3.2 Gauss- Elimination Method, Gauss-Jordan Method

1. Solve the system of equations by (i) Gauss- Elimination Method (ii) Gauss-Jordan Method

$$10x - 2y + 3z = 23, \quad 2x + 10y - 5z = -33, \quad 3x - 4y + 10z = 41$$

Solution:

(i) Gauss- Elimination Method

The given system is equivalent to

$$\begin{bmatrix} 10 & -2 & 3 \\ 2 & 10 & -5 \\ 3 & -4 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 23 \\ -33 \\ 41 \end{bmatrix}$$

AX=B

$$[A, B] = \begin{bmatrix} 10 & -2 & 3 & 23 \\ 2 & 10 & -5 & -33 \\ 3 & -4 & 10 & 41 \end{bmatrix}$$

Now, we will make the matrix A as a upper triangular.

$$[A, B] \sim \begin{bmatrix} 10 & -2 & 3 & 23 \\ 0 & 52 & -28 & -188 \\ 0 & -34 & 91 & 341 \end{bmatrix}$$

$$R_2 \leftrightarrow 5R_2 - R_1,$$

$$R_3 \leftrightarrow 10R_3 - 3R_1$$

$$\sim \begin{bmatrix} 10 & -2 & 3 & 23 \\ 0 & 52 & -28 & -188 \\ 0 & 0 & 3780 & 11340 \end{bmatrix} --- (1)$$

$$R_3 \leftrightarrow 52R_3 + 34R_2$$

This is an upper triangular matrix,

Now, using back substitution method.

$$3780z = 11340$$

$$z = \frac{11340}{3780} = 3$$

$$52y - 28z = -188$$

$$52y - 28(3) = -188$$

$$52y - 84 = -188$$

$$52y = -188 + 84$$

$$52y = -104$$

$$y = -\frac{104}{52} = -2$$

$$10x - 2y + 3z = 23$$

$$10x - 2(-2) + 3(3) = 23$$

$$10x + 4 + 9 = 23$$

$$10x + 13 = 23$$

$$10x = 23 - 13$$

$$10x = 10$$

$$x = 1$$

Hence, the solution is ,

$$x = 1, y = -2, z = 3$$

(ii) Gauss- Jordan method

Take the equation (1)

$$[A, B] \sim \begin{bmatrix} 10 & -2 & 3 & 23 \\ 0 & 52 & -28 & -188 \\ 0 & 0 & 3780 & 11340 \end{bmatrix}$$

Now, we will make the matrix A is diagonal matrix.

$$\sim \begin{bmatrix} 12600 & -2520 & 0 & 17640 \\ 0 & 7020 & 0 & -14040 \\ 0 & 0 & 3780 & 11340 \end{bmatrix}$$

$$R_1 \leftrightarrow 1260R_1 - R_3,$$

$$R_2 \leftrightarrow 135R_2 + R_3$$

$$\sim \begin{bmatrix} 88452000 & 0 & 0 & 88452000 \\ 0 & 7020 & 0 & -14040 \\ 0 & 0 & 3780 & 11340 \end{bmatrix}$$

$$R_1 \leftrightarrow 7020R_1 + 2520R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

Hence, the solution is ,

$$x = 1, y = -2, z = 3$$

2. Solve the following system of equations by Gauss elimination method.

Solution

The given system is equivalent to

$$\begin{bmatrix} 5 & 4 \\ 3 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ 12 \end{bmatrix}$$

AX=B

$$[A, B] = \begin{bmatrix} 5 & 4 & 15 \\ 3 & 7 & 12 \end{bmatrix}$$

Now, we will make the matrix A as a upper triangular.

$$[A, B] \sim \begin{bmatrix} 5 & 4 & 15 \\ 0 & 23 & 15 \end{bmatrix}$$

$$R_2 \leftrightarrow 5R_2 - 3R_1,$$

This is an upper triangular matrix,

Now, using back substitution method.

$$23\, y = 15$$

$$y = \frac{15}{23} = 0.6522$$

$$5x + 4y = 15$$

$$5x + 4(0.6522) = 15$$

$$5x + 2.6088 = 15$$

$$5x = 15 - 2.6088$$

$$5x = 12.3912$$

$$x = 2.4783$$

Hence, the solution is ,

$$x = 2.4783, y = 0.6522$$

3. Using the Gauss-Jordan Method solve the following equations

$$10x + y + z = 12, \;\; 2x + 10y + z = 13, \;\;\; x + y + 5z = 7$$

Solution

Interchanging the first and the last equation then,

$$[A, B] = \begin{bmatrix} 1 & 1 & 5 & 1 \\ 2 & 10 & 1 & 13 \\ 10 & 1 & 1 & 12 \end{bmatrix}$$

$$[A,B] \sim \begin{bmatrix} 1 & 1 & 5 & 7 \\ 0 & 8 & -9 & -1 \\ 0 & -9 & -49 & -58 \end{bmatrix}$$

$$R_2 \leftrightarrow R_2 - 2R_1,$$

$$R_3 \leftrightarrow R_3 - 10R_1$$

$$\sim \begin{bmatrix} 1 & 1 & 5 & 7 \\ 0 & 1 & -1.125 & -0.125 \\ 0 & -9 & -49 & -58 \end{bmatrix}$$

$$R_2 \leftrightarrow \frac{R_2}{8}$$

$$\sim \begin{bmatrix} 1 & 0 & 6.125 & 7.125 \\ 0 & 1 & -1.125 & -0.125 \\ 0 & 0 & -59.125 & -59.125 \end{bmatrix}$$

$$R_1 \leftrightarrow R_1 - R_2,$$

$$R_3 \leftrightarrow R_3 + 9R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 6.125 & 7.125 \\ 0 & 1 & -1.125 & -0.125 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$R_3 \leftrightarrow \frac{R_3}{-59.125}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$R_1 \leftrightarrow R_1 - 6.125R_3,$$

$$R_2 \leftrightarrow R_2 + 1.125R_3$$

Hence, the solution is ,

$$x = 1, y = 1, z = 1$$