm and

$\sigma_2 = -\frac{100}{20}$ = -5cm

Mohr's circle is drawn as:

Take any point A and draw a horizontal line through A on both sides of A as shown in fig. Take AB = $\sigma_1$ =10cm towards right of A and AC = $\sigma_2$ =-5cm towards left of A to some suitable scale. Bisect BC at O. With O as centre and radius equal to CO or OB, draw a circle. Through O draw a line OE making an angle 2θ(i.e.,2×30 =60$^o$) with OB.

From E, draw ED perpendicular to AB.Join AE and CE. Then the normal and tangential stresses on the oblique plane are given by AD and ED respectively. The resultant stress on the oblique plane is given by AE



By measurements:

Length AD = 6.25cm

Length ED = 6.5cm and

Length AE = 9.0cm

Then normal stress = Length AD × Scale

= 6.25× 20 = **125N/mm²**

Tangential or shear stress = Length ED × Scale

= 6.5×20 = **130 N/mm²**.

Resultant stress = Length AE × Scale.

= 9×20= **180N/mm²**

### 1.26.3. Mohr's Circle when a Body is subjected two mutually perpendicular principal Tensile Stresses Accompanied by a simple shear stress.

Consider a rectangular body subjected to two mutually perpendicular principal tensile stresses of unequal intensities accompanied by a simple shear stress. It is required to find the resultant stress on an oblique plane.

Let $\sigma_1$ = Major tensile stress

$\sigma_2$ = Minor tensile stress and

$r$ = Shear stress across face BC and AD

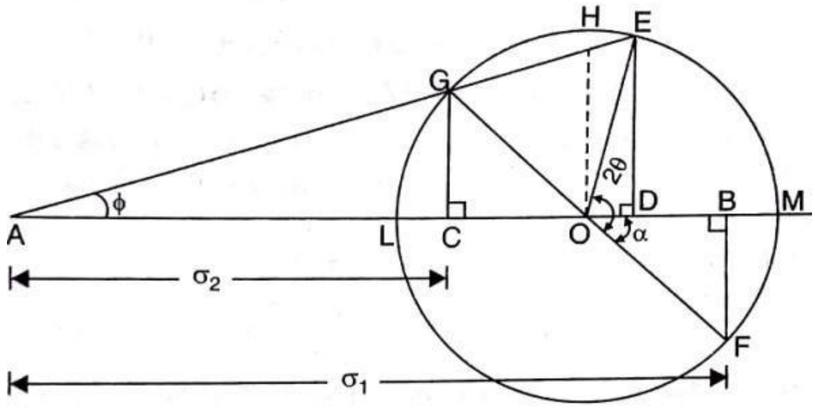
$\theta$ = Angle made by the oblique plane with the axis of minor tensile stress.

According to the principle of shear stress, the faces AB and CD will also be subjected to a shear stress of $r$

Mohr's Circle is drawn as follows:

Take any point A and draw a horizontal line through A. Take AB = $\sigma_1$ and AC = $\sigma_2$ towards right from A to some suitable scale. Draw perpendiculars at B and C and cut off BF and CG equal to shear stress $r$ to the same scale. Bisect BC at O. Now with O as centre and radius equal to OG or OF draw a circle. Through O, draw a line OE making an angle 2θ with OF as shown in Fig.

From E, draw ED perpendicular on CB.Join AE. Then the normal and tangential stresses on the oblique plane are given by AD and ED respectively. The resultant stress on the oblique plane is given by AE




From Fig.

Length AD = Normal stress on oblique plane

Length ED = Tangential stress on oblique plane.

Length AE = Resultant stress on oblique plane.

Radius of Mohr's circle = $\frac{\sigma_1 - \sigma_2}{2}$

Angle $\phi$ = obliquity

**Problem1.27.** A rectangular block of material is subjected to a tensile stress of 65N/mm$^2$ on one plane and a tensile stress of 35N/mm$^2$ on the plane right angles on the former. Each of the above stresses is accompanied by a shear stress of 25N/mm$^2$. Determine the Normal and Tangential stress a plane inclined at 45$^o$ to the axis of major stress.

**Given Data**

Major principal stress, $\sigma_1$ = 65N/mm$^2$

Minor principal stress, $\sigma_2$ = 35N/mm$^2$

Shear stress, $r$ = 25N/mm$^2$

Angle of oblique plane with the a×is of minor principal stress,

$\theta = 90^o - 45^o = 45^o$

**To Find**

The Normal stress and Tangential stress.

**Solution**

Scale. Let 1cm =10N/mm$^2$

Then $\sigma_1 = \frac{65}{10} = 6.5$cm ,

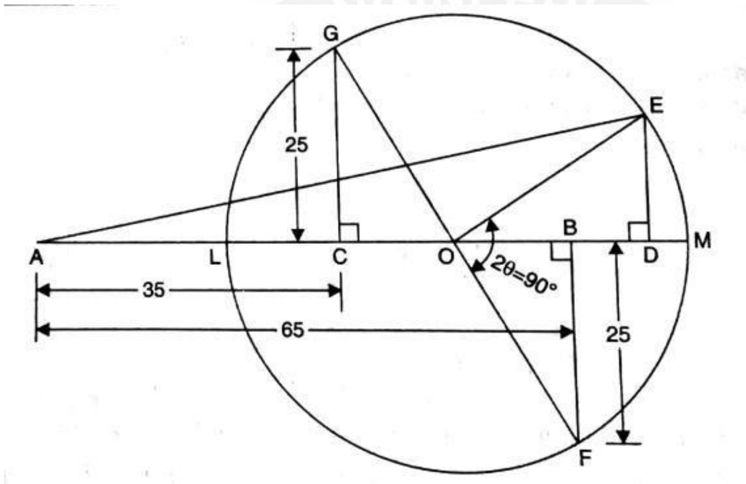
$\sigma_2 = \frac{35}{10} = 3.5$cm and

$r = \frac{25}{10} = 2.5$cm

Mohr's circle is drawn as:

Take any point A and draw a horizontal line through A on both sides of A as shown in fig. Take AB = $\sigma_1$ =6.5cm and AC = $\sigma_2$ =3.5cm towards right of A to some suitable scale. Draw perpendiculars at B and C and cut off BF and CG equal to shear stress $r$ = 2.5cm to the same scale. Bisect BC at O. Now with O as centre and radius equal to OG or OF draw a circle. Through O, draw a line OE making an angle 2θ(i.e 2×45 =90) with OF as shown in Fig.

From E, draw ED perpendicular on CB.Join AE. Then the normal and tangential stresses on the oblique plane are given by AD and ED respectively. The resultant stress on the oblique plane is given by AE




By measurements :

Length AD = 7.5 cm and

Length ED = 1.5 cm

Then normal stress = Length AD × Scale

= 7.5× 10 = **75N/mm$^2$**

Tangential or shear stress= Length ED × Scale

= 1.5×10 = **15 N/mm$^2$**.

## IMPORTANT TERMS

| | | |
|---|---|---|
| Stress($\sigma$) | $\sigma = \frac{p}{A}$ | $p = Load$<br>$A = area\ of\ cross\ section$ |
| Strain(e) | $e = \frac{dl}{l}$ | $dl = change$ in length<br>$l = original\ length$ |
| Lateral strain | $= \frac{dd}{d} = \frac{dt}{t} = \frac{db}{b}$ | d = diameter<br>t = thickness<br>b = width |
| Young's Modulus(E) | $E = \frac{\sigma}{e}$ | $\sigma = stress$<br>$e = strain$ |
| Shear modulus (or)<br>Modulus of rigidity(C) | $C = \frac{r}{\varphi}$ | $r = shear\ stress$<br>$\varphi = shear\ strain$ |

| | | |
|---|---|---|
| Total change in length of a bar | $dl = \frac{p}{E}[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3} + \cdots]$ | For same material (E = same) with different length and diameter |
| Total change in length | $dl = p[\frac{L_1}{A_1E_1} + \frac{L_2}{A_2E_2} + \cdots]$ | For different material with different length and diameter |
| For composite bar | Total load P = $p_1 + p_2 + \cdots$<br>Strain $e_1 = e_2 = \cdots$<br>Change in length are same | |
| Total change in length of uniform taper rod | $dl = \frac{4PL}{\pi E d_1 d_2}$ | P = load act on the section<br>L = length of the section<br>E = Young's modulus<br>$d_1$, $d_2$ = lager & smaller dia. |
| Total change in length of uniform taper rectangular bar | $dl = \frac{PL}{E\,t\,(a-b)} \log_e \frac{a}{b}$ | t = thickness of bar<br>a = width at bigger end<br>b = width at smaller end |
| Factor of safety | $F.S = \frac{Ultimate\ Stress}{Working\ Stress}$ | |
| Poisson's ratio(μ) | $\mu = \frac{Lateral\ strain}{Linear\ strain}$ | $e_{latl} = \mu \times e$ |
| For three dimensional stress system | $e_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{A_2} - \mu\frac{\sigma_3}{A_3}$ | Similar for other direction |
| Total change in length due to self-weight | $dl = \frac{wl^2}{2E}$ | w = weight per unit volume of bar |
| Volumetric strain($e_v$)<br>= | $e_v = \frac{dl}{l}(1 - 2\mu)$ | For one dimension rectangular bar |
| | $e_v = \frac{1}{E}(\sigma_x + \sigma_y + \sigma_z)(1 - 2\mu)$ | For three dimension cuboid<br>$\sigma_x = \frac{Load\ in\ x\ direction}{Area\ in\ x\ direction}$<br>Similar for $\sigma_y$ $\sigma_z$ |
| | $e_v = \frac{dl}{l} - 2\frac{dd}{d}$ | For cylindrical rod |
| Bulk modulus(K) | $K = \frac{\sigma}{(dV/V)}$ | |
| Relation between elastic constant E, K, C | $E = 3K(1 - 2\mu)$<br><br>$E = 2C(1 + \mu)$ | |

PRINCIPAL STRESSES AND STRAINS

*A member subjected to a direct stress in one plane*

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|---|---|---|---|
| Direct stress($\sigma$) | $\sigma = \frac{P}{A}$ | | P = load applied |
| Normal stress | $\sigma_n = \sigma \cos^2 \theta$ | $\sigma_n = r\sin 2\theta$ | |
| Tangential (or) shear stress | $\sigma_t = \frac{\sigma}{2}\sin 2\theta$ | $\sigma_t = -r\cos 2\theta$ | |

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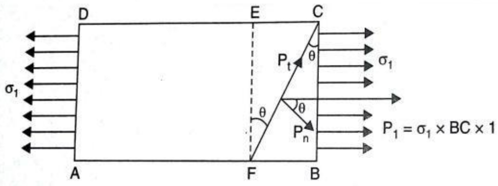
| Max. Normal stress | $= \sigma$ | | A = area of cross section<br>$\theta$ = angle of oblique plane with the normal cross section of the bar<br><br>$r = shear\ stress$ |
| Max. shear (or) Tangential stress | $= \frac{\sigma}{2}$ | | |
| A member subjected to two like stress in mutually perpendicular direction | | | |
| Normal stress | $\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} cos2\theta$ | | $\sigma_1 = Major\ tensile\ stress$<br>$\sigma_2 = Minor\ tensile\ stress$<br>$\theta$ = angle of oblique plane with the normal cross section of the bar<br><br><u>When compressive stress put – ve sign</u><br><br><u>When tensile force is given, we have to find tensile stress = force/that cross section area</u> |
| Tangential (or) shear stress | $\sigma_t = \frac{\sigma_1 - \sigma_2}{2} sin2\theta$ | | |
| Resultant stress | $\sigma_R = \sqrt{\sigma_n{}^2 + \sigma_t{}^2}$ | | |
| Position of obliquity | $\emptyset = \tan^{-1} \frac{\sigma_t}{\sigma_n}$ | | |
| Max. shear stress | $(\sigma_t)_{max} = \frac{\sigma_1 - \sigma_2}{2}$ | | |
| A member subjected to two like stress in mutually perpendicular direction with shear stress | | | |
| Normal stress | $\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} cos2\theta + rsin2\theta$ | | $\sigma_1 = Major\ tensile\ stress$<br>$\sigma_2 = Minor\ tensile\ stress$<br>$\theta$ = angle of oblique plane with the normal cross section of the bar<br><br><u>When compressive stress put – ve sign</u><br><br><u>When tensile force is given, we have to find tensile stress = force/that cross section area</u><br><br><u>When inclined stress is given it should be resolved into tensile stress and shear stress</u> |
| Tangential (or) shear stress | $\sigma_t = \frac{\sigma_1 - \sigma_2}{2} sin2\theta - rcos2\theta$ | | |
| Resultant stress | $\sigma_R = \sqrt{\sigma_n{}^2 + \sigma_t{}^2}$ | | |
| Position of principal plane | $tan2\theta = \frac{2r}{\sigma_1 - \sigma_2}$ | | |
| Max. shear (or) Tangential stress | $(\sigma_t)_{max} = \sqrt{(\frac{\sigma_1 - \sigma_2}{2})^2 + r^2}$ | | |
| Position of max. shear (or) Tangential stress | $tan2\theta = \frac{\sigma_2 - \sigma_1}{2r}$ | | |
| Major principal stress | $\frac{\sigma_1 + \sigma_2}{2} + \sqrt{(\frac{\sigma_1 - \sigma_2}{2})^2 + r^2}$ | | |
| Minor principal stress | $\frac{\sigma_1 + \sigma_2}{2} - \sqrt{(\frac{\sigma_1 - \sigma_2}{2})^2 + r^2}$ | | |
| Mohr,s Circle | | | |

| | |
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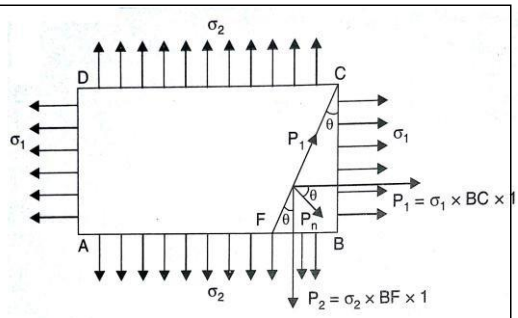
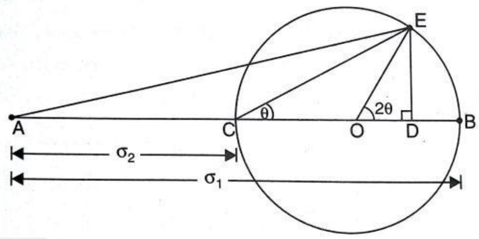
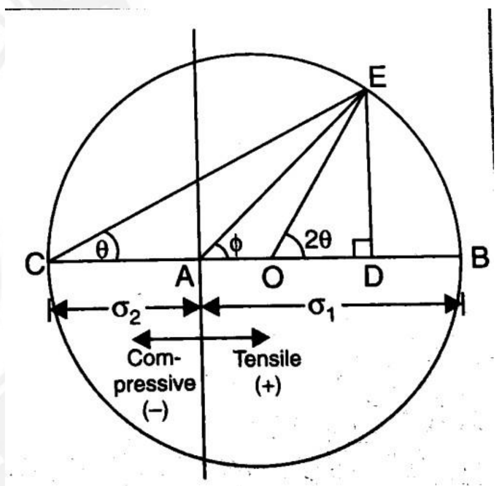
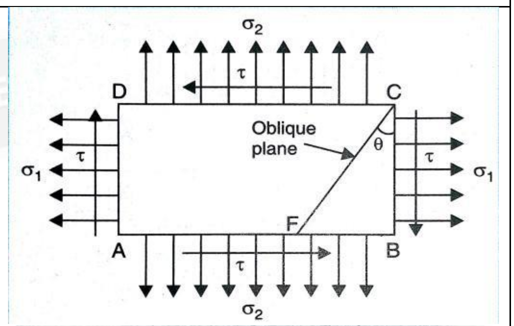
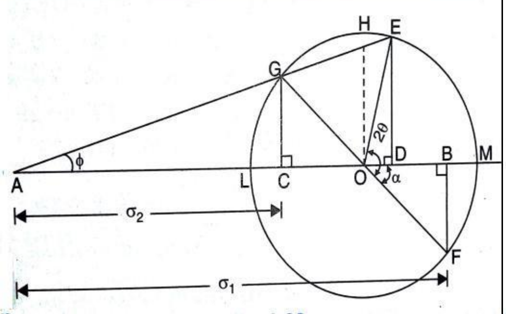
| A body subjected to two mutually perpendicular principal tensile stresses<br><br>Step1: select suitable scale<br>Step2: to draw a horizontal line AB = $\sigma_1$<br>Step3: to draw AC = $\sigma_2$<br>Step4: draw a circle with BC as diameter with O as centre<br>Step4: draw a line OE making an angle $2\theta$ with OB<br>Step5: from E to draw ED perpendicular to AB<br>Result:<br>Length AD = Normal stress<br>Length ED = Tangential (or) shear stress<br>Length AE = Resultant stress<br>Length OC = OB = Radius of mohr's circle = Max.shear stress<br>Angle of obliquity = 2Ø = ∠EAD | <br><br><br> |
| A body subjected to two mutually perpendicular principal tensile stresses which are unlike (Tensile and compressive)<br><br>All the above procedure are same but *step3* will be varied. Because for compressive stress is in –ve sign, hence to draw a line AC in negative directon | <br> |
| A body subjected to two mutually perpendicular principal tensile stresses with simple shear stress<br>Step1: select suitable scale<br>Step2: to draw a horizontal line AB = $\sigma_1$<br>Step3: to draw AC = $\sigma_2$<br>Step4: draw a perpendicular at B and C as BF and CG = $r$<br>Step5: joint the point G & F which intersect line BC at O.<br>Step6: draw a circle with O as centre and OG = OF as radius. Step7: draw a line OE making an angle $2\theta$ with OF<br>Step8: from E to draw ED perpendicular to AB<br>Result:<br>Length AD = Normal stress<br>Length ED = Tangential (or) shear stress<br>Length AE = Resultant stress<br>Length OG = OF = Radius of mohr's circle = Max.shear stress<br>Angle of obliquity = 2Ø = ∠EAD<br>Length AM= Max. Normalstress | <br><br><br> |

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|---|---|
| Length AL =Min. Normal stress | |

## THEORETICAL QUESTIONS

**TWO MARKS:**

1. Define stress and its types
2. Define strain.
3. Define tensile stress and tensile strain.
4. Define the three Elastic moduli.
5. Define shear strain and Volumetric strain