## NORMAL DISTRIBUTIONS AND STANDARD (Z) SCORES

### THE NORMAL CURVE

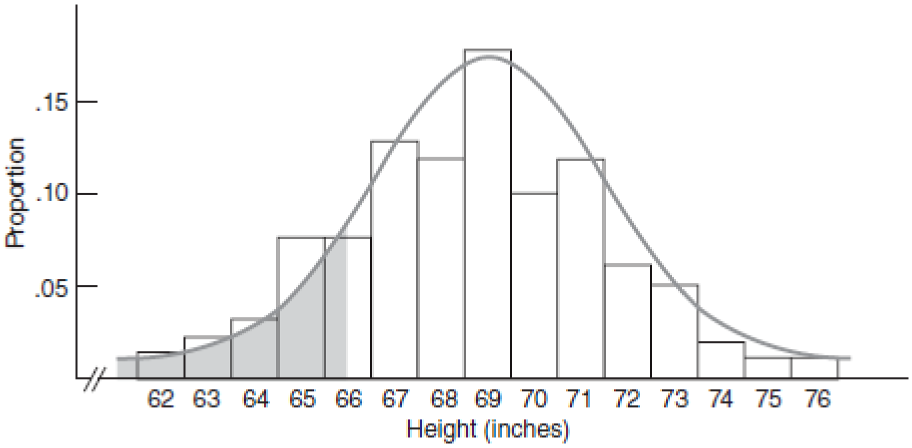
The normal distribution is a continuous probability distribution that is symmetrical on both sides of the mean, so the right side of the center is a mirror image of the left side.

**Properties of the Normal Curve**

- The normal curve is a theoretical curve defined for a continuous variable, as described in Section 1.6, and noted for its symmetrical bell-shaped form, as revealed in below figure.
- Because the normal curve is symmetrical, its lower half is the mirror image of its upper half.
- The normal curve peaks above a point midway along the horizontal spread and then tapers off gradually in either direction from the peak (without actually touching the horizontal axis, since, in theory, the tails of a normal curve extend infinitely far).
- The values of the mean, median (or 50th percentile), and mode, located at a point midway along the horizontal spread, are the same for the normal curve.

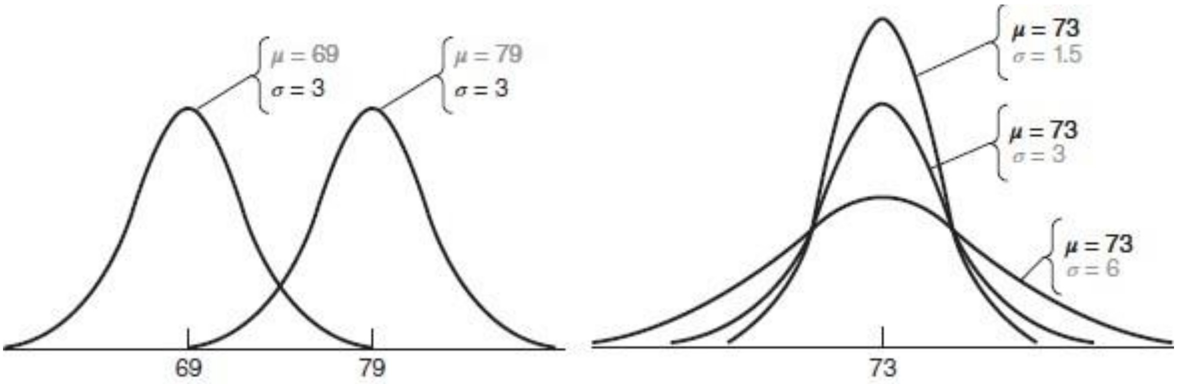
**Properties of a normal distribution**

- The mean, mode and median are all equal.
- The curve is symmetric at the center (i.e. around the mean, μ).
- Exactly half of the values are to the left of center and exactly half the values are to the right.
- The total area under the curve is 1




**Different Normal Curves**

As a theoretical exercise, it is instructive to note the various types of normal curves that are produced by an arbitrary change in the value of either the mean (μ) or the standard deviation (σ). Obvious differences in appearance among normal curves are less important than you might suspect. Because of their common mathematical origin, every normal curve can be interpreted in exactly the same way once any distance from the mean is expressed in standard deviation units.




A. Different Means, Same Standard Deviation    B. Same Mean, Different Standard Deviations

## z SCORES

A z score is a unit-free, standardized score that, regardless of the original units of measurement, indicates how many standard deviations a score is above or below the mean of its distribution.

A z score can be defined as a measure of the number of standard deviations by which a score is below or above the mean of a distribution. In other words, it is used to determine the distance of a score from the mean. If the z score is positive it indicates that the score is above the mean. If it is negative then the score will be below the mean. However, if the z score is 0 it denotes that the data point is the same as the mean.

To obtain a z score, express any original score, whether measured in inches, milliseconds, dollars, IQ points, etc., as a deviation from its mean (by subtracting its mean) and then split this deviation into standard deviation units (by dividing by its standard deviation),

$$z = \frac{X - \mu}{\sigma}$$

Where X is the original score and μ and σ are the mean and the standard deviation, respectively, for the normal distribution of the original scores. Since identical units of measurement appear in both the numerator and denominator of the ratio for z, the original units of measurement cancel each other and the z score emerges as a unit-free or standardized number, often referred to as a standard score.

A z score consists of two parts:

1. A positive or negative sign indicating whether it’s above or below the mean; and
2. A number indicating the size of its deviation from the mean in standard deviation units.

## STANDARD NORMAL CURVE

If the original distribution approximates a normal curve, then the shift to standard or z scores will always produce a new distribution that approximates the standard normal curve. This is the one normal curve for which a table is actually available. Although there is an infinite number of a different normal curve, each with its own mean and standard deviation, there is only one standard normal curve, with a mean of 0 and a standard deviation of 1.

For a standard normal curve

**Mean = 0**

$$\text{Mean of } z = \frac{X-\mu}{\sigma} = \frac{\mu-\mu}{\sigma} = \frac{0}{\sigma} = 0$$

**Standard deviation = 1**

$$\text{Standard deviation of } z = \frac{X-\mu}{\sigma} = \frac{\mu+1\sigma-\mu}{\sigma} = \frac{1\sigma}{\sigma} = 1$$

**Standard Normal Table**

The standard normal table consists of columns of z scores coordinated with columns of proportions

**Using the Top Legend of the Table**

Notice that columns are arranged in sets of three, designated as A, B, and C in the legend at the top of the table. When using the top legend, all entries refer to the upper half of the standard normal curve. The entries in column A are z scores, beginning with 0.00 and ending with 4.00

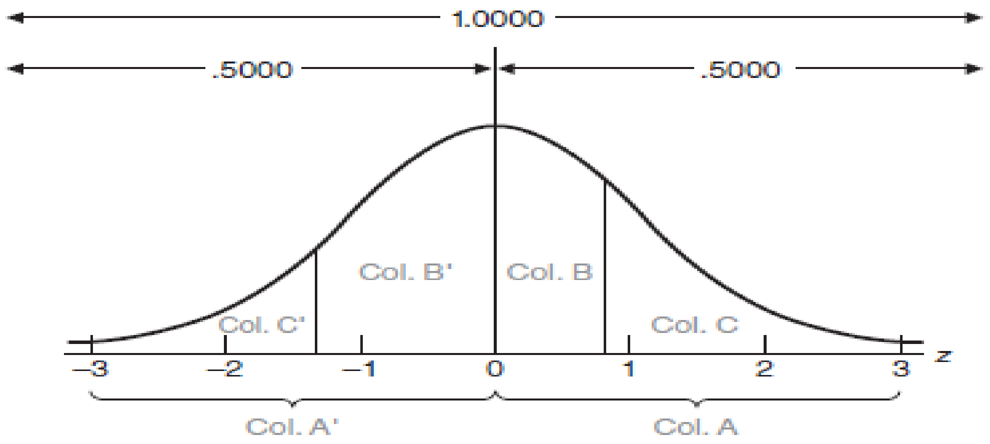
Given a z score of zero or more, columns B and C indicate how the z score splits the area in the upper half of the normal curve. As suggested by the shading in the top legend, column B indicates the proportion of area between the mean and the z score, and column C indicates the proportion of area beyond the z score, in the upper tail of the standard normal curve.

**Using the Bottom Legend of the Table**

Now the columns are designated as A′, B′, and C′ in the legend at the bottom of the table. When using the bottom legend, all entries refer to the lower half of the standard normal curve. A negative z score, columns B′ and C′ indicate how that z score splits the lower half of the normal curve. As suggested by the shading in the bottom legend of the table, column B′ indicates the proportion of area between the mean and the negative z score, and column C′ indicates the proportion of area beyond the negative z score, in the lower tail of the standard normal curve.

**TABLE 5.1**
**PROPORTIONS (OF AREAS) UNDER THE STANDARD NORMAL CURVE FOR VALUES OF *z* (FROM TABLE A OF APPENDIX C)**

| A | B | C | A | B | C | A | B | C |
|---|---|---|---|---|---|---|---|---|
| z | | | z | | | z | | |
| 0.00 | .0000 | .5000 | 0.40 | .1554 | .3446 | 0.80 | .2881 | .2119 |
| 0.01 | .0040 | .4960 | 0.41 | .1591 | .3409 | 0.81 | .2910 | .2090 |
| • | • | • | • | • | • | • | • | • |
| | | | | | | • | • | • |
| | | | | | | • | • | • |
| | | | | | | • | • | • |
| • | • | • | • | • | • | • | • | • |
| | | | | | | 0.99 | .3389 | .1611 |
| | | | | | | 1.00 | .3413 → | .1587 |
| • | • | • | • | • | • | 1.01 | .3438 | .1562 |
| | | | | | | • | • | • |
| | | | | | | • | • | • |
| • | • | • | • | • | • | • | • | • |
| 0.38 | .1480 | .3520 | 0.78 | .2823 | .2711 | 1.18 | .3810 | .1190 |
| 0.39 | .1517 | .3483 | 0.79 | .2852 | .2148 | 1.19 | .3830 | .1170 |
| −z | | | −z | | | −z | | |
| A′ | B′ | C′ | A′ | B′ | C′ | A′ | B′ | C′ |

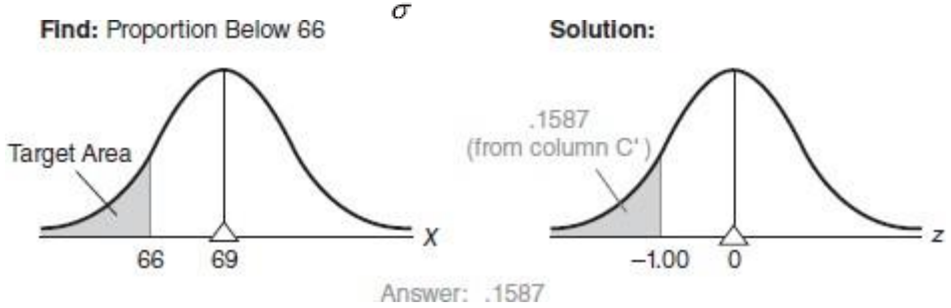



## FINDING PROPORTIONS

### Finding Proportions for One Score

- Sketch a normal curve and shade in the target area,
- Plan your solution according to the normal table.
- Convert X to z

$$z = \frac{X - \mu}{\sigma}$$




### Finding Proportions between Two Scores

- Sketch a normal curve and shade in the target area, (example, find proportion between 245 to 255)
- Plan your solution according to the normal table.
- Convert X to z by expressing 255 as

$$z = \frac{255 - 270}{15} = \frac{-15}{15} = -1.00$$

and by expressing 245 as

$$z = \frac{245 - 270}{15} = \frac{-25}{15} = -1.67$$