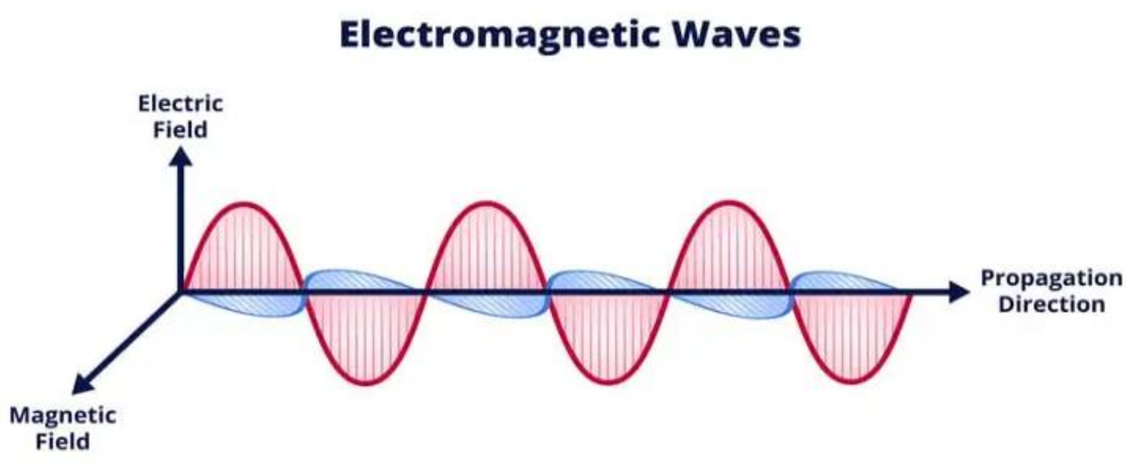
Plane Waves in Lossy Media

Wave Propagation in Lossy Dielectrics

- A lossy dielectric can be described as a medium where some fraction of the electromagnetic wave power is lost as the wave propagates. This power loss is due to poor conduction.
- Attenuation constant (α) is the measure of the spatial rate of decay of the electromagnetic wave in the medium, measured in nepers per meter (Np/m) or in decibels per meter (dB/m).
- Phase constant (**β**) is the measure of the phase shift per length and is called the phase constant or wave number.



Electromagnetic wave propagation

None of the dielectrics available today are lossless—every dielectric offers some kind of loss to the electromagnetic wave propagation going through it. The wave propagation in a lossy dielectric can be considered the general case of wave propagation. Let's take a closer look at electromagnetic wave propagation in lossy dielectrics.

Explaining Wave Propagation in Lossy Dielectrics

A lossy dielectric can be described as a medium where some fraction of the electromagnetic wave power is lost as the wave propagates. This power loss is due to poor conduction. A lossy dielectric offers a partially conducting medium with conductivity $\sigma \neq 0$. The lossy dielectric can be represented with the conductivity, permeability, and permittivity parameters as follows:

Lossy dielectrics ($\sigma \neq 0, \varepsilon = \varepsilon_r\varepsilon_o, \mu = \mu_r\mu_o$)

Maxwell's equations governing a linear, isotropic, homogenous, charge-free lossy dielectric can be given by equations (1) to (4):

$$\nabla \cdot \mathbf{E}_s = 0 \qquad (1)$$

$$\nabla \cdot \mathbf{H}_s = 0 \qquad (2)$$

$$\nabla \times \mathbf{E}_s = -j\omega\mu\mathbf{H}_s \qquad (3)$$

$$\nabla \times \mathbf{H}_s = (\sigma + j\omega\varepsilon)\mathbf{E}_s \qquad (4)$$

By taking the curl on both sides of equations (3) and (4), we can obtain Helmholtz's equations or the wave equations given by equations (5) and (6), respectively.

$$\nabla^2\mathbf{E}_s - \gamma^2\mathbf{E}_s = 0 \qquad (5)$$

$$\nabla^2\mathbf{H}_s - \gamma^2\mathbf{H}_s = 0 \qquad (6)$$

Where the following is true:

$$\gamma = \alpha + j\beta \qquad (7)$$

γ, α, and β are propagation constant, attenuation constant and phase constant respectively

In terms of dielectric material property, **α** and **β** can be given as:

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]} \quad (8)$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]} \quad (9)$$

With these considerations, if we assume that an electromagnetic wave is propagating along +az direction and Es is the electric field that has only an x component, then the following is true:

$$\mathbf{E}_s = E_{xs}(z)\mathbf{a}_x \quad (10)$$

Substituting equation (10) into equation (5), the scalar wave equation is:

$$E_{xs}(z) = E_o e^{-\gamma z} + E'_o e^{\gamma z} \quad (11)$$

According to the assumption that the field must be finite at infinity, $E'_o = 0$. Equation (11) can be rewritten using factor $e^{-\alpha z}$ as:

$$\mathbf{E}(z, t) = E_o e^{-\alpha z}\cos(\omega t - \beta z)\mathbf{a}_x \quad (12)$$

From equation (12), the magnitude of E versus t can be plotted. Only the x component of E travels along the +z direction. H(z,t) can also be obtained in a similar way, and can be given as:

$$\mathbf{H}(z, t) = \text{Re}\,(H_o e^{-\alpha z} e^{j(\omega t - \beta z)}\,\mathbf{a}_y) \quad (13)$$

where

$$H_o = \frac{E_o}{\eta} \quad (14)$$

η is the intrinsic impedance of the medium, given by:

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = |\eta|\angle\theta_\eta = |\eta|e^{j\theta_\eta} \quad (15)$$

Substituting (14) and (15) into (13), H can be written as:

$$\mathbf{H} = \frac{E_o}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)\, \mathbf{a}_y \quad (16)$$

The Attenuation Constant, Phase Constant, and Loss Tangent

From equation (16), we can conclude that as the wave propagates through a lossy dielectric along the z-direction, the amplitude is attenuated by a factor $e^{-\alpha z}$ and that is why α is called the attenuation constant, or attenuation factor, of the medium.

The attenuation constant (α) is the measure of the spatial rate of decay of the electromagnetic wave in the medium, and is measured in nepers per meter (Np/m) or in decibels per meter (dB/m). The phase constant (**β**) is the measure of the phase shift per length and is called the phase constant or wave number.

From equations (12) and (16), we can say that at any instance of time, E and H are out of phase by θ_n due to the complex intrinsic impedance of the medium. E leads H by θ_n. In a lossy dielectric medium, the ratio of the conduction current density Js to displacement current density J_{ds} is given by equation (17), where tan θ is called loss tangent and θ is the loss angle of the medium.

$$\frac{|\mathbf{J}_s|}{|\mathbf{J}_{ds}|} = \frac{|\sigma \mathbf{E}_s|}{|j\omega\varepsilon \mathbf{E}_s|} = \frac{\sigma}{\omega\varepsilon} = \tan\theta \quad (17)$$

Wave propagation in the lossy dielectric is a generalized approach to understanding the propagation through other types of mediums.