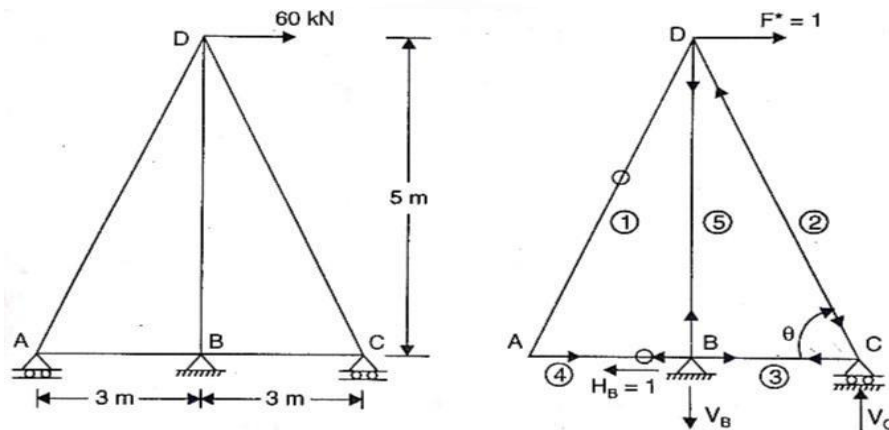


ANALYSIS OF INDETERMINATE PIN-JOINTED FRAMES BY FLEXIBILITY METHOD.

NUMERICAL PROBLEMS ON PIN-JOINTED FRAMES;

PROBLEM NO:01

Analysis the truss loaded as shown in fig, by using Matrix Flexibility method. And find the member forces. A and E are the same for all members.



Solution:

- Static indeterminacy:**

Degree of internal indeterminacy; $I = m - (2j - 3) = 5 - (2 \times 4 - 3) = 0$

Degree of external indeterminacy; $E = r - R = 4 - 3 = 1$

The structure is externally indeterminate to one degree. Treating support A as redundant, the primary structure is shown in fig,

To get the B matrix :

The B matrix would be in 2 parts, B_X and B_W . To get the B_X matrix let us apply a unit force ($F_X = 1$) at D as shown and get the member forces by method of joints.

Reactions: $H_B = 1$

Equating moments about C to zero,

$$V_B \times 3 = 1 \times 5 ; V_B = 1.667$$

Hence, $V_C = 1.667$

At joint A, $F_{AB} = F_{AD} = 0$

At joint B, $F_{BA} = V_B = 1.667$ (tension)

At joint C, $F_{CD} \sin \theta = V_C = 1.667$

$$F_{CD} = 1.667 / \sin \theta = 1.944 \text{ (comp)}$$

$$F_{BC} = F_{CD} \cos \theta = 1.00 \text{ (tallies)}$$

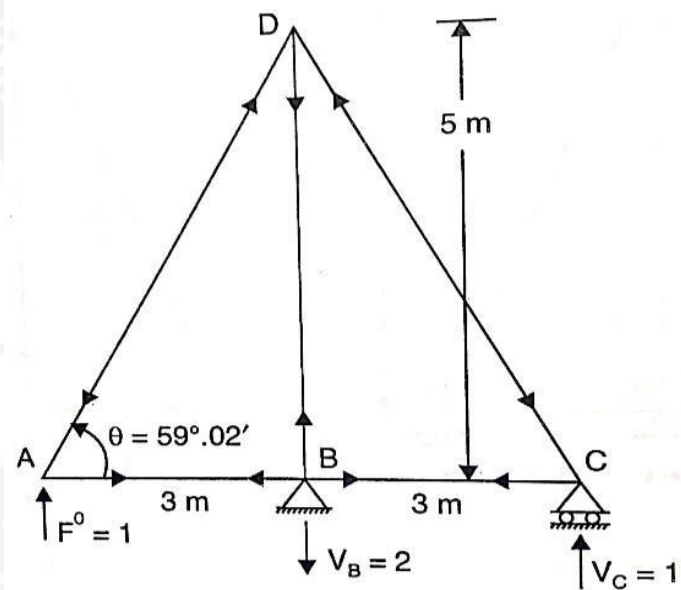
Thus, $B_X = [0 \quad -1.944 \quad 1.00 \quad 0 \quad 1.667]^T$

B_W is obtained by applying a unit force at the location of the redundant F_X

Equating to zero, moments about B,

$$V_C \times 3 = 1 \times 3; V_C = 1$$

From $\Sigma V = 0; V_B = 2$



At joint A, $F_{CD} \sin \theta = 1; F_{AD} = 1.667$ (comp)

$$F_{AB} = F_{AD} \cos \theta; F_{AB} = 0.60 \text{ (tension)}$$

At joint B, $F_{AB} = F_{BC} = 0.60$ (tension)

$$F_{BD} = 2 \text{ (tension)}$$

At joint C, $F_{CD} \sin \theta = 1; F_{CD} = 1.667$ (comp)

$$FBC = FCD \cos \theta = 0.60 \text{ (tallies)}$$

Hence $B_W = [-1.166 \quad -1.166 \quad 0.60 \quad 0.60 \quad 2.00]^T$

$$= \begin{bmatrix} 0 & -1.166 \\ -1.944 & -1.166 \\ 1.00 & 0.60 \\ 0 & 0.60 \\ 1.667 & 2.00 \end{bmatrix}$$

$$F_X = B_X^T \cdot F \cdot B_X$$

$$= [-1.166 \quad -1.166 \quad 0.6 \quad 0.6 \quad 2.0] \frac{1}{AE} \begin{bmatrix} 5.83 & & & & \\ & 5.83 & & & \\ & & 3.0 & & \\ & & & 3.0 & \\ & & & & 5.0 \end{bmatrix} \begin{bmatrix} -1.166 \\ -1.166 \\ 0.60 \\ 0.60 \\ 2.00 \end{bmatrix}$$

$$= [-6.798 \quad -6.798 \quad 1.8 \quad 1.8 \quad 10.0] \frac{1}{AE} \begin{bmatrix} -1.166 \\ -1.166 \\ 0.60 \\ 0.60 \\ 2.00 \end{bmatrix} = \frac{38.01}{AE}$$

$$F_W = B_X^T \cdot F \cdot B_W$$

$$= [-1.166 \quad -1.166 \quad 0.6 \quad 0.6 \quad 2.0] \frac{1}{AE} \begin{bmatrix} 5.83 & & & & \\ & 5.83 & & & \\ & & 3.0 & & \\ & & & 3.0 & \\ & & & & 5.0 \end{bmatrix} \begin{bmatrix} 0 \\ -1.944 \\ 1.00 \\ 0 \\ 1.667 \end{bmatrix}$$

$$= [-6.798 \quad -6.798 \quad 1.80 \quad 1.80 \quad 10.00] \frac{1}{AE} \begin{bmatrix} 0 \\ -1.944 \\ 1.00 \\ 0 \\ 1.667 \end{bmatrix} = \frac{31.59}{AE}$$

- Displacement Matrix (X):

$$= -\frac{AE}{38.01} \times \frac{31.59}{AE} \times 60 = -49.87 \text{ kN}$$

• **Final Moments (P):**

$$\mathbf{P} = \boldsymbol{\mu} + \mathbf{F}$$

$$\{\mathbf{P}\} = \begin{bmatrix} 0 & -1.667 \\ -1.944 & -1.166 \\ 1.00 & 0.6 \\ 0 & 0.6 \\ 1.667 & 2.0 \end{bmatrix} \begin{Bmatrix} 60 \\ -49.87 \end{Bmatrix} = \begin{Bmatrix} 58.15 \\ -58.25 \\ 30.08 \\ -29.92 \\ -0.26 \end{Bmatrix}$$

