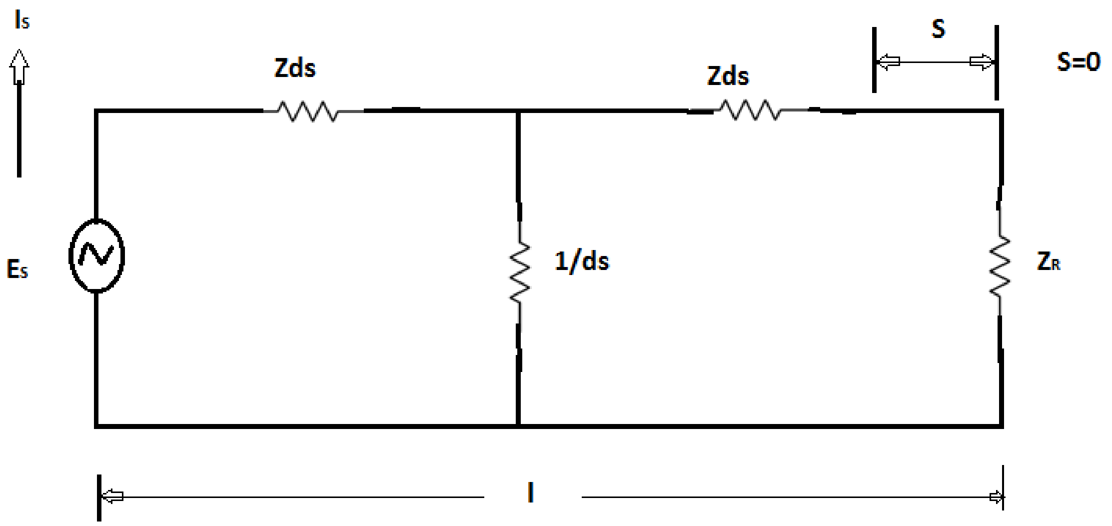
PHYSICAL SIGNIFICANCE OF TRANSMISSION LINE (or) INFINITE LINE (or) THE TWO STANDARD FORM FOR INPUT IMPEDANCE OF THE TRANSMISSION LINE TERMINATED BY AN IMPEDANCE Z_R.

Equation for the current and voltage may be written for the sending end current 'I_S' of a line of length 'l' is,



The sending current equation is given by,

$I_S = I_R \cosh\sqrt{ZY}.l + \frac{E_R}{Z_O} \sinh\sqrt{ZY}.l$ $[E_R = I_R Z_R]$

Sub E_R value in above equ,

$I_S = I_R \cosh\sqrt{ZY}.l + \frac{I_R Z_R}{Z_O} \sinh\sqrt{ZY}.l$

$I_S = I_R \left[\cosh\sqrt{ZY}.l + \frac{Z_R}{Z_O} \sinh\sqrt{ZY}.l\right]$ ....(1)

The sending voltage equation is given by,

$E_S = E_R \cosh\sqrt{ZY}.l + I_R Z_O \sinh\sqrt{ZY}.l$

$[E_R = I_R Z_R]$

$[I_R = \frac{E_R}{Z_R}]$

Sub I_R value in above equ,

$E_S = E_R \cosh\sqrt{ZY}.l + \frac{E_R Z_O}{Z_R} \sinh\sqrt{ZY}.l$

$E_S = E_R\left[\cosh\sqrt{ZY}.l + \frac{Z_O}{Z_R}\sinh\sqrt{ZY}.l\right]$ ....(2)

Since, We know that,

Propagation Constant $\gamma = \sqrt{ZY}$

Characteristic Impedance $Z_O = \sqrt{\frac{Z}{Y}}$

Sub γ value in equ (1) and (2),

From equ (1),

$I_S = I_R\left[\cosh\gamma l + \frac{Z_R}{Z_O}\sinh\gamma l\right]$ ....(3)

From equ (2),

$E_S = E_R\left[\cosh\gamma l + \frac{Z_O}{Z_R}\sinh\gamma l\right]$ ....(4)

Input Impedance $Z_S = \frac{E_S}{I_S}$ [E = IZ]

$$Z_S = \frac{E_R\left[\cosh\gamma l + \frac{Z_O}{Z_R}\sinh\gamma l\right]}{I_R\left[\cosh\gamma l + \frac{Z_R}{Z_O}\sinh\gamma l\right]}$$

$$Z_S = Z_R\left[\frac{\frac{Z_R\cosh\gamma l + Z_O\sinh\gamma l}{Z_R}}{\frac{Z_O\cosh\gamma l + Z_R\sinh\gamma l}{Z_O}}\right]$$

$E_R = I_R Z_R$

$Z_R = \frac{E_R}{I_R}$

$$Z_S = Z_R\left[\frac{Z_R\cosh\gamma l + Z_O\sinh\gamma l}{Z_R}\ X\ \frac{Z_O}{Z_O\cosh\gamma l + Z_R\sinh\gamma l}\right]$$

$$Z_S = Z_O\left[\frac{Z_R\cosh\gamma l + Z_O\sinh\gamma l}{Z_O\cosh\gamma l + Z_R\sinh\gamma l}\right] \quad ....(5)$$

This is the first standard form of input impedance of the transmission line.

$\text{Cosh}\theta = \frac{e^{\theta} + e^{-\theta}}{2}$

$\text{Sinh}\theta = \frac{e^{\theta} - e^{-\theta}}{2}$

Sub the above formula in equ (5),

$$Z_S = Z_O \left[\frac{Z_R\left(\frac{e^{\gamma l} + e^{-\gamma l}}{2}\right) + Z_O\left(\frac{e^{\gamma l} - e^{-\gamma l}}{2}\right)}{Z_O\left(\frac{e^{\gamma l} + e^{-\gamma l}}{2}\right) + Z_R\left(\frac{e^{\gamma l} - e^{-\gamma l}}{2}\right)} \right]$$

$$Z_S = \frac{2Z_O}{2} \left[\frac{Z_R\left(e^{\gamma l} + e^{-\gamma l}\right) + Z_O\left(e^{\gamma l} - e^{-\gamma l}\right)}{Z_O\left(e^{\gamma l} + e^{-\gamma l}\right) + Z_R\left(e^{\gamma l} - e^{-\gamma l}\right)} \right]$$

$$Z_S = Z_O \left[\frac{Z_R e^{\gamma l} + Z_R e^{-\gamma l} + Z_O e^{\gamma l} - Z_O e^{-\gamma l}}{Z_O e^{\gamma l} + Z_O e^{-\gamma l} + Z_R e^{\gamma l} - Z_R e^{-\gamma l}} \right]$$

$$Z_S = Z_O \left[\frac{e^{\gamma l}\,[Z_R + Z_O] + e^{-\gamma l}\,[Z_R - Z_O]}{e^{\gamma l}\,[Z_R + Z_O] - e^{-\gamma l}\,[Z_R - Z_O]} \right]$$

$$Z_S = Z_O \frac{[Z_R + Z_O]}{[Z_R + Z_O]} \left[\frac{e^{\gamma l} + e^{-\gamma l}\left[\frac{Z_R - Z_O}{Z_R + Z_O}\right]}{e^{\gamma l} - e^{-\gamma l}\left[\frac{Z_R - Z_O}{Z_R + Z_O}\right]} \right]$$

$$Z_S = Z_O \left[\frac{e^{\gamma l} + e^{-\gamma l}\left[\frac{Z_R - Z_O}{Z_R + Z_O}\right]}{e^{\gamma l} - e^{-\gamma l}\left[\frac{Z_R - Z_O}{Z_R + Z_O}\right]} \right] \qquad \ldots\ldots\ldots(6)$$

This is the second standard form of input impedance of the transmission line.