## STRUCTURES FOR IIR SYSTEMS:

IIR Systems are represented in four different ways

1. Direct Form Structures Form I and Form II
2. Cascade Form Structure
3. Parallel Form Structure
4. Lattice and Lattice-Ladder structure.

## DIRECT FORM-I :

***Challenge: Obtain the direct form-I, direct form-II,Cascade and parallel form realization of the system y(n)=-0.1y(n-1)+0.2y(n-2)+3x(n)+3.6x(n-1)+0.6x(n-2) [April/May-2015]***

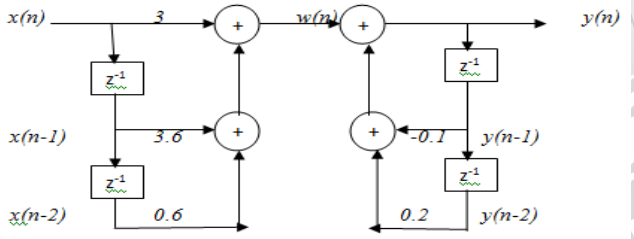
**Solution:**

**Direct Form I:**

$Let\ 3x(n)+3.6x(n-1)+0.6x(n-2)=w(n)$

$y(n)=-0.1y(n-1)+0.2y(n-2)+w(n)$

*The direct form I realization is*

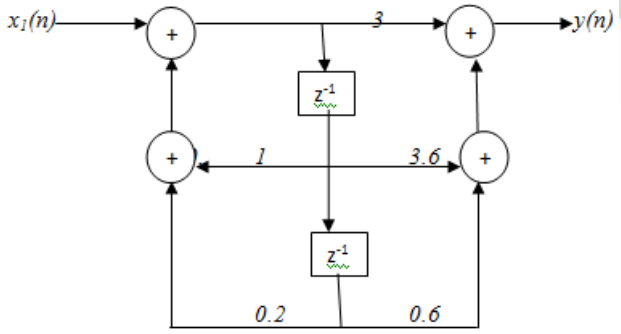



**Direct form II:**

From the given difference equation we have

$$H(z)=\frac{Y(z)}{X(z)}=\frac{3+3.6z^{-1}+0.6z^{-2}}{1+0.1z^{-1}-0.2z^{-2}}$$

The above system function can be realized in direct form II



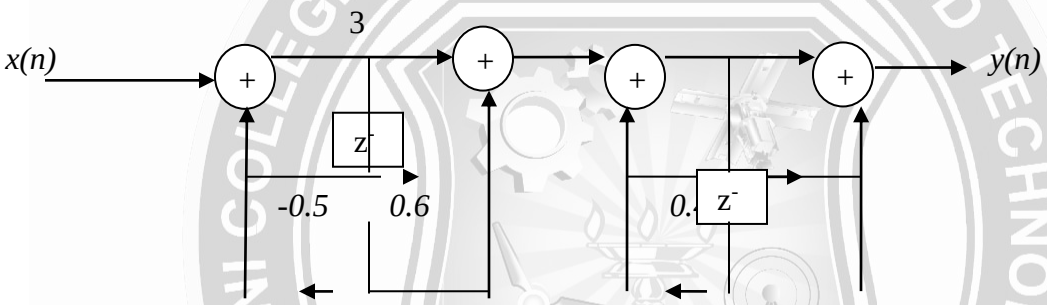
$$\frac{Y(z)}{X(z)} = \frac{3+3.6z^{-} +0.6z^{-}}{1+0.1z^{-} -0.2z^{-}}$$

$$= \frac{(3+0.6z^{-})(1+z^{-})}{(1+0.5z^{-})(1-0.4z^{-})}$$

$$H(z) = \frac{3+0.6z^{-}}{1+0.5z^{-}}$$

$$H(z) = \frac{1+z^{-}}{1-0.4z^{-1}}$$

*Now we realize* $H_1(z)$ *and* $H_2(z)$ and cascade both to get realization of H(z)

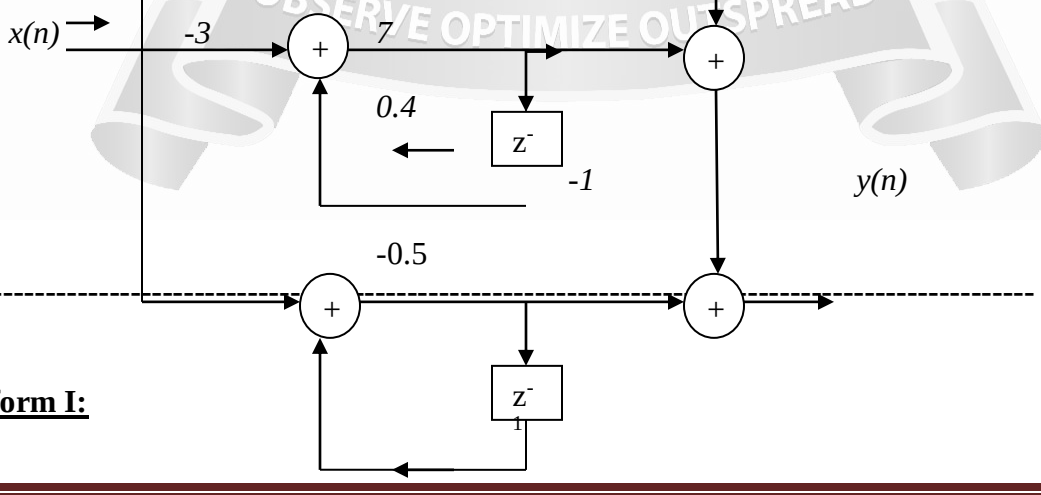



**Parallel form:**

$$H(z) = \frac{3+3.6z^{-1}+0.6z^{-2}}{1+0.1z^{-1}-0.2z^{-2}}$$

$$= -3 + \frac{7}{1-0.4z^{-1}} - \frac{1}{1+0.5z^{-1}}$$

$$= c + H_1(z) + H_2(z)$$




**Direct form I:**

**H.W: Obtain the direct form-I realization for the system described by the following difference equations.**

(*i*) $y(n) = 2y(n-1) + 3y(n-2) + x(n) + 2x(n-1) + 3x(n-2)$

(*ii*) $y(n) = 0.5y(n-1) + 0.06y(n-2) + 0.3x(n) + 0.5x(n-1)$

**Obtain the direct form-I realization for the system described by difference equation**

$y(n) = 0.5y(n-1) - 0.25y(n-2) + x(n) + 0.4x(n-1)$

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**Direct form II**

**H.W: Determine the direct form II realization for the following system**

(*i*) $y(n) + y(n-1) - 4y(n-3) = x(n) + 3x(n-2)$

(*ii*) $y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n) + \frac{1}{2}x(n-1)$ **[May/June-14]**

**Determine the direct form II realization for the following system**

$y(n) = -0.1y(n-1) + 0.72y(n-2) + 0.7x(n) - 0.252x(n-2)$

**CASCADE FORM:**

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**H.W: For the system function** $H(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$ **obtain cascade structure.**

**Realize the system with difference equation** $y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n) + \frac{1}{3}x(n-1)$ **in cascade form.**

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**Parallel form:**

**H.W: Realize the system given by difference equation**

$y(n) = -0.1y(n-1) + 0.72y(n-2) + 0.7x(n) - 0.252x(n-2)$ **in parallel form.**

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**Analog filter design:**

There are two types of analog filter design are,

- Butterworth Filter
- Chebyshev Filter.

## *Analog Low pass Butterworth Filter:*

| N | Denominator of H(s) |
|---|---|
| 1 | $S+1$ |
| 2 | $s^2 + \sqrt{2}s + 1$ |
| 3 | $(s+1)(s^2 + s + 1)$ |
| 4 | $(s^2 + 0.76537s + 1)(s^2 + 1.8477s + 1)$ |
| 5 | $(s+1)(s^2 + 0.61803s + 1)(s^2 + 1.61803s + 1)$ |
| 6 | $(s^2 + 1.931855s + 1)(s^2 + \sqrt{2}s + 1)(s^2 + 0.51764s + 1)$ |

# Problem

***Obtain the parallel form realisation of the system government by the equation*** $y(n) = -3/8\, y(n-1) + 3/32\, y(n-2) + 1/64\, y(n-3) + x(n) + 3x(n-1) + 2x(n-2)$

- Soln.: Taking z-transform on both sides

$$Y(z) = -\frac{3}{8}Y(z)z^{-1} + \frac{3}{32}Y(z)z^{-2} + \frac{1}{64}Y(z)z^{-3} + X(z) + 3X(z)z^{-1} + 2X(z)z^{-2}$$

$$Y(z)\left[1 + \frac{3}{8}z^{-1} - \frac{3}{32}z^{-2} - \frac{1}{64}z^{-3}\right] = X(z)\left[1 + 3z^{-1} + 2z^{-2}\right]$$

$$\frac{Y(z)}{X(z)} = \frac{\left[1 + 3z^{-1} + 2z^{-2}\right]}{\left[1 + \frac{3}{8}z^{-1} - \frac{3}{32}z^{-2} - \frac{1}{64}z^{-3}\right]}$$

# Problem Cont..

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z[z+2][z+1]}{[z-0.25][z+0.5][z+0.125]}$$

$$\frac{H(z)}{z} = \frac{[z+2][z+1]}{[z-0.25][z+0.5][z+0.125]}$$

$$\frac{H(z)}{z} = \frac{A}{[z-0.25]} + \frac{B}{[z+0.5]} + \frac{C}{[z+0.125]}$$

# Problem Cont..

$$A=\left[z-0.25\right]\frac{H(z)}{z}\Big|_{z=0.25}=\left[z-0.25\right]\frac{\left[z+2\right]\left[z+1\right]}{\left[z-0.25\right]\left[z+0.5\right]\left[z+0.125\right]}\Big|_{z=0.25}$$

- A=10

$$B=\left[z+0.5\right]\frac{H(z)}{z}\Big|_{z=-0.5}=\left[z+0.5\right]\frac{\left[z+2\right]\left[z+1\right]}{\left[z-0.25\right]\left[z+0.5\right]\left[z+0.125\right]}\Big|_{z=-0.5}$$

- B=2.666

$$C=\left[z+0.125\right]\frac{H(z)}{z}\Big|_{z=-0.125}=\left[z+0.125\right]\frac{\left[z+2\right]\left[z+1\right]}{\left[z-0.25\right]\left[z+0.5\right]\left[z+0.125\right]}\Big|_{z=-0.125}$$

- C=-11.67

# Problem Cont..

$$\frac{H(z)}{z} = \frac{10}{[z-0.25]} + \frac{2.666}{[z+0.5]} - \frac{11.67}{[z+0.125]}$$

$$H(z) = 10\frac{z}{[z-0.25]} + 2.666\frac{z}{[z+0.5]} - 11.67\frac{z}{[z+0.125]}$$

$$H(z) = 10\left[\frac{1}{1-0.25z^{-1}}\right] + 2.666\left[\frac{1}{1+0.5z^{-1}}\right] - 11.67\left[\frac{1}{1+0.125z^{-1}}\right]$$

$$H(z) = H_1(z) + H_2(z) + H_3(z)$$

# Problem Cont..

$$H_1(z) = \frac{Y_1(z)}{X(z)} = \frac{10}{1 - 0.25\,z^{-1}}$$

$$Y_1(z) = 10\,X(z) + 0.25\,z^{-1}\,Y_1(z)$$

$$H_2(z) = \frac{Y_2(z)}{X(z)} = \frac{2.66}{1 + 0.5\,z^{-1}}$$

$$Y_2(z) = 2.66\,X(z) - 0.5z^{-1}\,Y_2(z)$$

$$H_3(z) = \frac{Y_3(z)}{X(z)} = \frac{-11.67}{1 + 0.125\,z^{-1}}$$

$$Y_3(z) = -11.67\,X(z) - 0.125z^{-1}\,Y_3(z)$$

# Problem Cont..