

5.2 BOUNDARY LAYER THICKNESS

Boundary Layer thickness (δ)

The velocity within the boundary layer increases from zero at the boundary surface to the velocity of the main stream asymptotically. Therefore the thickness of the boundary layer is arbitrarily defined as that distance from the boundary in which the velocity reaches 99 per cent of the velocity of the free stream ($u = 0.99U_\infty$). It is denoted by the symbol (δ). This definition however gives an approximate value of the boundary layer thickness and hence δ is generally termed as nominal thickness of the boundary layer.

The boundary layer thickness for greater accuracy is defined as in terms of certain mathematical expression which are the measure of the boundary layer on the flow. The commonly adopted definitions of the boundary layer thickness are:

1. Displacement thickness (δ^*)
2. Momentum thickness (θ)
3. Energy thickness (δ_e)

Displacement thickness (δ^*)

The displacement thickness can be defined as the distance measured perpendicular to the boundary by which the main/free stream is displaced on account of formation boundary layer.

or

It is an additional “Wall thickness” that would have to be added to compensate for the reduction in flow rate on account of boundary layer formation”.

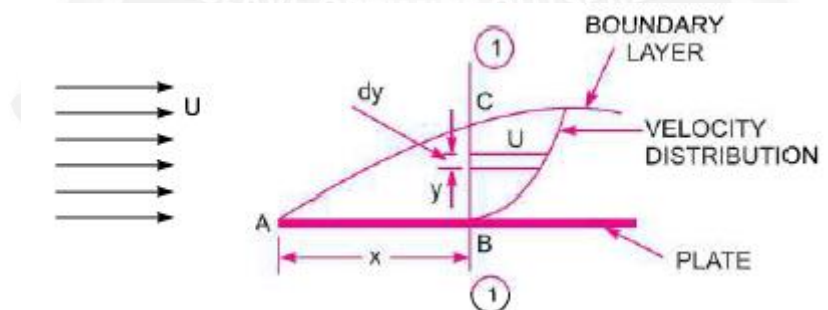


Figure 5.2.1 Displacement thickness

[Source: “Fluid Mechanics and Hydraulics Machines” by Dr.R.K.Bansal, Page: 613]

Let fluid of density ℓ flow past a stationary plate with velocity U as shown above. Consider an elementary strip of thickness dy at a distance y from the plate.

Assumed unit width, the mass flow per second through the elementary strip

$$= \rho u dy \text{ ----- (i)}$$

Mass of flow per second through the elementary strip (unit width) if the plate were not there

$$= \rho U dy \text{ ----- (ii)}$$

Reduce the mass flow rate through the elementary strip

$$\begin{aligned} &= \rho U dy - \rho u dy \\ &= \rho (U - u) dy \end{aligned}$$

Total momentum of mass flow rate due to introduction of plate

$$= \int_0^{\delta} \rho (U - u) dy \text{ ----- (iii)}$$

(If the fluid is incompressible)

Let the plate is displaced by a distance (δ^*) and velocity of flow for the distance (δ^*) is equal to the main/free stream velocity (i.e. U). Then, loss of the mass of the fluid/sec. flowing through the distance (δ^*) .

$$= \rho U \sigma^* \text{ ----- (iv)}$$

Equating eqns. (iii) and (iv) we get

$$= \rho U \sigma^* = \int_0^{\sigma} \rho (U - u) dy$$

or

$$\sigma^* = \int_0^{\sigma} \left(1 - \frac{u}{U} \right) dy$$

Momentum Thickness (θ)

This is defined as the distance which the total loss of momentum per second be equal to if it were passing a stationary plate. It is denoted by θ .

It may also be defined as the distance, measured perpendicular to the boundary of the solid body by which the boundary should be displaced to compensate for reduction in momentum of the flowing fluid on account of boundary layer formation.

Refer to diagram of displacement thickness above,

Mass of flow per second through the elementary strip = $\rho u dy$

Momentum/Sec. of this fluid inside the boundary layer

$$= \rho u dy \times U = \rho u^2 dy$$

Momentum/sec. of the same mass of fluid before entering boundary layer $= \rho u U dy$

Loss of Momentum/sec. $= \rho u U dy - \rho u^2 dy = \rho u (U - u) dy$

\therefore Total loss of momentum/sec

$$= \int_0^\delta \rho u (U - u) dy \text{ ----- (i)}$$

Let θ = Distance by which plate is displaced when the fluid is flowing with a constant velocity U . then loss of momentum/Sec. of fluid flowing through distance θ with a velocity U .

$$= \rho \theta U^2 \text{ ----- (ii)}$$

Equating eqns. (i) and (ii), we have

$$\rho \theta U^2 = \int_0^\delta \rho u (U - u) dy$$

OR

$$\theta = \int_0^\delta \frac{u}{U} \left(1 - \frac{u}{U} \right) dy$$

Energy Thickness (δ^{**})

Energy thickness is defined as the distance measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in K.E of the flowing fluid on account of boundary layer formation. It is denoted by (δ^{**})

Refer to the above displacement thickness diagram,

Mass of flow per second through the elementary strip $= \rho u dy$

K.E of this fluid inside the boundary layer

$$= \frac{1}{2} m u^2 = \frac{1}{2} (\rho u dy) u^2$$

K.E of the same mass of fluid before entering the boundary layer

$$\frac{1}{2} (\rho u dy) u^2$$

Loss of K.E. through elementary strip

$$\begin{aligned} & \frac{1}{2}(\rho u dy)u^2 - \frac{1}{2}(\rho u dy)u^2 \\ & = \frac{1}{2}\rho u(U^2 - u^2)dy \text{ ----- (i)} \end{aligned}$$

∴ Total loss of K.E of fluid

$$= \int_0^\delta \frac{1}{2}\rho u(U^2 - u^2)dy$$

Let (δ^{**}) = Distance by which the plate is displaced to compensate for the reduction in K.E

Then loss of K.E. through (δ^{**}) of fluid flowing with velocity

$$U = \frac{1}{2}(\rho U \delta_e)U^2 \text{ ----- (ii)}$$

Equating eqns (i) and (ii), we have

$$\frac{1}{2}(\rho u dy)u^2 = \int_0^\delta \frac{1}{2}\rho u(U^2 - u^2)dy$$

$$\delta_e = \frac{1}{U^3} \int_0^\delta u(U^2 - u^2)dy$$

or

$$\int_0^\delta \frac{u}{U} \left(1 - \frac{u^2}{U^2}\right) dy$$