

### 3.3 ANALYSIS OF THREE HINGED PARABOLIC ARCHES

#### Three hinged arch

The three hinged arches are statically determinate structures in which the horizontal movement at the support is prevented to a certain extent or it is wholly prevented. This type of arch has two hinges at the end supports and also an supplementary intermediate hinge at the crown.

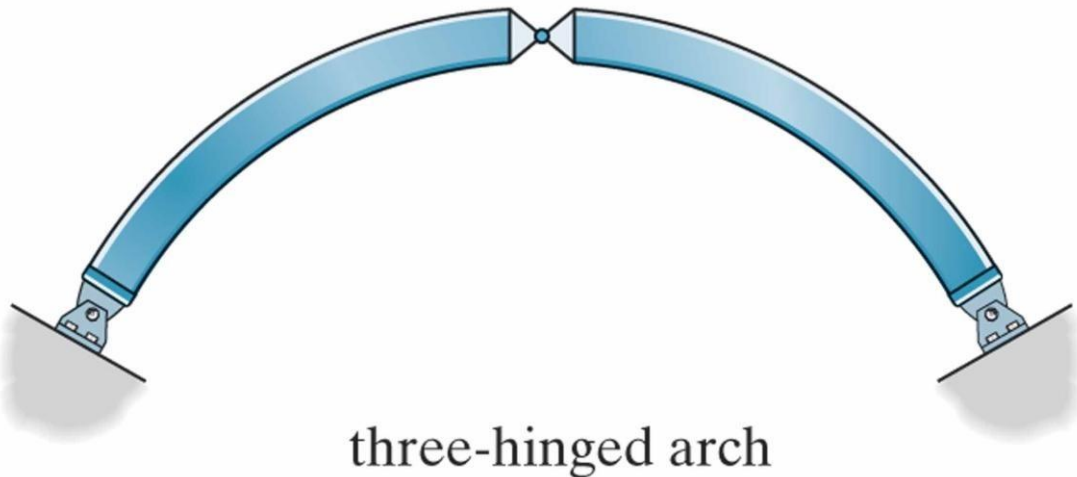


Fig. 3.3.1

#### Three hinged parabolic arches

An arch which is hinged at three points and whose axis represents a parabolic shape is known as a three hinged parabolic arch.

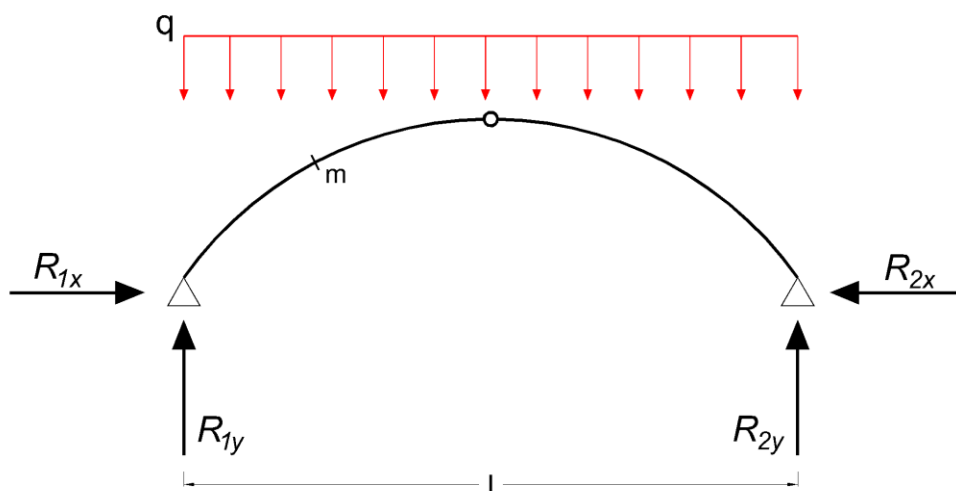


Fig. 3.3.2 Three hinged parabolic arches

## Analysis of 3-hinged arches

It is the process of determining external reactions at the support and internal quantities such as normal thrust, shear and bending moment at any section in the arch.

### Procedure to find reactions at the supports

Step 1. Sketch the arch with the loads and reactions at the support.

Apply equilibrium conditions namely  $\sum F_x = 0$ ,  $\sum F_y = 0$  and  $\sum M = 0$

Apply the condition that BM about the hinge at the crown is zero (Moment of all the forces either to the left or to the right of the crown).

### Example:

A 3hinged arch of span 40m and rise 8m carries concentrated load of 200kN and 150kN at distance of 8m and 16m from the left end and an UDL of 50k/N on the right half of the span.

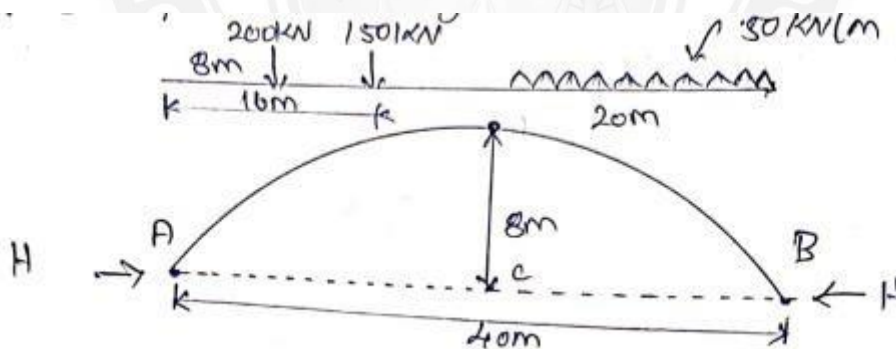


Fig. 3.3.3

## SOLUTION

### a) vertical reactions $V_A$ and $V_B$

Taking moment about A

$$(200 \times 8) + (150 \times 16) + (50 \times 20 \times (20 + 20/2)) - V_B \times 40 = 0$$

$$1600 + 2400 + 30000 - V_B \times 40 = 0$$

$$V_B = 850 \text{ KN}$$

$$\text{Total load} = V_A + V_B$$

$$200 + 150 + (50 \times 20) = 850 + V_A$$

$$V_A = 500 \text{ KN}$$

### b) Horizontal thrust (H)

Taking moment about C

$$H \times 8 - V_A(20) + (200 \times 12) + (150 \times 4) = 0$$

$$8H - 4000 + 2400 + 600 = 0$$

$$H = 325 \text{ KN}$$

### Example:

A parabolic 3 hinged arch carries loads as shown in fig. Determine the resultant reactions at supports. Find the bending moment normal thrust and radial shear at D, 5m from A. what is the max bending.

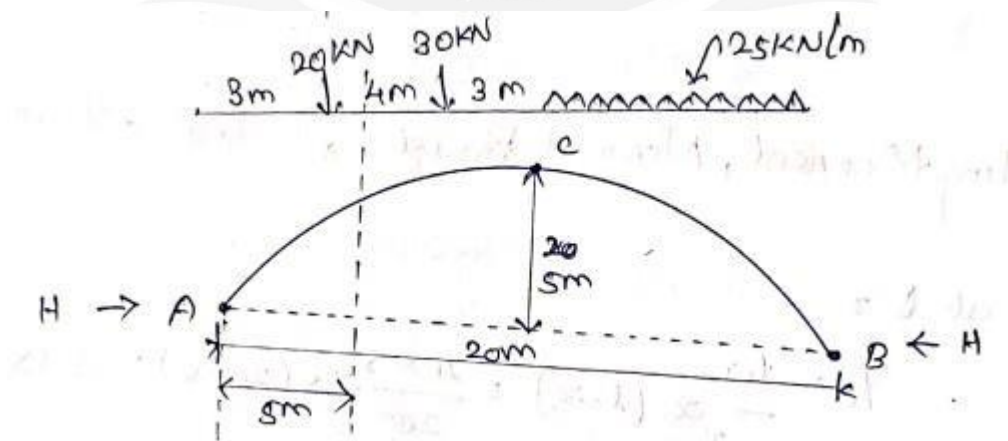


Fig. 3.3.4

**SOLUTION**

Taking moment about A

$$(20 \times 3) + (30 \times 7) + [25 \times 10 \times (10 + 10/2)] - V_B \times 20 = 0$$

$$V_B = 201 \text{ KN}$$

$$V_A = 99 \text{ KN}$$

**Horizontal pull**

$$(H \times 5) + (20 \times 7) + (30 \times 3) - V_A \times 10 = 0$$

$$5H - 140 + 90 - 990 = 0$$

$$H = 152 \text{ KN}$$

**Resultant Reaction (RA and RB)**

$$\begin{aligned} R_A &= \sqrt{H^2 + V_A^2} \\ &= \sqrt{152^2 + 99^2} \\ &= 181.39 \text{ KN} \end{aligned}$$

$$\begin{aligned} R_B &= \sqrt{H^2 + V_B^2} \\ &= \sqrt{152^2 + 201^2} \\ &= 252 \text{ KN} \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1} V_A / H \\ &= \tan^{-1} 99 / 152 \\ &= 33^\circ 4' 36''.6 \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1} V_B / H \\ &= \tan^{-1} 201 / 152 \\ &= 52^\circ 54' 9''.86 \end{aligned}$$

**Bending moment, Normal thrust, radial SF at D****BM at D**

$$\begin{aligned}
 Y_D &= 4r/l^2 x(1-x) \\
 &= 4 \times 5 / 20^2 \times 5(20-5) \\
 &= 3.75 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \text{BMD} &= +VA \times 5 + H_{YD} + 20 \times 2 \\
 &= 495 + 570 + 40 \\
 &= -11.5 \text{ KNm}
 \end{aligned}$$

**Slope of the arch at D**

$$\begin{aligned}
 \text{Slope} &= \tan^{-1} [4r/l^2 (1-2x)] \\
 &= \tan^{-1} [4 \times 5 / 20^2 (20-2 \times 5)] \\
 &= 26^\circ 33' 55''.18
 \end{aligned}$$

**Normal thrust**

$$\begin{aligned}
 P &= V_x \sin + H \cos \\
 V_x &= \text{net beam shear force}
 \end{aligned}$$

$$\begin{aligned}
 V_x &= VA - 20 \\
 &= 99 - 20 \\
 &= 79 \text{ KN}
 \end{aligned}$$

$$\begin{aligned}
 P &= 79 \sin 26^\circ 33' 55''.18 + 152 \cos 26^\circ 33' 55''.18 \\
 &= 171.28 \text{ KN}
 \end{aligned}$$

**Radial shear force**

$$\begin{aligned}
 F &= vx \cos - H \sin \\
 &= 79 \cos - 152 \sin \\
 &= -2.683 \text{ KN}
 \end{aligned}$$

**Max BM in CB**

$$BM_x = VB_x - Wx^2/2 - Hyx$$

$$\begin{aligned}
 Y_x &= 4r/l^2 x(1-x) \\
 &= 4 \times 5/20^2 \times x(20-x) \\
 &= 0.05x(20-x)
 \end{aligned}$$

$$\begin{aligned}
 M_x &= 201x - 25 \times x^2/2 - 152(0.05x(20-x)) \\
 &= 201x - 12.5x^2 - 7.6x(20-x) \\
 &= 201x - 12.5x^2 - 152x + 7.6x^2
 \end{aligned}$$

$$M_x = 49x - 4.9x^2$$

**Diff W.r to x**

$$dM/dx = 49 - 9.8x$$

**BM to be max**

$$9.8x - 49 = 0$$

$$x = 5\text{m}$$

$$\begin{aligned}
 M_x &= 49(x5) - 4.9(5^2) \\
 &= 122.5 \text{ KN}
 \end{aligned}$$

**Example:**

A symmetrical three hinged parabolic arch of span 40m and rise 8m carries a UDL of 30kN/m, over the left half of the span. The hinges are provide at the support and at the centre of the arch. Calculation the reaction at the support.als calculate the bending moment, radial shear and normal thrust a distance of 10m from the left support.

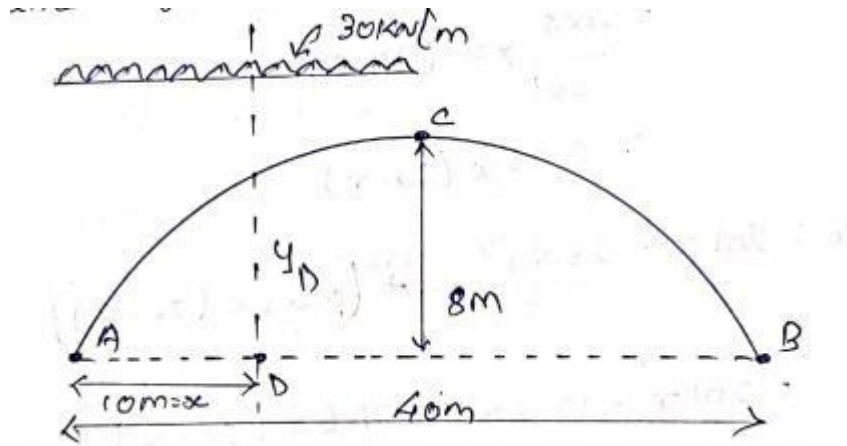


Fig. 3.3.5

**Solution****Vertical Components  $V_A$  and  $V_B$** **Taking moment about A**

$$V_B \times 40 - 30 \times 20^2 / 2 = 0$$

$$V_B = 150 \text{ kN}$$

$$V_A = \text{total load} - V_B$$

$$= 30 \times 20 - 150$$

$$= 450 \text{ kN}$$

**Horizontal components**

$$V_A \times 20 - H \times 8 - 30 \times 20 \times 20 / 2 = 0$$

$$H = 375 \text{KN}$$

### Resultant Reaction RA, RB

$$\begin{aligned} R_A &= \sqrt{H^2 + V_A^2} \\ &= \sqrt{450^2 + 375^2} \\ &= 585.71 \text{KN} \end{aligned}$$

$$\begin{aligned} R_B &= \sqrt{H^2 + V_B^2} \\ &= \sqrt{150^2 + 375^2} \\ &= 403.89 \text{KN} \end{aligned}$$

### Bending moment at 10m from A

$$\begin{aligned} Y &= 4r/l^2 x(1-x) \\ &= 4 \times 8/40^2 \times 10(40-10) \\ &= 6 \text{m} \end{aligned}$$

### Bending moment at 10m

$$\begin{aligned} &= V_A(10) - H_A(Y) - 30 \times 10 \times 10/2 \\ &= 450(10) - (375y) - 30(50) \\ &= 3000 - 375Y \\ &= 3000 - 375(6) \\ &= 750 \text{KNm} \end{aligned}$$

### Radial shear force at x=10m

$$R = V_x \cos \theta - H \sin \theta$$



$$\begin{aligned}
 V_x &= V_A - 30 \times 10 \\
 &= 450 - 300 \\
 &= 150 \text{ KN}
 \end{aligned}$$

**slope at D**

$$\begin{aligned}
 \theta &= \tan^{-1} [4r/l^2 (l - 2x)] \\
 \theta &= \tan^{-1} [4 \times 8 / 40^2 (40 - 2 \times 10)] \\
 &= 21^\circ 48'
 \end{aligned}$$

$$\begin{aligned}
 R &= 150 \cos 21^\circ 48' - 375 \sin 21^\circ 48' \\
 &= 0
 \end{aligned}$$

**Normal thrust**

$$\begin{aligned}
 P &= V_x \sin \theta + H \cos \theta \\
 &= 150 \sin 21^\circ 48' + 375 \cos 21^\circ 48' \\
 &= 403.89 \text{ KN}
 \end{aligned}$$

**Example:**

A three hinged parabolic arch of 40m span has abutments at unequal levels. The highest point of the arch is 4m above the left support and 9m above the right support abutments. The arch is subjected to an UDL of 15KN/m over its entire horizontal span. Find the horizontal thrust and bending moment at the point 8m from the left support.

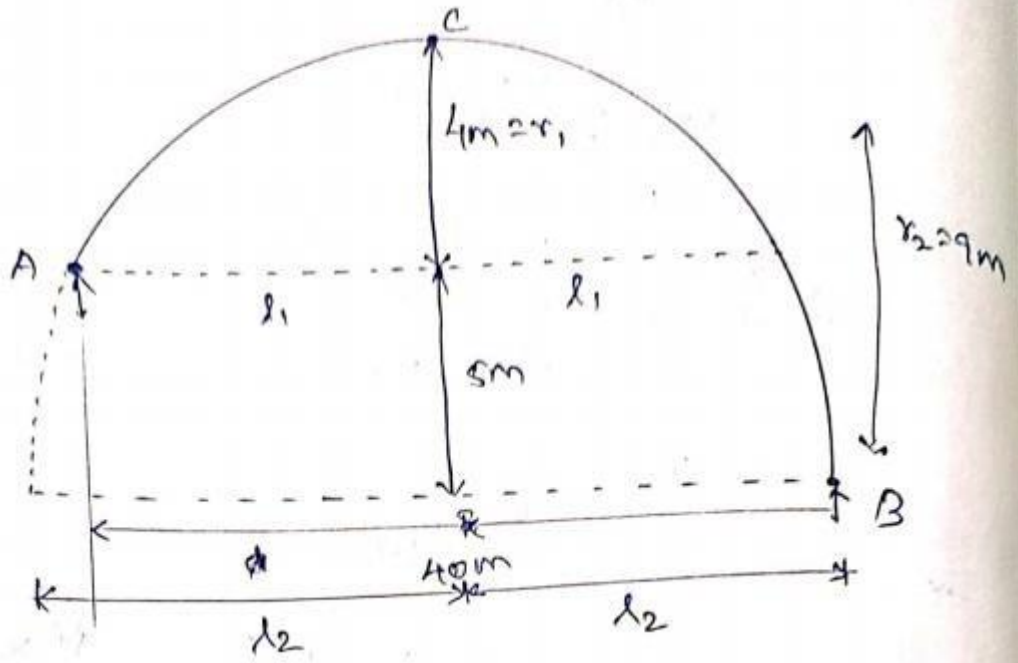


Fig. 3.3.6

**Solution**

**Reaction A,B and H**

**Find  $l_1, l_2$**

$$l_1/l_2 = \sqrt{r_1/r_2}$$

$$l_1/40-l_1 = \sqrt{4/9}$$

$$l_1 = (40-l_1) \times 2/3$$

$$l_1 = 16\text{m}$$

$$l_2 = 40-l_1$$

$$= 40-16$$

$$= 24\text{m}$$

considering left slab of C

$$V_A(16) - 4H - 15 \times 16 \times 16 / 2 = 0$$

$$16V_A - 4H - 1920 = 0$$

$$4V_A - H - 480 = 0 \quad \text{_____ (1)}$$

Considering the right slab of C

$$-V_B(24) + H(9) + 15 \times 24 \times 24 / 2 = 0$$

$$-24V_B + 9H + 4320 = 0$$

$$-8V_B + 3H + 1440 = 0 \quad \text{_____ (2)}$$

$$V_A + V_B = 600$$

$$V_B = 600 - V_A \quad \text{_____ (3)}$$

Sub 3 in 2

$$-8(600 - V_A) + 3H + 1440 = 0$$

$$8V_A - 4800 + 3H + 1440 = 0$$

$$8V_A + 3H - 3360 = 0 \quad \text{_____ (4)}$$

1&4 solve

$$4V_A - H - 480 = 0$$

$$V_A - 3H - 3360 = 0$$

$$\times 4 = -4V_A + 12H + 13440 = 0$$

$$11H + 12960 = 0$$

$$H = 480 \text{ KN}$$

$$8V_A - 2H - 960 = 0$$

$$8V_A + 3H - 3360 = 0$$

$$- \quad - \quad +$$

$$-5H = 2400$$

$$4V_A - H - 480 = 0$$

$$4V_A - 480 - 480 = 0$$

$$V_A = 240 \text{ KN}$$

$$V_B = 360 \text{ KN}$$

### Bending moment $x = 8 \text{ m}$

$$BM_x = V_A(8) - 15 \times 8 \times 8/2 - H_y$$

$$Y = 4r/l^2 x(1-x)$$

$$Y = 4 \times 4 / (2 \times 16)^2 \times 8(2 \times 16 - 8)$$

$$Y = 3 \text{ m}$$

$$BM = 240 \times 8 - 15 \times 32 - 480 \times 3$$

$$= 0$$

### Radial shear

$$R = V_x \cos \theta - H \sin \theta$$

$$V_x = V_A - (15 \times 8)$$

$$= 240 - (15 \times 8)$$

$$= 120 \text{ KN}$$

$$\theta = \tan^{-1} [4r/l^2 (1-2x)]$$

$$\theta = \tan^{-1} [4 \times 4 / (2 \times 16)^2 (32 - 2 \times 8)]$$

$$= 14^\circ 2'$$

$$F = 120 \cos 14^\circ 2' - 480 \sin 14^\circ 2'$$

$$F = 0$$

## Normal thrust

$$\begin{aligned}
 P_N &= V_x \sin\theta + H \cos\theta \\
 &= 120 \sin 14^\circ 2' + 480 \cos 14^\circ 2' \\
 &= 494.77 \text{ KN}
 \end{aligned}$$

## Example:

A three hinge arch is circular 25m in span with a central rise of 5m. It is loaded with a concentration load of 10KN at 7.5m from the left hand hinge. Find the horizontal thrust, reaction at each end hinge bending moment under the load.

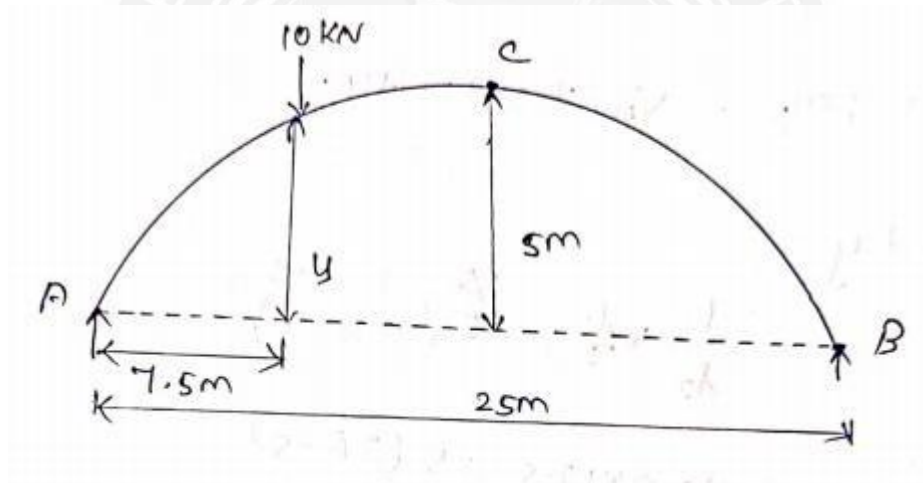


Fig. 3.3.7

**solution**

**Vertical reaction  $V_A$  and  $V_B$**

**Taking moment about A**

$$V_B \times 25 - 10 \times 7.5 = 0$$

$$V_B = 3 \text{ KN}$$

$$V_A = 7 \text{ KN}$$

**Horizontal thrust**

$$V_B \times 12.5 - H \times 5 = 0$$

$$3 \times 12.5 - H(5) = 0$$

$$H = 7.5 \text{ KN}$$

**Reaction RA and RB**

$$R_A = \sqrt{H^2 + V_A^2}$$

$$= \sqrt{7^2 + 7.5^2}$$

$$= 10.26 \text{ KN}$$

$$R_B = \sqrt{V_B^2 + H^2}$$

$$= \sqrt{3^2 + 7.5^2}$$

$$= 8.08 \text{ KN}$$

**Bending moment under the load**

$$\text{BMD} = V_A(7.50) - H y$$

**Find y**

$$1/2 \times 1/2 = 5(2R - 5)$$

$$12.5 \times 12.5 = 5(2R - 5)$$

$$R = 18.125 \text{ m}$$

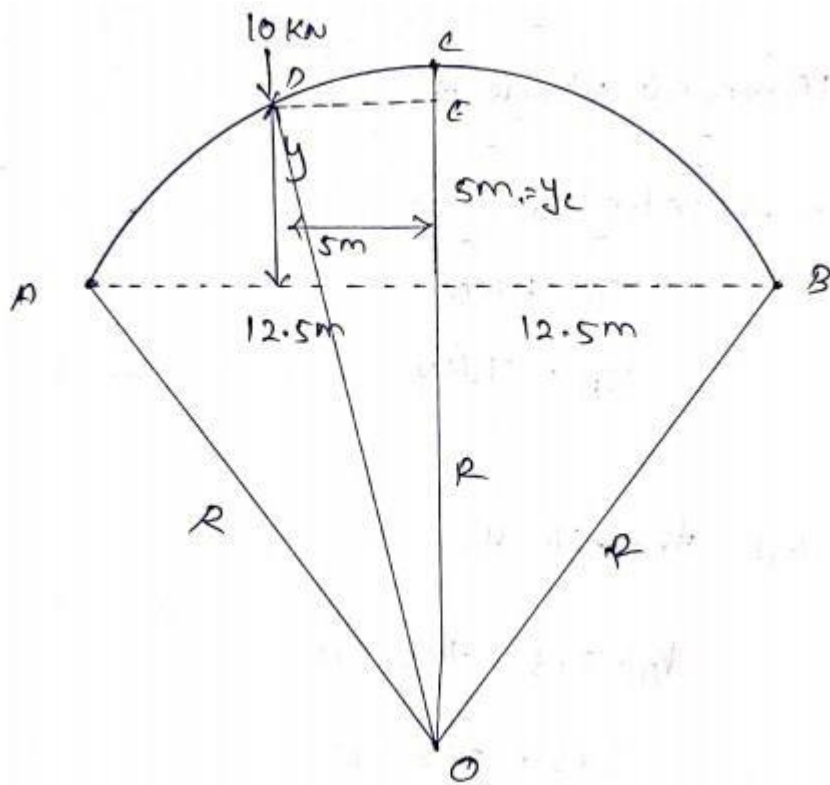


Fig. 3.3.8

$$R^2 = (R - Y_c + y)^2 + x^2$$

$$18.125^2 = (18.125 - 5 + y)^2 + 5^2$$

$$303.515 = (13.125 + Y)^2$$

$$17.421 = 13.125 + y$$

$$Y = 4.3\text{ m}$$

$$\text{BMD} = 7(7.5) - 7.5(4.3)$$

$$= 20.25\text{ kNm}$$