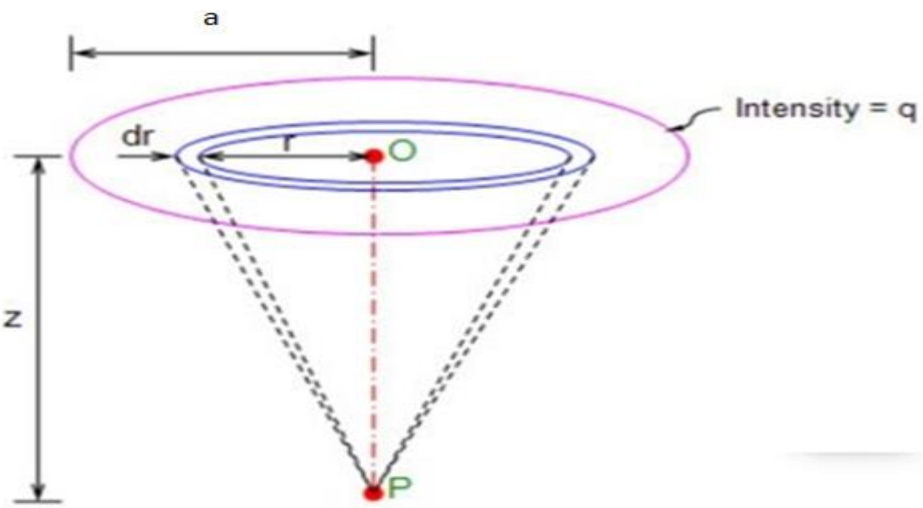
2) Stress due to uniformly loaded circular area:



The Boussinesq equation for vertical stress due to a point load can be extended to find the vertical stress at any point beneath the centre of a uniformly loaded circular area.

Let q= intensity of the load per unit area

and R=the radius of the loaded area

Let us consider an elementary ring of radius r and thickness 'dr' or δr of the loaded area.

The load on the elementary ring= q δa= q(2πr)dr

But we know that

$$\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{\left(1+\frac{r^2}{z^2}\right)}\right]^{5/2}$$

$$\Delta\sigma_z = \frac{3\mathrm{q}(2\pi \mathrm{r})\mathrm{dr}}{2\pi z^2}\left[\frac{1}{\left(1+\frac{r^2}{z^2}\right)}\right]^{5/2}$$

$$\Delta\sigma_z = \frac{3q(r)dr}{(r^2+z^2)^{5/2}} z^3$$

The vertical stress due to full load is given by

$\sigma_z = 3qz^3 \int_0^a \frac{rdr}{(r^2+z^2)^{5/2}}$ -------------(1)

According to Boussinesq equation

$$R = \sqrt{r^2+z^2} = (r^2+z^2)^{1/2}$$

Replacing R=n

$$n = (r^2+z^2)^{1/2}$$

$$n^2 = (r^2+z^2) ------(2)$$

Diff eqn (2)   2n.dn=2rdr

ndn=rdr

when r=0 eqn (2) $\longrightarrow$ $n^2 = z^2$

when r=0 eqn (2) $\longrightarrow$ $n^2 = (a^2+z^2)$

$$n = (a^2+z^2)^{1/2}$$

Substituting the new limit in terms of n in equation(1)

$$\sigma_z = 3qz^3 \int_z^{(a^2+z^2)^{1/2}} \frac{rdr}{(r^2+z^2)^{5/2}}$$

$$\sigma_z = 3qz^3 \int_z^{(a^2+z^2)^{1/2}} \frac{ndn}{(n^2)^{5/2}}$$

$$\sigma_z = 3qz^3 \int_z^{(a^2+z^2)^{1/2}} \frac{ndn}{(n)^5}$$

$$\sigma_z = 3qz^3 \int_{z}^{(a^2+z^2)^{1/2}} \frac{dn}{(n)^4}$$

$$\sigma_z = 3qz^3 \int_{z}^{(a^2+z^2)^{1/2}} n^{-4}\, dn$$

$$\sigma_z = 3qz^3 \left[\frac{n^{-4+1}}{-4+1}\right]_z^{(a^2+z^2)^{1/2}}$$

$$\sigma_z = 3qz^3 \left[\frac{n^{-3}}{-3}\right]_z^{(a^2+z^2)^{1/2}}$$

$$\sigma_z = \frac{3qz^3}{-3} [n^{-3}]_z^{(a^2+z^2)^{1/2}}$$

$$\sigma_z = \frac{3qz^3}{-3}\left[\left((a^2+z^2)^{\frac{1}{2}}\right)^{-3} - z^{-3}\right]$$

$$\sigma_z = \frac{3qz^3}{-3}\left[\frac{1}{(a^2+z^2)^{\frac{3}{2}}} - \frac{1}{z^3}\right]$$

$$\sigma_z = \frac{3qz^3}{-3}\left[\frac{1}{\left[z^2\left(\frac{a}{z^2}^2+1\right)\right]^{3/2}} - \frac{1}{z^3}\right]$$

$$\sigma_z = \frac{3qz^3}{-3z^3}\left[\frac{1}{\left[\left(\frac{a}{z^2}^2+1\right)\right]^{3/2}} - 1\right]$$

$$\sigma_z = \frac{3qz^3}{3z^3}\left[1 - \frac{1}{\left[\left(\frac{a}{z^2}^2+1\right)\right]^{3/2}}\right]$$

$$\sigma_z = q\left[1 - \frac{1}{\left[\left(\frac{a}{z^2}^2 + 1\right)^{3/2}\right]}\right]$$

$$\sigma_z = qK_B$$

$$K_B = \left[1 - \frac{1}{\left[\left(\frac{a}{z^2}^2 + 1\right)^{3/2}\right]}\right]$$

K_B = Influence factor (Boussinesq) for uniformly distributed circular area.

Problem:

1)A circular area 6m in diameter ,carries uniformly distributed load of 10KN/m^2.Determine the vertical stress at a depth of 2m,4m,8m.Plot the variation of vertical stress with depth

Solution:

$$\sigma_z = q\left[1 - \frac{1}{\left[\left(\frac{a}{z^2}^2 + 1\right)^{3/2}\right]}\right]$$

a=3m,q=10KN/m^2

at z= 2m,

$$\sigma_z = 10\left[1 - \frac{1}{\left[\left(\frac{3}{2^2}^2 + 1\right)^{3/2}\right]}\right]$$

$$\sigma_z = 8.29KN/m^2$$

at z=4m

$$\sigma_z = 10\left[1 - \frac{1}{\left[\left(\frac{3}{4^2}^2 + 1\right)^{\frac{3}{2}}\right]}\right]$$

$$\sigma_z = 4.88\, KN/m^2$$

At z=8m

$$\sigma_z = 10\left[1 - \frac{1}{\left[\left(\frac{3}{8^2}^2 + 1\right)^{3/2}\right]}\right]$$

$$\sigma_z = 1.79 KN/m^2$$