

## UNIT:5 MASS TRANSFER

### Definition:

Transfer of mass as a result of particle concentration difference in a mixture.

Air is a mixture of various gases. Whenever we have a multi component system with a concentration gradient, one constituent of the mixture gets transported from the region of higher concentration to the region of lower concentration till the concentration gradient reduces to zero. This phenomenon of the transport of mass as a result of concentration gradient is called 'Mass Transfer'.

### Difference of Heat transfer and Mass Transfer

Heat Transfer		Mass transfer
Temperature Gradient	❖ a	Concentration Gradient
Occurs from higher temperature to lower temperature	❖ b	Occurs from higher Concentration to lower concentration

### Modes of Mass Transfer

There are basically three modes of mass transfer:

i. Diffusion mass Transfer

- ❖ occurs due to concentration difference
- ❖ Transport of matter in microscopic level
- ❖ Occurs between higher concentration and lower concentration

Eg. Osmosis, Reverse osmosis, Leakage of air from automobile and leakage of LPG from tanks

ii. Convective Mass Transfer

- ❖ occurs due to concentration difference and velocity
- ❖ Concentration of particles at its surface differs from its concentration in a gas moving over the surface

Eg. Drying of clothes, evaporization of water from swimming pool.

iii. Phase change Mass Transfer

- ❖ occurs due to simultaneous effect of convection and diffusion mass transfer

Eg. Burnt gases from chimney rise by convection and then mixes with air by diffusion

**Important Terms in concentration :**

a. Mass concentration or mass density ( $\rho$ )

$$= \frac{\text{Mass of a component}}{\text{Unit volume}} = \frac{m_A}{V}$$

b. Molar Concentration or molar density ( $C_A$ )

$$= \frac{\text{Mass concentration of a component}}{\text{Molecular weight of a component}} = \frac{\rho_A}{M_A}$$

c. Mass fraction ( $x_A$ )

$$= \frac{\text{Mass concentration of a component}}{\text{Mass density of mixture}} = \frac{\rho_A}{\rho}$$

d. Mole fraction ( $m_A$ )

$$= \frac{\text{Mole concentration of a component}}{\text{Mole concentration of mixture}} = \frac{C_A}{C}$$

e. Mass flow rate ( $\dot{m}_A$ ) =  $N_A (M_A)$

where,  $N_A$  = Molar mass rate of flow in kg-mol/sec

We also Know that ,  $(\rho_A) = \frac{P_A}{RT}$  , where, R = Characteristic Gas Constant

also,  $(C_A) = \frac{P_A}{\bar{R}T}$  , where,  $\bar{R}$  = Universal gas constant = 8314.3 j/kg-mol K

**Problem 1:**

The composition of dry atmospheric air on a molar basis is 78.1%  $N_2$ , 20.9%  $O_2$ , and 1% Ar.

Neglecting other constituents, Assuming atmospheric pressure 1bar and temperature 27°C.

Find the mass fractions of the constituents of air.

Solution:

Since, | **Mass fraction ( $x_A$ )** =  $\frac{\text{Mass concentration of a component}}{\text{Mass density of mixture}} = \frac{\rho_A}{\rho}$

To find Molar concentration of  $N_2$ ,  $O_2$ , Ar,

$$C_{N_2} = \frac{P_{N_2}}{\bar{R}T} = \frac{0.781 \times 1 \times 10^5}{8314 \times 300} = 0.0313 \text{ kg mole/m}^3$$

$$C_{O_2} = \frac{P_{O_2}}{\bar{R}T} = \frac{0.209 \times 1 \times 10^5}{8314 \times 300} = 0.0084 \text{ kg mole/m}^3$$

$$C_{Ar} = \frac{P_{Ar}}{\bar{R}T} = \frac{0.01 \times 1 \times 10^5}{8314 \times 300} = 0.0004 \text{ kg mole/m}^3$$

To find mass Densities of N<sub>2</sub>, O<sub>2</sub>, Ar,

$$\rho_{N_2} = M_{N_2} \times C_{N_2} = 28 \times 0.0313 = 0.8764 \text{ kg/m}^3$$

$$\rho_{O_2} = M_{O_2} \times C_{O_2} = 32 \times 0.0084 = 0.2688 \text{ kg/m}^3$$

$$\rho_{Ar} = M_{Ar} \times C_{Ar} = 18 \times 0.0004 = 0.0072 \text{ kg/m}^3$$

Over all mass density are,

$$\rho = \rho_{N_2} + \rho_{O_2} + \rho_{Ar} = 0.8764 + 0.2688 + 0.0072 = 1.1524 \text{ kg/m}^3$$

Mass Fractions of Constituents of air are,

$$x_{N_2} = \frac{\rho_{N_2}}{\rho} = \frac{0.8764}{1.1524} = 0.7605$$

$$x_{O_2} = \frac{\rho_{O_2}}{\rho} = \frac{0.2688}{1.1524} = 0.2334$$

$$x_{Ar} = \frac{\rho_{Ar}}{\rho} = \frac{0.0072}{1.1524} = 0.00625$$

### Fick's law of diffusion

The molar flux (Rate of Mass transfer) is directly proportional to concentration difference and inversely proportional to separation.

$$\text{Molar flux} \propto \frac{\text{Concentration Difference}}{\text{Separation}}$$

$$\frac{N_a}{A} \propto \frac{C_{a_2} - C_{a_1}}{dx}$$
$$\frac{N_a}{A} = -D_{ab} \frac{(C_{a_2} - C_{a_1})}{dx}$$

where,  $D_{ab}$  = Diffusion coefficient or Diffusivity (m<sup>2</sup>/sec)

And  $C_a$  = concentration or molecules per unit volume of the particles

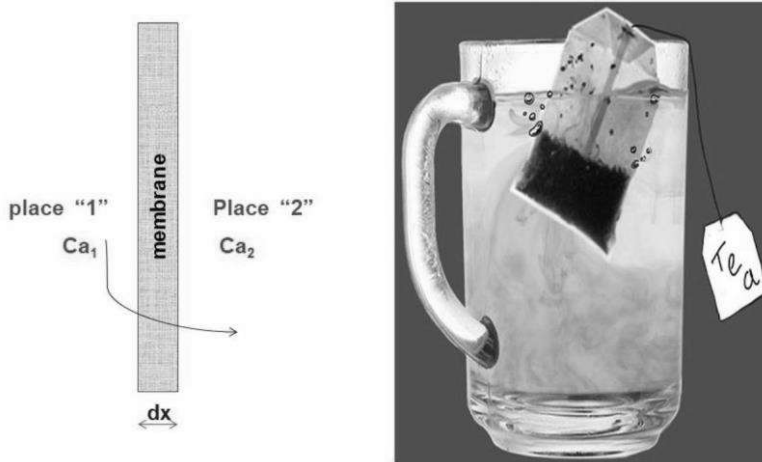
$$= \text{Solubility} \times \text{Pressure}$$

A = Area through which the mass is flowing in m<sup>2</sup>

-ve sign indicates that the diffusion takes place in the direction opposite to that of increasing concentration

**Types of Diffusion Mass transfer**

**Type A: Steady State diffusion of a component “a” through the membrane**



**Problem 2:**

Hydrogen diffuses through a plastic membrane of 1mm thick. The molar concentration of hydrogen on either side of the plastic membrane are 0.02 kg- mol/m<sup>3</sup>, 0.005 kg- mol/m<sup>3</sup>. Diffusion coefficient of H<sub>2</sub> through plastic 10<sup>-9</sup> m<sup>2</sup>/sec. determine molar flux and mass flux.

Solution:

Molar flux:

From HMT data book, pg. no. 175

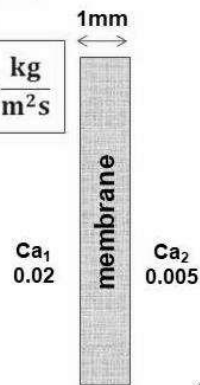
$$\frac{N_a}{A} = -D_{ab} \frac{(C_{a2} - C_{a1})}{dx} \text{ or } \frac{N_a}{A} = D_{ab} \frac{(C_{a1} - C_{a2})}{dx}$$

$$\frac{N_a}{A} = 10^{-9} \frac{(0.02 - 0.005)}{1 \times 10^{-3}} = \frac{N_a}{A} = 1.5 \times 10^{-8} \frac{\text{kg}}{\text{m}^2\text{s}}$$

Mass flux:

$$\frac{\dot{m}_a}{A} = \left[ \frac{N_a}{A} \right] M_{wt} = \frac{\dot{m}_a}{A} = 1.5 \times 10^{-8} \times 2.016$$

$$\frac{\dot{m}_a}{A} = 3.024 \times 10^{-8} \frac{\text{kg}}{\text{m}^2\text{s}}$$



### Problem 3:

Oxygen at 25°C and pressure of 2 bar flows through a rubber pipe of inside diameter 25 mm and wall thickness 2.5 mm. The diffusivity of oxygen through the rubber tube is  $0.21 \times 10^{-9}$  m<sup>2</sup>/sec and the solubility of oxygen in rubber is  $3.12 \times 10^{-3}$  kg. mole/m<sup>3</sup> bar. Find the loss of oxygen by diffusion / m length of the pipe. Molar proportion of oxygen in air is 21%.

Given:  $D_{ab} = 0.21 \times 10^{-9}$  m<sup>2</sup>/sec ;  $dx = 2.5$  mm =  $2.5 \times 10^{-3}$  m ;  $D = 25$  mm =  $25 \times 10^{-3}$  m

Solution:

$$C_{a1} = \text{Solubility} \times \text{Pressure} = 3.12 \times 10^{-3} \times 2 = \boxed{C_{a1} = 6.24 \times 10^{-3} \frac{\text{kg} - \text{mol}}{\text{m}^3}}$$

Since, molar proportion of oxygen in air is given in percentage, and we know that the atmospheric pressure is 1 bar,

$$P_{O_2} = 0.21 \times 1 = 0.21 \text{ bar}$$

Therefore,

$$C_{a2} = \text{Solubility} \times \text{Pressure} = 3.12 \times 10^{-3} \times 0.21 = \boxed{C_{a2} = 6.552 \times 10^{-4} \frac{\text{kg} - \text{mol}}{\text{m}^3}}$$

Since,

Molar Flux,

$$\frac{N_a}{A} = D_{ab} \frac{(C_{a1} - C_{a2})}{dx} \text{ or } \frac{N_a}{A} = 0.21 \times 10^{-9} \frac{(6.24 \times 10^{-3} - 6.552 \times 10^{-4})}{2.5 \times 10^{-3}}$$

$$\boxed{\frac{N_a}{A} = 34.6914 \times 10^{-10} \frac{\text{kg} - \text{mol}}{\text{m}^2 \text{ s}}}$$

Since the surface area of cylinder is,

$$A = \pi DL = \pi \times 25 \times 10^{-3} \times 1 \text{ or } \boxed{A = 0.07854 \text{ m}^2}$$

Therefore,  $N_a = 34.6914 \times 10^{-10} \times 0.07854$

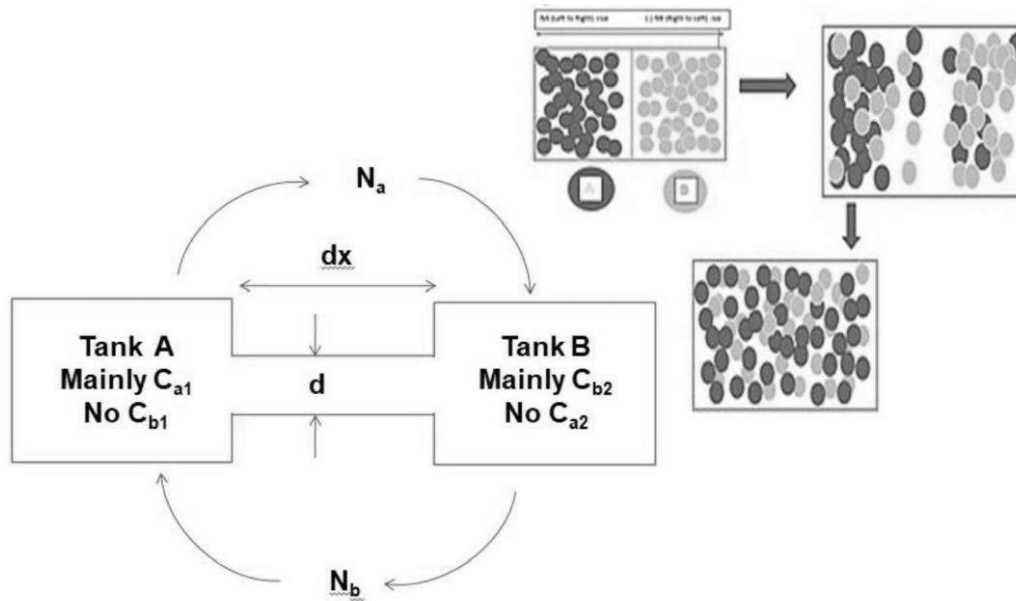
$$N_a = 3.6846 \times 10^{-10} \frac{\text{kg} - \text{mol}}{\text{s}}$$

But, mass flow rate,

$$\dot{m}_a = M_{wt} N_a = 32 \times 3.6846 \times 10^{-10} = 1.1788 \times 10^{-9} \text{ kg/s}$$

$$\boxed{\therefore \text{Loss of oxygen per meter length} = 1.1788 \times 10^{-9} \text{ kg/s}}$$

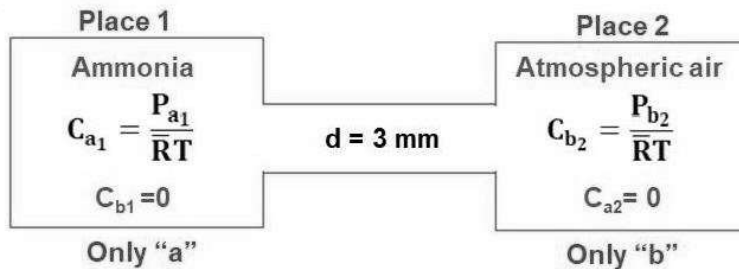
**Type B: Steady State equimolar counter diffusion of a component “a” and “b”**



**Problem 4:**

Ammonia and air experiences diffusion through 3 mm diameter, 20 mm long pipe. Total pressure is 1 atm and temperature 25°C. Determine the diffusion rate of ammonia and air

**Solution**



**Given that,**

$$P_{a1} = 1 \text{ atm} = 1 \text{ bar}, \bar{R} = 8314.3 \text{ J/kg mole.K (known)}, T = 25^\circ\text{C}$$

**Therefore,**

$$C_{a1} = \frac{P_{a1}}{RT} = \frac{1 \times 10^5}{8314.3 \times 298} = 0.04036 \frac{\text{kg - mole}}{\text{m}^3}, C_{a2} = 0$$

From HMT Data book, pg No.181

Diffusion Coefficient [for Ammonia and Air]  $D_{ab} = 21.60 \times 10^{-6} \text{ m}^2/\text{s}$

Since molar flux,

$$\frac{N_a}{A} = D_{ab} \left\{ \frac{C_{a1} - C_{a2}}{dx} \right\}$$

$$\begin{aligned} \frac{N_a}{A} &= 2.161 \times 10^{-5} \frac{(0.04036 - 0)}{20} \\ &= \boxed{4.3611 \times 10^{-8} \frac{\text{kg - mole}}{\text{m}^2 \text{sec}}} \end{aligned}$$

Since the cross-sectional area,

$$A = \frac{\pi d^2}{4} = \frac{\pi (3 \times 10^{-3})^2}{4} = \boxed{7.0685 \times 10^{-6} \text{ m}^2}$$

Therefore,

$$\begin{aligned} N_a &= 4.3611 \times 10^{-8} \times 7.0685 \times 10^{-6} \\ &= 3.0827 \times 10^{-13} \frac{\text{kg - mol}}{\text{s}} = N_b \end{aligned}$$

Therefore, diffusion rate of Ammonia,

$$\begin{aligned} \dot{m}_a &= (M_{wt})_a N_a = (M_{wt})_{\text{NH}_3} N_{\text{NH}_3} = 3.0827 \times 10^{-13} \times 17.03 \\ &= \boxed{5.248 \times 10^{-12} \frac{\text{kg}}{\text{s}}} \end{aligned}$$

∴ From HMT Data book Pg.No 184

Molecular wt of  $\text{NH}_3 = 17.03$

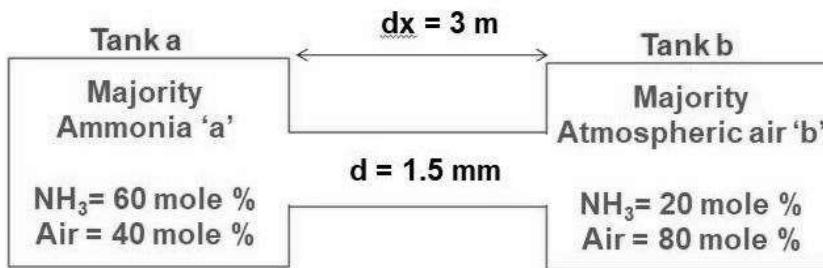
And, diffusion rate of air,

$$\begin{aligned} \dot{m}_b &= (M_{wt})_b N_b = (M_{wt})_{\text{air}} N_{\text{air}} = 3.0827 \times 10^{-13} \times 28.96 \\ &= \boxed{8.92 \times 10^{-12} \frac{\text{kg}}{\text{s}}} \end{aligned}$$

∴ From HMT Data book Pg.No 184  
Molecular wt of air = 28.96

### Problem 5:

Two large tanks, maintained at the same temperature and pressure are connected by a circular 0.15 m diameter direct, which is 3 m in length. One tank contains a uniform mixture of 60 mole % ammonia and 40 mole % air and the other tank contains a uniform mixture of 20 mole % ammonia and 80 mole % air. The system is at 273 K and  $1.013 \times 10^5$  Pa. Determine the rate of ammonia transfer between the two tanks. Assuming a steady state mass transfer.



Tank a,

$$P_{a_1} = \frac{60}{100} \times 1.1013 = 0.6078 \text{ bar} \quad P_{b_1} = \frac{40}{100} \times 1.1013 = 0.4052 \text{ bar}$$

Tank b,

$$P_{a_2} = \frac{20}{100} \times 1.1013 = 0.2026 \text{ bar} \quad P_{b_2} = \frac{80}{100} \times 1.1013 = 0.8104 \text{ bar}$$

$$A = \frac{\pi d^2}{4} = \frac{\pi(0.15)^2}{4} = 0.017671 \text{ m}^2$$

$$C_{a_1} = \frac{P_{a_1}}{RT} = \frac{0.6078 \times 10^5}{8314.3 \times 273} = 0.02677 \frac{\text{kg} - \text{mole}}{\text{m}^3}$$

$$C_{a_2} = \frac{P_{a_2}}{RT} = \frac{0.2026 \times 10^5}{8314.3 \times 273} = 0.008925 \frac{\text{kg} - \text{mole}}{\text{m}^3}$$

From HMT Data book, pg No.181

$$\text{Diffusion Coefficient [for Ammonia and Air]} D_{ab} = 21.60 \times 10^{-6} \text{ m}^2/\text{s}$$



Since molar flux,

$$\begin{aligned} \frac{N_a}{A} &= D_{ab} \left\{ \frac{C_{a1} - C_{a2}}{dx} \right\} \\ &= 21.61 \times 10^{-6} \left\{ \frac{0.02677 - 0.008925}{3} \right\} \\ &= 1.28543 \times 10^{-7} \frac{\text{kg} - \text{mol}}{\text{m}^2 \text{sec}} \end{aligned}$$

Therefore,

$$\begin{aligned} N_a &= 1.28543 \times 10^{-7} \times A = 1.28543 \times 10^{-7} \times A \times 0.017671 = \\ N_a &= 2.27149 \times 10^{-9} \frac{\text{kg} - \text{mole}}{\text{m}^3} = N_b \end{aligned}$$

Therefore the rate of ammonia transfer between two tanks,

$$\dot{m}_a = \dot{m}_{NH_3} = M_{wt} N_a = 2.27149 \times 10^{-9} \times 17.03$$

$$= 3.8683 \times 10^{-8} \frac{\text{kg}}{\text{s}}$$

∴ From HMT Data book Pg.No 184  
Molecular wt of  $NH_3 = 17.03$

Therefore the rate of air transfer between two tanks,

$$\dot{m}_b = \dot{m}_{air} = (M_{wt})_b N_b = 2.27149 \times 10^{-9} \times 28.96$$

$$= 65.782 \times 10^{-9} \frac{\text{kg}}{\text{s}}$$

∴ From HMT Data book Pg.No 184  
Molecular wt of air = 28.96

**Type C: Steady State evaporation of a component “a” into a stagnant air**

**Assumptions:**

- ❖ Water vapor and air behaves as ideal gases
- ❖ System is held at isothermal conditions
- ❖ Evaporation Process is steady

**Problem 6:**

Determine the rate of water from bottom of a test tube of 10 mm diameter, 150 mm long, into a dry stagnant air at 25°C

Points to be remembered:

From Dalton's law,

$$P_{a1} + P_{b1} = P_{atm}$$

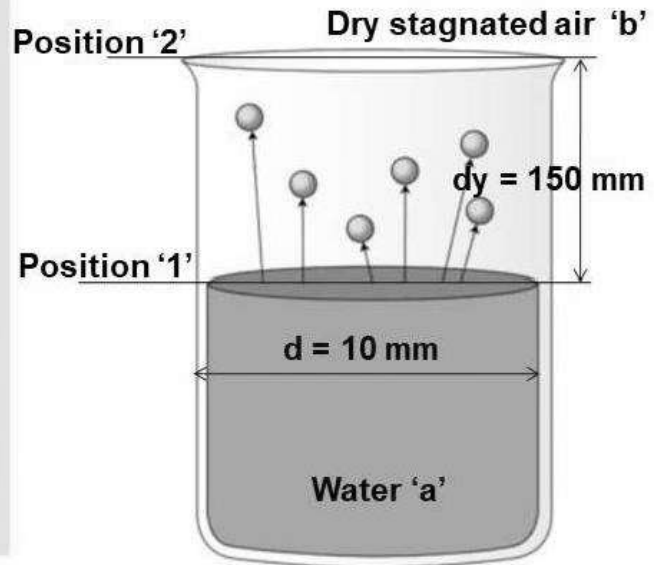
$$P_{a2} + P_{b2} = P_{atm}$$

$$P_{a2} = \phi P_{a1}$$

Where  $\phi$  = Relative Humidity

$$P_{a2} = 0$$

(dry air or  $\phi = 0$  or no humidity)



Since the Temperature of air is given as 25°C,

The partial pressure of water vapor ( $p_{a1}$ ) corresponds to the saturation pressure at 25°C in steam tables.

Therefore,  $P_{a1} = 0.03166$  bar

Since,

Dry air or  $\phi = 0$  or no humidity

$$P_{a2} = 0$$

And since,

$$P_{a1} + P_{b1} = P_{atm}$$

Therefore,

$$P_{b1} = 0.96834$$
 bar

From HMT Data book Pg.No. 181, Diffusion Coefficient

[for Water and Air at 25°C]  $D_{ab} = 25.83 \times 10^{-6}$  m<sup>2</sup>/s

From HMT Data book, pg No. 175,

$$\frac{N_a}{A} = \frac{D_{ab}}{dy} \frac{P_{atm}}{RT} \ln \left[ \frac{P_{b2}}{P_{b1}} \right]$$

Saturated Water and Steam

$T$ [°C]	$P_s$ [bar]	$v_g$ [m <sup>3</sup> /kg]
0.01	0.006112	206.1
1	0.006566	192.6
2	0.007054	179.9
3	0.007575	168.2
4	0.008129	157.3
5	0.008719	147.1
6	0.009346	137.8
7	0.01001	129.1
8	0.01072	121.0
9	0.01147	113.4
10	0.01227	106.4
11	0.01312	99.90
12	0.01401	93.83
13	0.01497	88.17
14	0.01597	82.89
15	0.01704	77.97
16	0.01817	73.38
17	0.01936	69.09
18	0.02063	65.08
19	0.02196	61.34
20	0.02337	57.84
21	0.02486	54.56
22	0.02642	51.49
23	0.02808	48.62
24	0.02982	45.92
25	0.03166	43.40
26	0.03360	41.03
27	0.03564	38.81

$$\therefore \frac{N_a}{A} = \frac{2.583 \times 10^{-5}}{0.15} \times \frac{1 \times 10^5}{8314.4 \times 298} \ln \left[ \frac{1}{0.96834} \right]$$

$$= 2.23598 \times 10^{-7} \frac{\text{kg} - \text{mol}}{\text{m}^2 \text{s}}$$

$$A = \frac{\pi d^2}{4} = \frac{\pi (0.01)^2}{4} = 78.539 \times 10^{-6} \text{m}^2$$

Therefore,

$$N_a = 2.2359 \times 10^{-7} \times 78.539 \times 10^{-6}$$

$$= 1.75607 \times 10^{-11} \frac{\text{kg} - \text{mol}}{\text{s}}$$

From HMT data Book, Pg.No. 184, Molecular Weight of steam = 18.016

Therefore, Rate of Evaporation is,

$$\dot{m}_a = N_a (M_{wt})_a = 1.75607 \times 10^{-11} \times 18.016$$

$$= 3.106929 \times 10^{-10} \frac{\text{kg}}{\text{s}}$$

### Problem 7:

A well of 40 m deep 9m diameter is exposed to atmospheric air at 25 C, 50% R.H. determine the rate of atmospheric evaporation of water from well.

### Solution:

Since the Temperature of air is given as 25°C,

The partial pressure of water vapor ( $p_{a1}$ ) corresponds to the saturation pressure at 25°C in steam tables.

Therefore,  $P_{a1} = 0.03166$  bar

and

$$P_{a2} = \phi P_{a1}$$

Since,

R.H or  $\phi = 50\%$

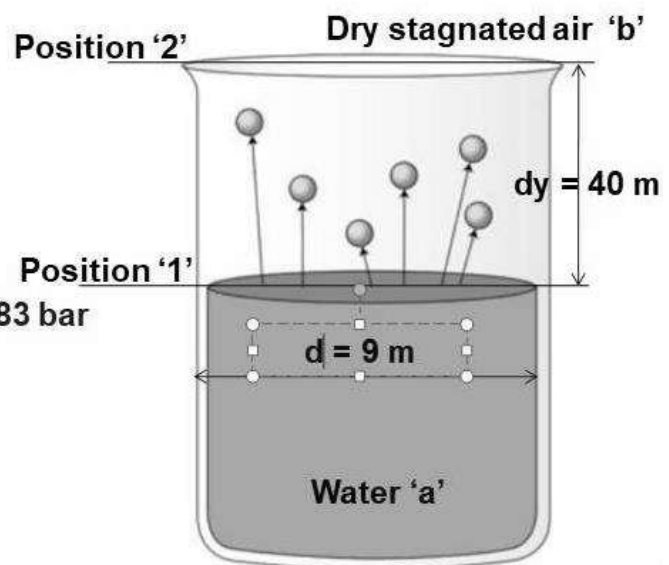
$$P_{a2} = 0.5 \times 0.03166 = 0.01583 \text{ bar}$$

And since,

$$P_{a1} + P_{b1} = P_{atm} = 1$$

Therefore,

$$P_{b1} = 0.96834 \text{ bar}$$



Since,

$$P_{a2} + P_{b2} = P_{atm}$$

Therefore,

$$P_{b2} = 0.98417 \text{ bar}$$

From HMT Data book Pg.No. 181, Diffusion Coefficient [for Water and Air at 25°C]

$$D_{ab} = 25.83 \times 10^{-6} \text{ m}^2/\text{s}$$

From HMT Data book, pg No. 175,

$$\frac{N_a}{A} = \frac{D_{ab} P_{atm}}{dy RT} \ln \left[ \frac{P_{b2}}{P_{b1}} \right]$$
$$\therefore \frac{N_a}{A} = \frac{2.583 \times 10^{-5}}{40} \times \frac{1 \times 10^5}{8314.4 \times 298} \ln \left[ \frac{0.9847}{0.96834} \right]$$
$$= 4.226 \times 10^{-10} \frac{\text{kg} - \text{mol}}{\text{m}^2 \text{s}}$$

and the area of the well,

$$A = \frac{\pi d^2}{4} = \frac{\pi(9)^2}{4} = 63.6175 \text{ m}^2$$

Therefore,

$$N_a = 2.6880 \times 10^{-8} \frac{\text{kg} - \text{mole}}{\text{m}^3}$$

From HMT data Book, Pg.No. 184, Molecular Weight of steam = 18.016

Therefore, Rate of Evaporation is,

$$\dot{m}_a = N_a (M_{wt})_a = 2.688 \times 10^{-8} \times 18.016$$
$$= 4.8441 \times 10^{-7} \frac{\text{kg}}{\text{s}}$$