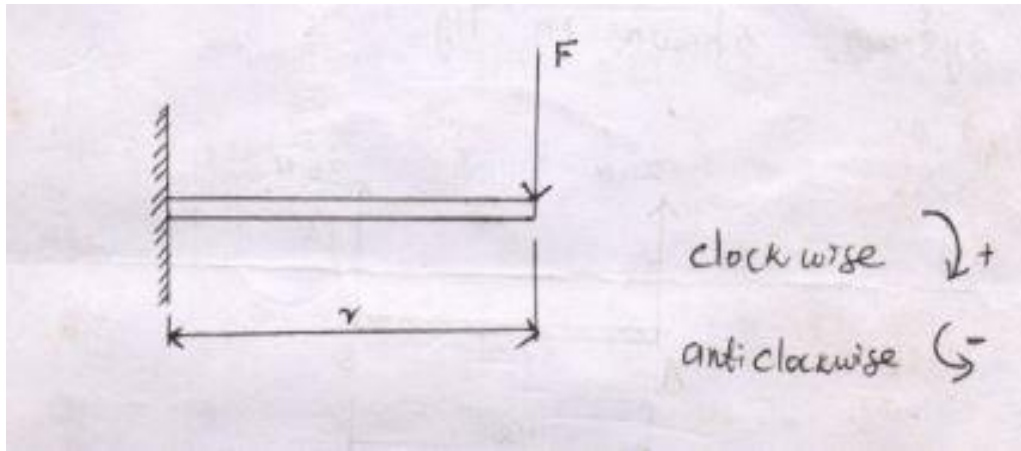


UNIT II

Statics of Rigid bodies in Two Dimensional

Moment of force:

Moment of force is defined as the product of the force and perpendicular distance of the line of the force from the point.



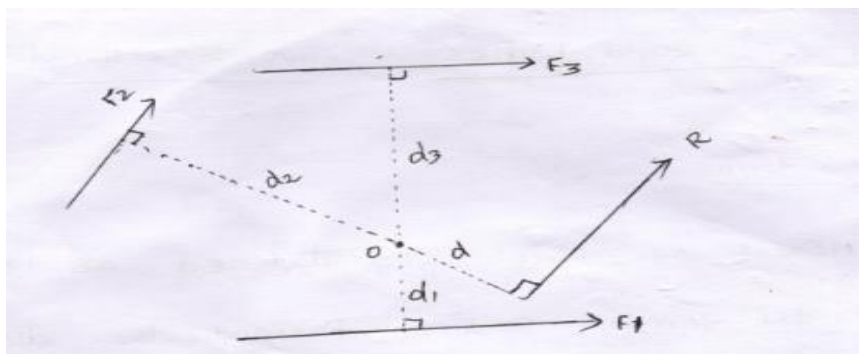
Moment = Force \times perpendicular distance.

$$M_o = F \times d \text{ N.m}$$

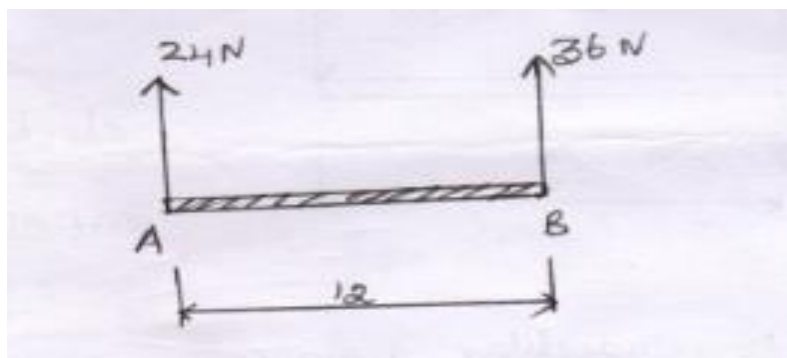
Varignon's Theorem:

The algebraic sum of the moment of any number of force about any point in their plane is equal to the moment of their resultant about the same point.

$$F_1 \times d_1 + F_2 \times d_2 + F_3 \times d_3 = R \times d$$



Find the resultant force for the parallel force System shown in fig.



Resultant force 'R'

$$R = 24 + 36$$

$$R = 60N$$

Location of resultant force:

Algebraic sum of moment of all force about a

$$\Sigma M_A = -36 \times 12$$

$$\Sigma M_A = -432 N.m$$

$$\Sigma M_A = 432 N.m(\text{clockwise})$$

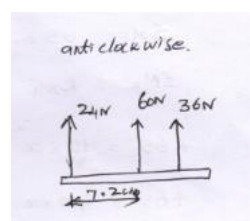
By virginal theorem

$$\Sigma M_A = R \times x$$

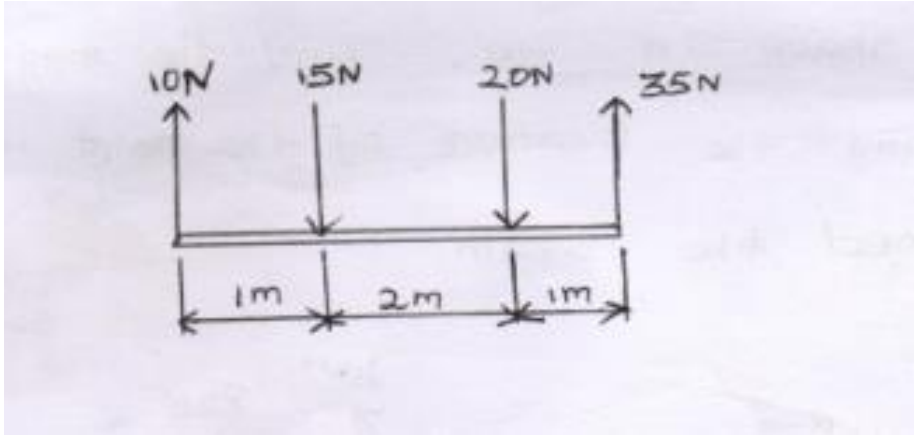
$$+432 = +60 \times x$$

$$x = \frac{+432}{+60}$$

$$x = 7.2 cm$$



2. Four parallel forces of magnitude 10N, 50N, 20N and 35N as shown in fig. Determine the magnitude and direction of the resultant. Find the distance of the resultant from point A.



Solution:-

Magnitude of resultant:-

$$R = 10 - 15 - 20 + 35$$

$$R = +10N$$

Locating of the resultant

$$\sum M_A = R \times x$$

$$\sum M_A = (15 \times 1) + (20 \times 3) + (-35 \times 4)$$

$$\sum M_A = -65 N.m$$

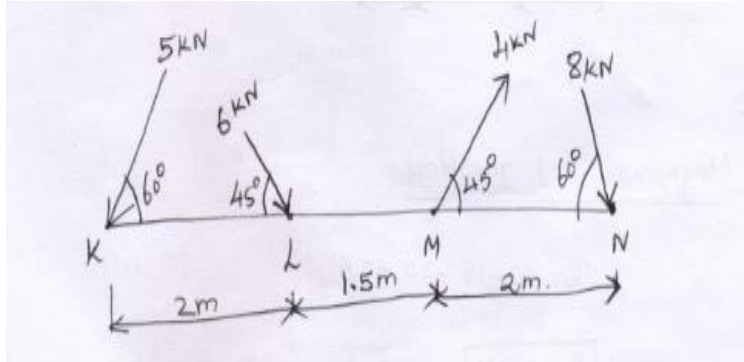
$$\sum M_A = R \times x$$

$$+65 = 10 \times x$$

$$x = (+65)/(+10)$$

$$x = 6.5m$$

1. A system of forces acts on a weightless beam as shown I fig. Find the magnitude of the resultant and the location of the point where the resultant met the beam.



Given:

$$\text{Load at } K = 5\text{ kN at } 60^\circ$$

$$L = 6\text{ kN at } 45^\circ$$

$$M = 4\text{ kN at } 45^\circ$$

$$N = 8\text{ kN at } 60^\circ$$

To find:

Resultant force & location

Soln:

$$\text{Resultant } R = \sqrt{(\sum F_H)^2 + (\sum F_V)^2}$$

$$\sum F_H = 0 \rightarrow \leftarrow$$

$$= -5 \cos 60 + 6 \cos 45 + 4 \cos 45 + 8 \cos 60$$

$$\sum F_H = 8.57 \text{ kN}$$

$$\sum F_V = 0 \uparrow + \downarrow -$$

$$= -5 \sin 60 + 6 \sin 45 + 4 \sin 45 + 8 \sin 60$$

$$\Sigma F_v = -12.67 \text{ KN}$$

$$R = \sqrt{(\Sigma FH)^2 + (\Sigma FV)^2}$$

$$R = \sqrt{(8.57)^2 + \Sigma(12.67)^2}$$

$$R = 15.3 \text{ Kn}$$

$$\text{Inclination of the resultant } \alpha = \tan^{-1} \left(\frac{\Sigma F_v}{\Sigma F_H} \right)$$

$$\alpha = \tan^{-1} \left(\frac{12.67}{8.57} \right)$$

$$\alpha = 55.92^\circ$$

To locate the resultant:

$$\Sigma M_k = 0 \downarrow + \uparrow -$$

$$\Sigma M_k = 0 + [+ \sin 45 \times 2] + [-4 \sin 45 \times 3.5] + [+8 \sin 60 \times 5.5]$$

$$\Sigma M_k = +36.69 \text{ KN.m (clockwise)}$$

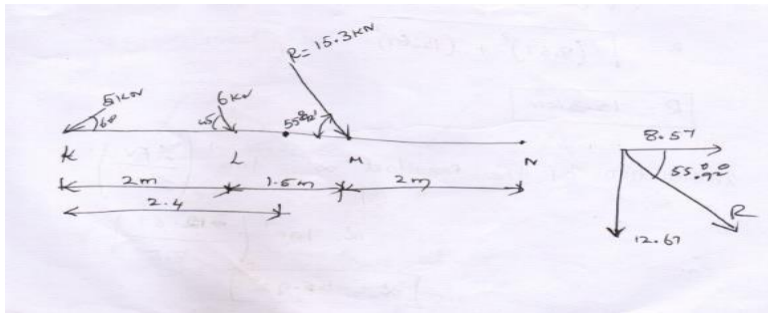
By varignon's Theorem

$$\Sigma M_k = R \times x$$

$$+36.69 = 15.3 \times x$$

$$x = \frac{+36.69}{15.3}$$

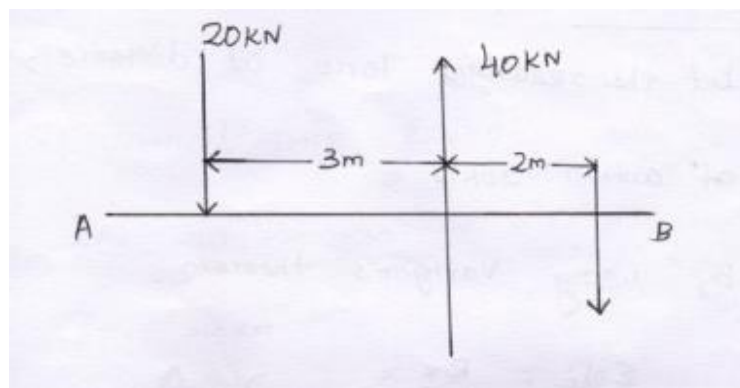
$$x = 2.4 \text{ m}$$



Problem:1

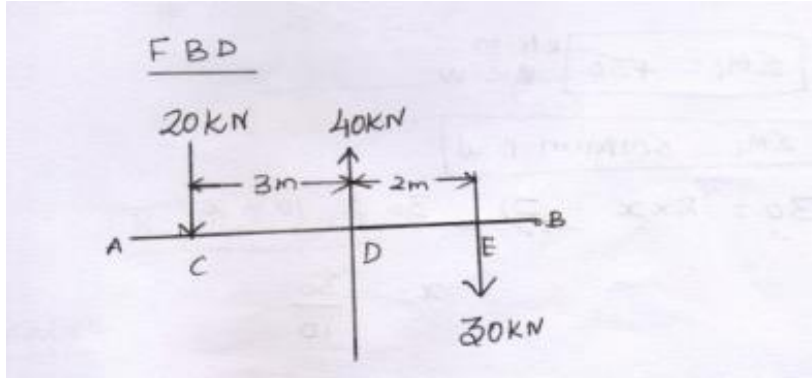
A coplanar parallel force system consisting of three forces acts on a rigid bar AB as shown fig. below

- Determine the simplest equivalent action for the force system.
- If an additional force of 10kN acts along the bar A to what be simplest equivalent action.



soln:

(a) simplest Equivalent force:



Sum of Horizontal force $\sum F_H = 0$

$$\sum F_H = 0$$

Sum of vertical force $\sum F_V = 0$

$$\sum F_V = 20 + 40 - 30 = -10\text{KN}$$

Magnitude of Resultant Force = R

$$R = \sqrt{(\sum F_H)^2 + (\sum F_V)^2}$$

$$= \sqrt{0^2 + (-10)^2}$$

$$R = \sqrt{100}$$

$$R = 10\text{N}$$

Line of Action:-

Let the resultant force at distance 'X' From the line of action 20kN

By using varignon's theorem

$$\sum M_c = R \times x$$

$$\sum M_c = (-40 \times 3) + (30 \times 5) = 120 + 150$$

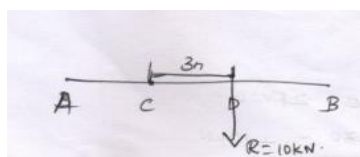
$$\sum M_c = +30\text{ Nm}$$

$$\sum M_c = 30 N.m \text{ c.w}$$

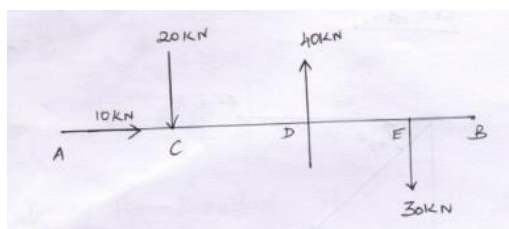
$$30 = R \times x \rightarrow 30 = 10 \times x$$

$$x = \frac{30}{10}$$

$$x = 3m$$



b) With additional force of 10kN from A to B



Sum of Horizontal force $\sum F_H = 0$

$$\sum F_H = 10kN$$

Sum of horizontal force $\sum F_v = 0$

$$\sum F_v = -20 + 20 - 30$$

$$\sum F_v = -10kN$$

Resultant Force 'R' $R = \sqrt{(\sum F_H)^2 + (\sum F_v)^2}$

$$R = \sqrt{(10)^2 + [-10]^2}$$

$$R = \sqrt{100 + 100} = \sqrt{200}$$

$$R = 14.14 \text{ KN}$$

Location

$$\sum M_c = \sum F_v$$

$$\sum M_c = (-40 \times 3) + (30 \times 5) = -30 \text{ kN.M}$$

$$\sum M_c = 30 \text{ KN.M} \quad \text{clockwise}$$

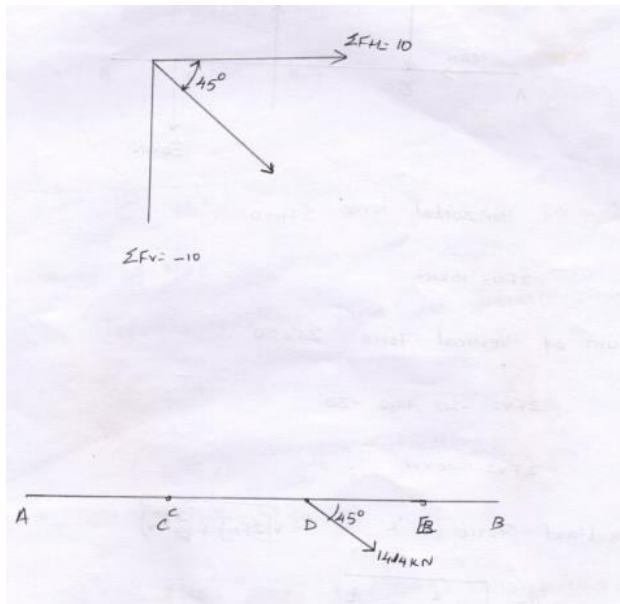
$$30 = 10 \times x$$

$$x = 13 \text{ m}$$

Location

$$\theta = \tan^{-1} \left(\frac{\sum F_v}{\sum F_h} \right) = \tan^{-1} \left(\frac{10}{10} \right)$$

$$\theta = 45^\circ$$



$$-80P = -4628.2$$

$$P = \frac{4628.2}{80}$$

$$P = 61.60N$$

ii) Magnitude of the Resultant force:

$$\text{Resultant } R = \sqrt{(\sum(F_H))^2 + (\sum(F_V))^2}$$

$$\sum F_H = -61.60 - 100 \cos 60$$

$$\sum F_H = -111.60 N$$

$$\sum F_v = 100 \sin 60$$

$$\sum F_v = 86.6N$$

$$R = \sqrt{[-111.60]^2 + [86.6]^2}$$

$$R = 141.26N$$

iii) Point of Application

By Varignon's theorem

$$\sum M_o = R \times x$$

$$\sum M_o = 61.60 \times 40 + [-100 \sin 60 \times 80] = 0$$

$$\sum M_o = 2464 - 6928$$

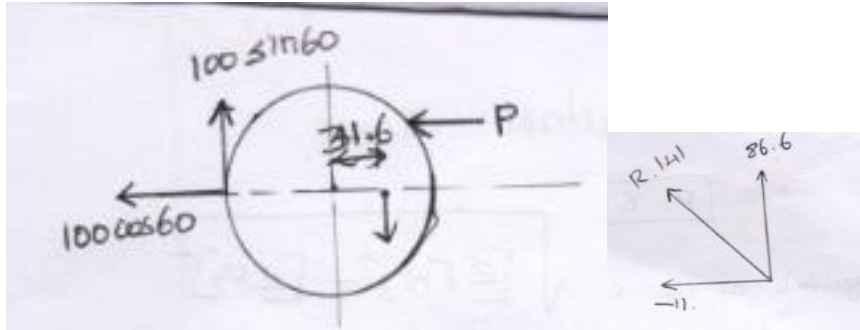
$$\sum M_o = -4464.2 \text{ Counts clockwise}$$

$$\sum M_o = 4464 \text{ Clockwise}$$

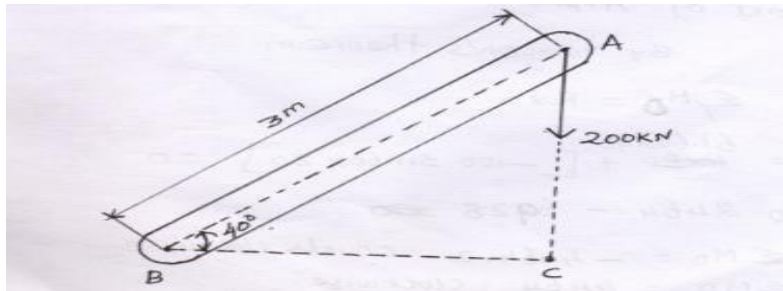
$$\sum M_o = R \times x$$

$$4464.2 = 141.26 \times x$$

$$X = 31.60 \text{ mm}$$



6. A 200kN vertical force is applied to the end of a lever which attached a shaft as B as shown in Fig Below. Determine the (i) magnitude of horizontal force (ii) The smallest force applied at which creates the same moment about B (iii) How far from the end B, at 400kN Vertical force must to create the same moment about B (iv) Replace the given system of force at B.



Vertical load at point A = 200kN

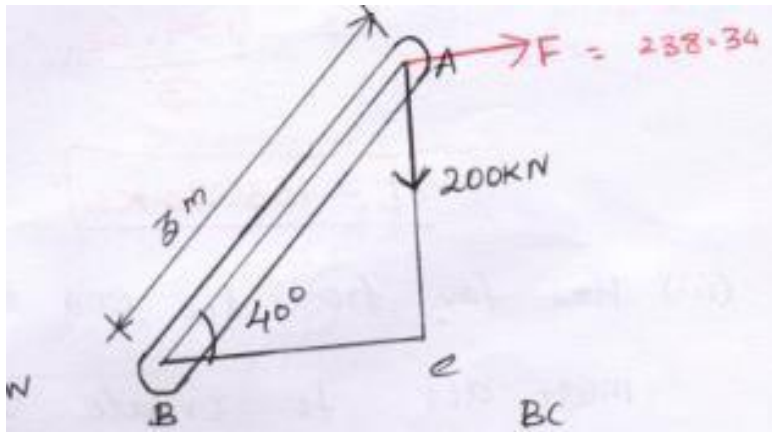
Length of bar L = 3m

Angle = 40°

Soln:

(i) The magnitude of horizontal force applied at 'A' which create same moment about 'B'

Take moment about 'B'



$$M_O = +200 \times BC$$

$$\cos \theta = \frac{BC}{3}$$

$$M_O = +200 \times 2.29$$

$$\cos \theta = \frac{BC}{3}$$

$$M_O = +459.62 \text{ KN.M}$$

$$BC = 2.29\text{m}$$

$\rightarrow F$

$$M_D = 459.62 \text{ KN.m}$$

$$\sin \theta = \frac{AC}{AB}$$

Take moment About 'o' horizontal force

$$\sin \theta = \frac{AC}{AB}$$

act forwards right

$$\sin 40 = \frac{AC}{3}$$

$$M_D = F \times AC$$

$$AC = 1.92\text{m}$$

$$459.62 = F \times 1.92$$

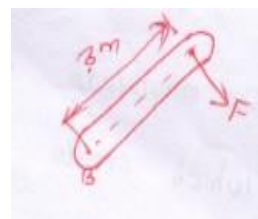
$$F = \frac{459.62 \text{ KN.m}}{1.92 \text{ m}}$$

$$F = 238.34 \text{ KN}$$

ii) The smallest force applied at which create the same moment about 'B'

moment About $B = 459.62 \text{ KN.m}$

$$M_B = F \times 3$$



$$459.62 = F \times 3$$

$$F = \frac{459.62}{3}$$

$$F = 153.20 \text{ KN}$$

(iii) How far from the end B, a 400KN vertical force must act to create the same moment about B.

Let 400KN Vertical force act at a distance of 'x' A to have same moment -459.62 KN.m clockwise

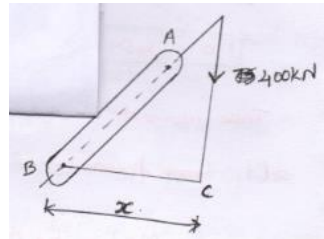
To have clockwise moment 400 N Vertical force on the right side of A

$$\text{Moment} = -459.62 \text{ KN.m}$$

$$-400 \times x = -459.62$$

$$x = (-459.62)/(-400)$$

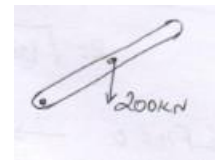
$$x = 1.149 \text{ m}$$



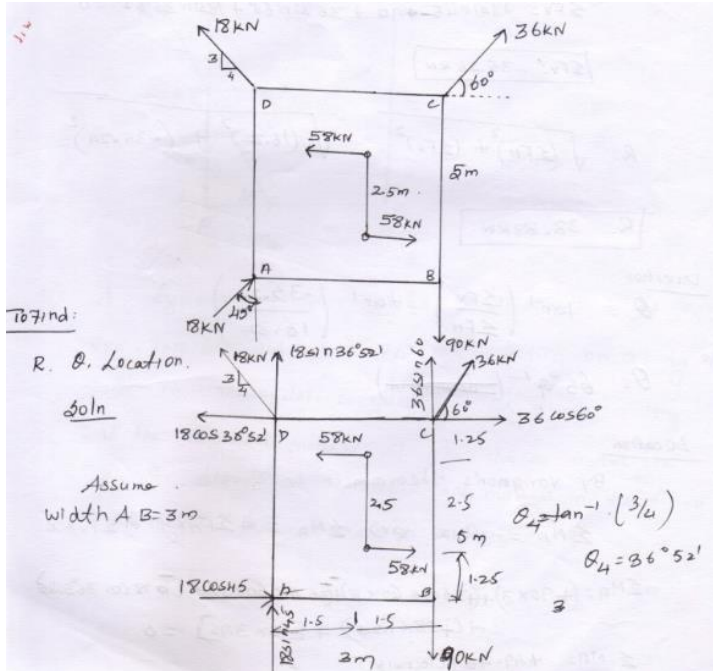
iv) Replace the given system of Force at B

$$\text{Downward load} = 200\text{KN}$$

$$\text{Moment at B} = 459.63 \text{ KN.m}$$



7. Determine the resultant of The calendar non concurrent force system shown in fig. below. Calculate its mangnitude and direction and locate its position with respect to the sides AB and AD



Resultant force

$$R = \sqrt{(\sum F_H)^2 + (\sum F_V)^2}$$

$$\sum F_H = 0 \rightarrow \leftarrow - F_H = F \cos \theta$$

$$\sum F_H = 18 \cos 45 + 36 \cos 60 - 18 \cos 36^\circ 52' - 58 + 58$$

$$\sum F_H = 16.32 \text{ KN}$$

$$\sum F_V = 0 \quad \uparrow \quad \downarrow$$

$$\sum F_V = 18 \sin 45 - 90 + 36 \sin 60^\circ + 18 \sin 36^\circ 52' = 0$$

$$\sum F_V = -35.26 \text{ KN}$$

$$R = \sqrt{(\sum F_H)^2 + (\sum F_V)^2} = \sqrt{(16.32)^2 + (-35.26)^2}$$

$$R = 38.88 \text{ KN}$$

Direction:-

$$\theta = \tan^{-1} \left(\frac{\Sigma F_V}{\Sigma F_H} \right) = \tan^{-1} \left(\frac{-35.26}{16.32} \right)$$

$$\theta = 65^\circ 9'$$

Location:

By varignon's theorem

$$\Sigma M_A = R \times x \text{ (or)} \Sigma M_A = \Sigma F_H \times y \text{ or } \Sigma F_V \times x$$

$$\begin{aligned} \Sigma M_A = (+90 \times 3) + (36 \cos 60 \times 5) + (-36 \sin 60 \times 3) \\ + (-18 \cos 36^\circ 52' \times 5) + (+58 \times 1.25) + (-58 \times 3.75) = 0 \end{aligned}$$

$$\Sigma M_A = +49.46 \text{ KN.M (clockwise)}$$

$$\Sigma M_A = 49.46 \text{ (clockwise)}$$

$$\Sigma M_A = \Sigma F_V \times x$$

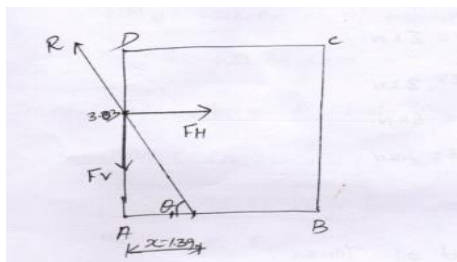
$$49.46 = 35.26 \times x$$

$$x = 1.39 \text{ m}$$

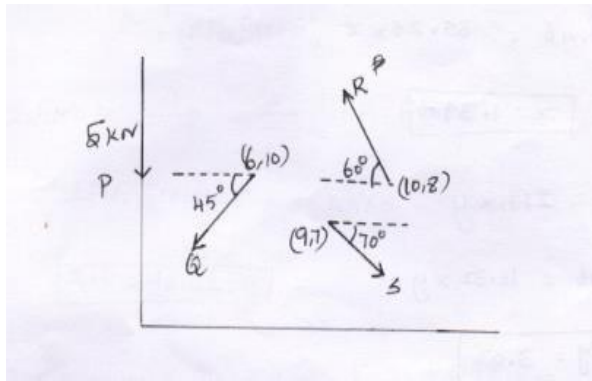
$$\Sigma M_A = \Sigma F_H \times y$$

$$49.46 = 16.32 \times y$$

$$y = 3.03$$



8. A system of four forces P, Q, R and S of magnitude 5KN, 8KN, 6KN And 4KN respectively acting on a body are shown in rectangular coordinates. As shown in fig find the moment of the forces about the origin O. also find the resultant moment of the forces about O. The distance are in meters.



Given:

Load on $P = 5\text{KN}$

Load on $Q = 3\text{KN}$

Load on $R = 6\text{KN}$

Load on $S = 4\text{KN}$

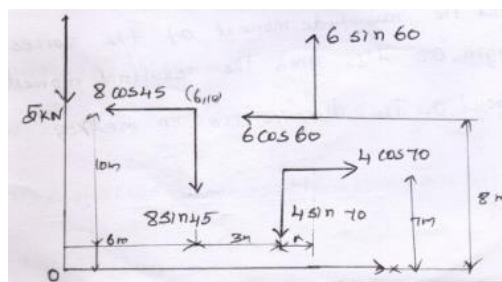
To Find:

1. moment of Forces

2. Resultant

Soln:-

Free body diagram



Moment of P

Moment of force ' P ' about the origin, M_P

$$M_P = 5 \times 0$$

$$M_p = 0$$

Moment of Q

Moment of force 'Q' about the origin, M_Q

$$M_Q = (8 \sin 45 \times 6) + (-8 \cos 45 \times 10)$$

$$M_Q = -22.64 \text{ KN.m}$$

$$M_Q = +2.64 \text{ KN C.W}$$

Moment of R

Moment of force R about the origin M_R

$$M_R = -75.96 \text{ KN.m}$$

$$M_R = 75.96 \text{ c.w}$$

Moment of S

Moment of force s about the Origin ' M_s '

$$M_s = (4 \cos \times 7) + (4 \sin 70 \times 9)$$

$$M_s = 43.40 \text{ KN.m}$$

