4.4 **Approximation of derivates using interpolation**

Newton's forward difference formula

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \cdots..\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 + \cdots..\right]$$

$$\left(\frac{d^3y}{dx^3}\right)_{x=x_0} = \frac{1}{h^3}\left[\Delta^3 y_0 - \frac{3}{2}\Delta^4 y_0 + \cdots..\right]$$

Newton's backward difference formula

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n + \cdots..\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \cdots..\right]$$

$$\left(\frac{d^3y}{dx^3}\right)_{x=x_n} = \frac{1}{h^3}\left[\nabla^3 y_n + \frac{3}{2}\nabla^4 y_n + \cdots..\right]$$

1. Find the first , second and third derivatives of $f(x) at\ \ x = 1.5$ if

x	1.5	2.0	2.5	3.0	3.5	4.0
$f(x)$	3.375	7.000	13.625	24.000	38.875	59.000

Solution

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1.5	3.375				
		3.625			
2.0	7.000		3		
		6.625		0.75	
					0
2.5	13.625		3.75		
		10.375		0.75	
3.0	24.000		4.5		0
		14.875			
				0.75	
3.5	38.875		5.25		
		20.125			
4.0	59.000				

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \cdots ..\right]$$

$$= \frac{1}{0.5}\left[3.625 - \frac{1}{2} * 3 + \frac{1}{3} * 0.75 - \frac{1}{4} * 0\right]$$

$$= 4.75$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 + \cdots ..\right]$$

$$= \frac{1}{(0.5)^2}\left[3 - 0.75 + \frac{11}{12} * 0\right] = 9$$

$$\left(\frac{d^3y}{dx^3}\right)_{x=x_0} = \frac{1}{h^3}\left[\Delta^3 y_0 - \frac{3}{2}\Delta^4 y_0 + \cdots ..\right]$$

$$= \frac{1}{(0.5)^3}[0.75] = 6$$

2. Compute $f'(0) and\ f''(4) from\ the\ data$

x	0	1	2	3	4
y	1	2.718	7.381	20.086	54.598

Solution

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	1				
		1.718			
1	2.718		2.945		
				5.097	
		4.663			8.668
2	7.381		8.042		
		12.705		13.765	
3	20.086		21.807		
		34.512			
4	54.598				

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \cdots ..\right]$$

$$= \frac{1}{1}\left[1.718 - \frac{1}{2} * 2.945 + \frac{1}{3} * 5.097 - \frac{1}{4} * 8.668\right]$$

$$= 1.718 - 1.4725 + 1.699 - 2.167 = -0.2225$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \cdots ..\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{1}\left[21.807 + 13.765 + \frac{11}{12} * 8.668.\right] = 43.5177$$