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DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

VII Semester

AU3008 Sensors and Actuators

UNIT – I - INTRODUCTION TO MEASUREMENTS AND SENSORS

1.9 Dynamic characteristics of first and second order transducers for standard test inputs

DYNAMIC CHARACTERISTICS:

Dynamic characteristics in measurements refer to the behavior of a measurement system when the ***input quantity being measured is changing over time***. Unlike static characteristics, which focus on steady-state conditions, dynamic characteristics provide insight into how a measurement system responds to rapid changes, fluctuations, or dynamic processes. Understanding these dynamic characteristics is essential for applications involving time-varying quantities or processes. Here are some key dynamic characteristics:

The various dynamic characteristics are:

i) Speed of response
ii) Measuring lag
iii) Fidelity
iv) Dynamic error
v) Overshoot
vi) Settling time
vii) Damped Oscillations
viii) Rise Time
ix) Fall Time

(i) Speed of response:

It is defined as the rapidity with which a measurement system responds to changes in the measured quantity.

(ii) Measuring lag:

The delay in the response of an instrument to the changes in the measured quantity is known as measuring lag. The measuring lags are of two types:

1) Retardation type: In this case the response of the measurement system begins immediately after the change in measured quantity has occurred.

2) Time delay lag: In this case the response of the measurement system begins after a dead time after the application of the input.

(iii) Fidelity:

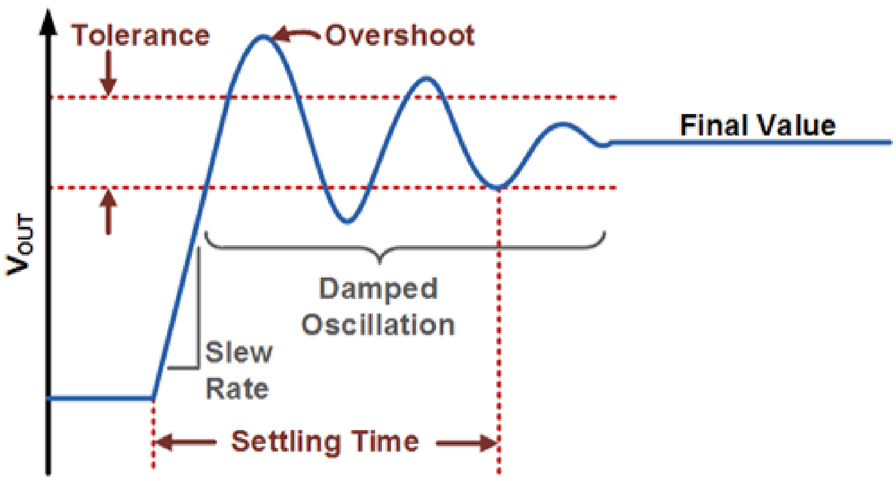
It is defined as the degree to which a measurement system indicates changes in the measurand quantity without dynamic error.

(iv) Dynamic error:

It is the difference between the true value of the quantity changing with time & the value indicated by the measurement system if no static error is assumed. It is also called measurement error.

(v) Overshoot:

The overshoot is evaluated as the maximum amount by which moving system moves beyond the steady state position.



(vi) Settling Time:

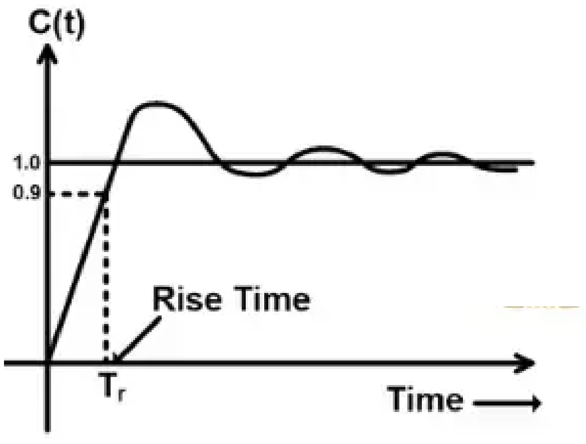
The settling time of a dynamic system is defined as the time required for the output to reach and steady within a given tolerance band.

(vii) Damped Oscillations:

Damped Oscillation is defined as the reduction in amplitude of an oscillating system due to the dissipation of energy.

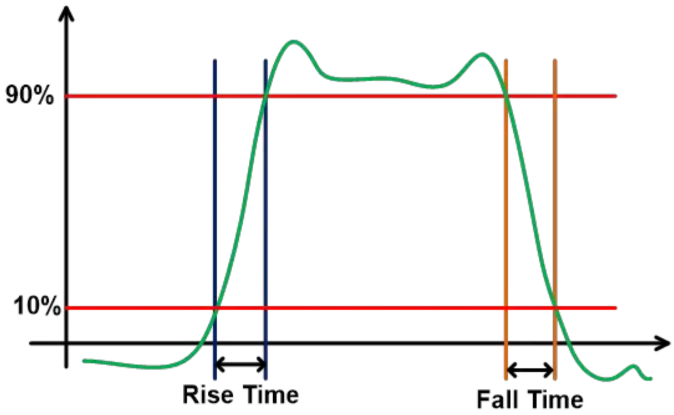
(viii) Rise Time:

Rise time is the time it takes for the measurement system's output to change from a specified percentage (e.g., 10% to 90%) of the final value in response to a step change in the input. It's a measure of the system's speed in responding to changes.



(ix) Fall Time:

Fall time is the time it takes for the measurement system's output to change from a specified percentage (e.g., 90% to 10%) of the initial value in response to a step change in the input.



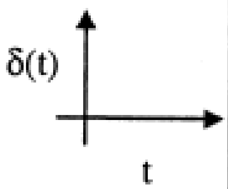
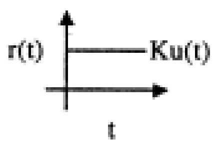
TEST INPUT

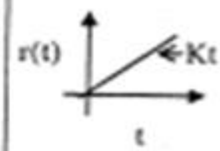
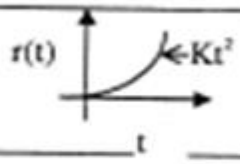
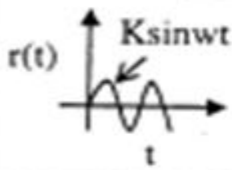
Normally, transducers are subjected to inputs which are random in nature. As it is not possible to predict the random input, the following test inputs are used to determine the dynamic behavior of the transducers:

- impulse input
- step input
- ramp input
- parabolic input
- sinusoidal input

The time function, laplace function and its pictorial representation of the test inputs are given in table 3.1.

Table 3.1: Test Inputs

	Name of the input	Time function	Laplace function	Pictorial representation
1	Impulse input If the area of the impulse is 1 then it is called unit impulse	$r(t) = \delta(t)$ $= 1$ for $t = 0$ $= 0$ for $t \neq 0$	1	
2	Step input	$r(t) = K$ for $t>0$ $= 0$ for $t<0$ If K=1 $r(t) = u(t) =$ unit step.	K/s	

	Name of the input	Time function	Laplace function	Pictorial representation
3	Ramp input	$r(t)=Kt$ for $t \geq 0$ $=0$ for $t \leq 0$	K/s^2	
4	Parabolic input	$r(t) = Kt^2$ for $t \geq 0$ $=0$ for $t \leq 0$	$2K/s^3$	
5	Sinusoidal input	$r(t)=K \sin wt$ for $t>0$ $= 0$ for $t \leq 0$	$\dfrac{kw}{s^2 + w^2}$	

ZERO-ORDER TRANSDUCER

The input-output relationship of a zero-order transducer is given by

$$y(t) = K\, r(t) \tag{3.10}$$

where r(t) is the input, y(t) is the output and K is the static-sensitivity of the transducer. The laplace transfer function of the zero-order transducer is given by

$$Y(s) / R(s) = K \tag{3.11}$$

The relationship clearly shows that the output varies exactly the same way as the input. Hence, a zero-order transducer response, represents ideal dynamic performance.

A potentiometer used for displacement measurements is an example for zero-order transducer. The output of a potentiometer is given by

$$e_0 = E \,.\, x/l$$
$$= K \,.\, x$$

where x is the displacement of the slider, l is the total length of the potentiometer, E is the excitation and e_o is the output in volts. The static sensitivity of the potentiometer is $K = E/l$ volts/cm. The

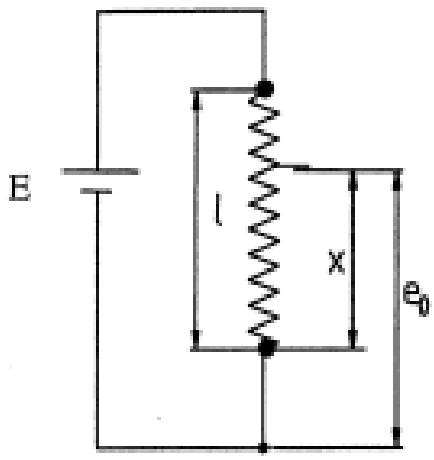


Fig 3.9
Potentiometer

potentiometer will behave as a zero-order instrument only when it is a pure resistance. It may be noted that displacement is considered as the input and not the force causing the displacement.

The response of zero-order transducers for some of the inputs are shown in fig. 3.10a & 3.10b

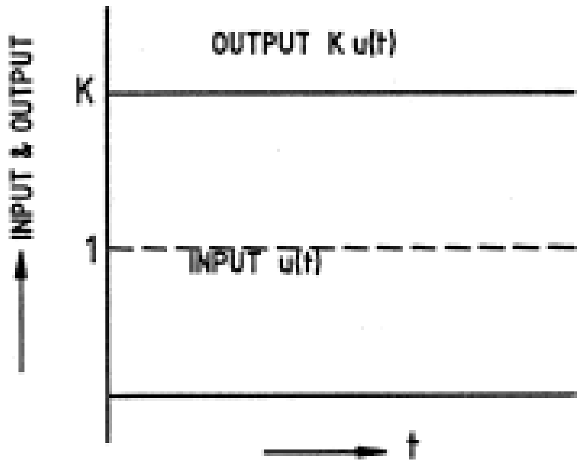


Fig 3.10a
Step response of zero-order transducer

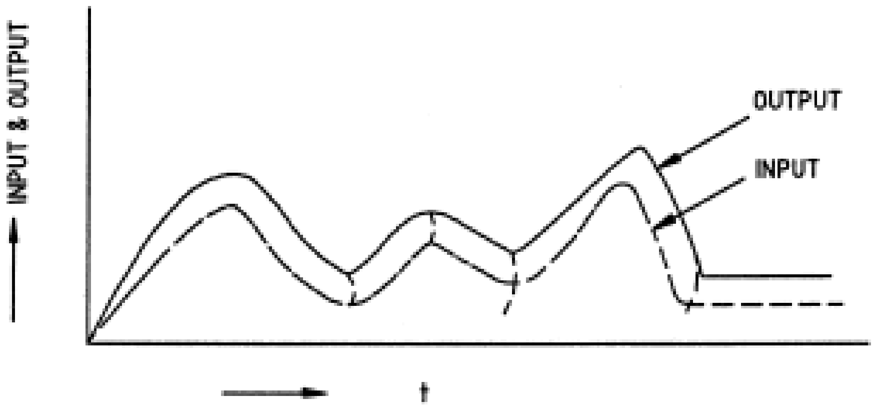


Fig 3.10b
Response of zero-order transducer for random input

3.6 FIRST-ORDER TRANSDUCER

The input-output relationship of a first-order transducer is given by

$$a_1 \frac{dy\ (t)}{dt} + a_0\, y\ (t) \ = \ b_0\, u\ (t)$$

where a_1, a_0 and b_0 are the parameters of the transducer. The Laplace transfer function of the first-order transducer is given by

$$\frac{y\ (s)}{u\ (s)} = \frac{b_0/a_0}{\left(\frac{a_1}{a_0} s + 1\right)} = \frac{K}{(\tau\, s + 1)} \qquad (3.13)$$

where $K = b_0 / a_0$ = static sensitivity

and $\tau = a_1 / a_0$ = time constant

The two parameters namely static sensitivity and time constant characterize the first-order transducer.

A thermocouple used for temperature measurements is a first-order transducer. Consider a thermocouple immersed in a fluid medium as shown in fig. 3.11. The heat balance equation is given by

$$UA\ (T_2 - T_1) \ = \ MS \frac{dT_1}{dt} \qquad (3.14)$$

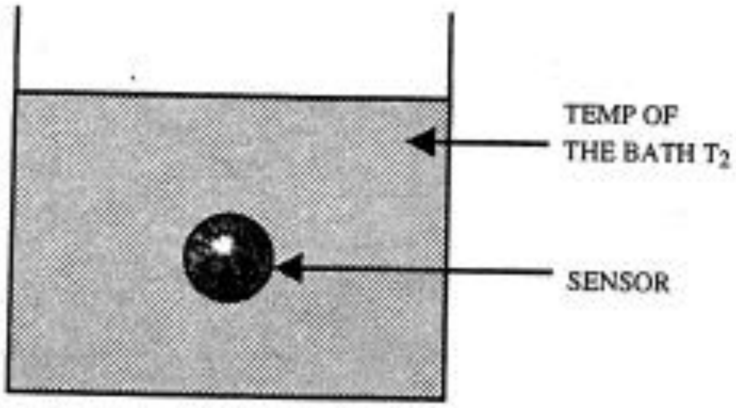


Fig 3.11
Example - I: order transducer

where U is the overall heat-transfer coefficient. A is the heat transfer area, T_2 is the temperature of the medium, T_1 is the temperature indicated by the thermocouple, M is the mass of the sensing portion of the thermocouple and S is the specific heat of the sensing bead.

The laplace transfer function is given by

$$\frac{T_1(s)}{T_2(s)} = \frac{1}{\tau s + 1} \qquad (3.15)$$

where $\tau = MS / UA$

The voltage output of a thermocouple is proportional to the temperature of the bead at the hot junction while the cold junction is kept constant at 0°C. Hence

$V = K\,T_1$

Where v is the thermocouple output in volts and K is proportionality constant.

The overall transfer function of the thermocouple is given by

$$\frac{V(s)}{T_2(s)} = \frac{V(s)}{T_1(s)}\,\frac{T_1(s)}{T_2(s)} \qquad (3.16)$$

$$= \frac{K}{\tau s + 1} \qquad (3.17)$$

which is a first order system.

It may be noted that when the hot junction of a thermocouple is kept inside a thermal wall in order to protect it from abrasive and corrosive effects of the surroundings, the transducer becomes a second order one.

Impulse Response of First Order Transducer

When a I order transducer is subjected to an Impulse input, the output of the transducer is known as the impulse response of the I order transducer.

This can be obtained as given below.

Let y(s) be the Laplace transform of the response. Then y(s), in general, is given as

$$y(s) = \frac{K}{1 + s\tau} u(s) \qquad (3.18)$$

where u(s) is the Laplace transform of the input.

If the input is assumed to be an unit impulse, then u(s) = 1. Therefore

$$y(s) = \frac{K}{1 + s\tau} 1 \qquad (3.19)$$

The time response can be determined by taking the Laplace inverse of y(s)

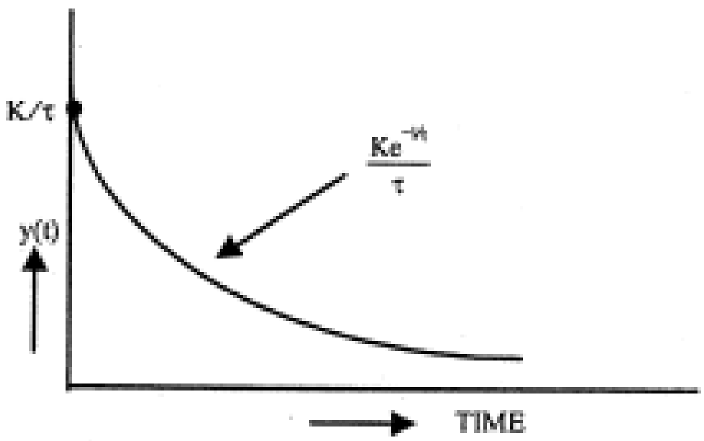


Fig 3.12a

Ideal impulse response of I order transducer

$$L^{-1} y(s) = y(t) = L^{-1} \frac{K}{1 + s\tau} = \frac{K}{\tau} e^{-t/\tau} \quad (3.20)$$

The variation of y(t) with respect to time is plotted in fig. 3.12a. The actual output, however, cannot rise from zero to K/τ in zero time but will take a small but negligible time to reach the peak. The practical input and output curves of impulse response of I order instrument / transducer is shown in fig. 3.12b.

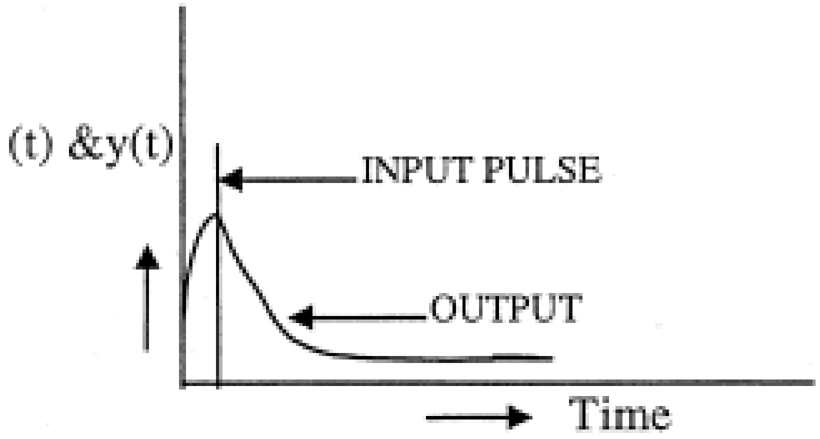


Fig 3.12b

Practical impulse response of I order transducer

The impulse response of three first order transducers with different time-constants are portrayed in fig. 3.13.

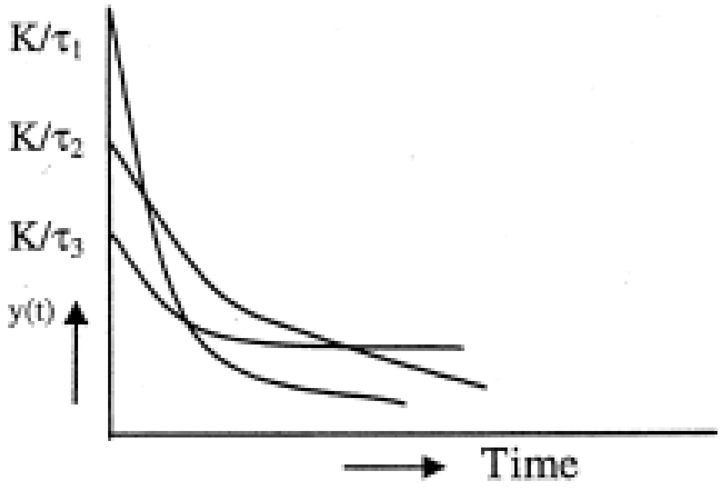


Fig 3.13

Impulse response for different time constants - I order transducer

As the time-constant becomes larger, the response becomes flatter. When the time-constant is very small the system approaches zero-order and the output is similar to the input except the change in magnitude.

The impulse response is also useful in determining the transfer-function of the transducer in situations where the transfer function is not already known. It may be noted that when the transducer is excited with an unit impulse, laplace transform of the system output is same as the transfer function of the transducer itself.

3.6.2 Step-Response Of First-Order Transducer

When a first-order transducer is excited by a unit-step input,

$$\alpha(s) = 1/s$$

$$y(s) = \frac{k}{(1+s\tau)} u(s)$$

Now

$$y(s) = \frac{K}{s(1+s\tau)}$$

$$y(t) = L^{-1} y(t)$$

$$= L^{-1} \frac{K}{s(1+s\tau)}$$

From the Laplace transform table,

$$y(t) = K(1 - e^{-t/\tau}) \qquad (3.21)$$

The variation of y(t) with respect to time is plotted in fig. 3.14a. The output rises at a faster rate initially and slowly afterwards.

The transducer takes theoretically infinite time to reach the steady-state value of the output, K. however, in practice the final value is assumed to have reached in four or five times the time-constant of the transducer. Actually the output would have reached 95.0% of the final value in three times the time-constant, 98.2% in 4 times the time-constant and 99.3% in five times the time-constant. The output will reach 63.2% of the final value when $t = \tau$ i.e. in one time constant. It is the normal practice to define a

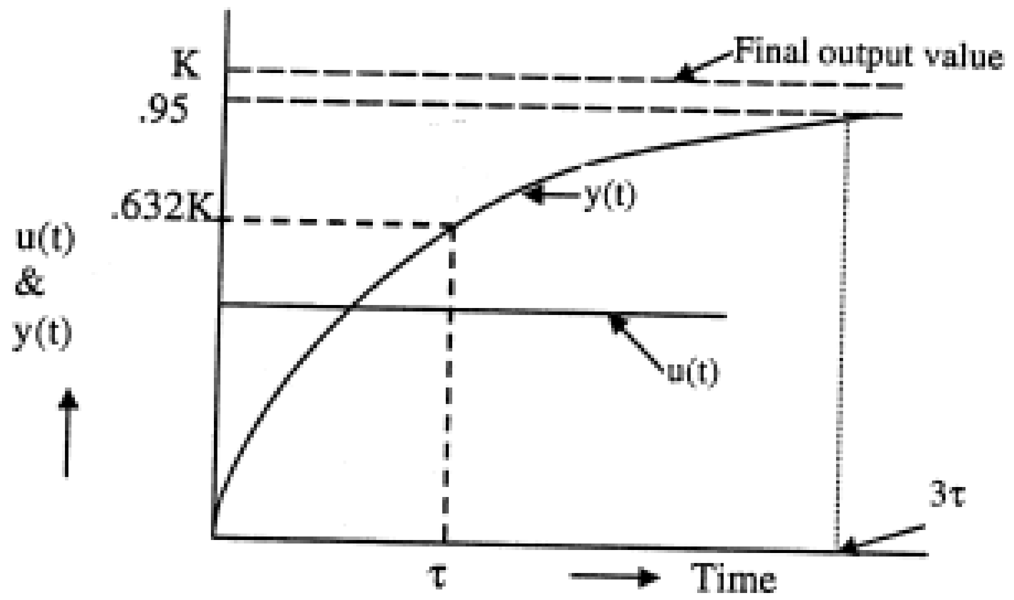


Fig 3.14a
Step response of I order transducer

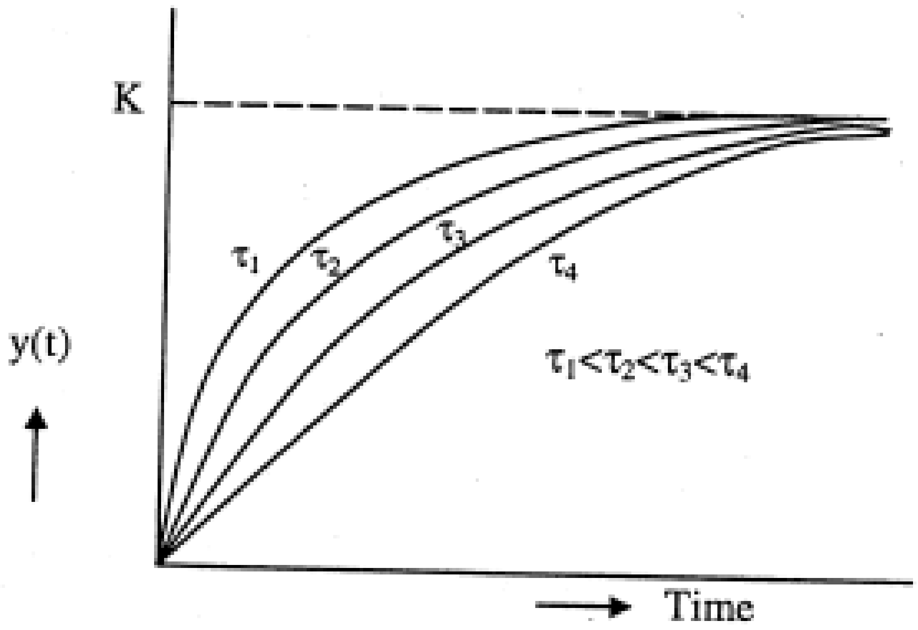


Fig 3.14b
Step response for different time constants

reading time for a first-order transducer as 4 times the time constant.

Reading time of transducer is = 4τ.

In the expressions derived thus for in this section, it is conveniently assumed that the initial conditions are zero. However, when the initial conditions are non-zero, the expression for the step-response will get modified as

$$y(t) = y(o) + (K - y(o))(1 - e^{-t/\tau}) \quad (3.22)$$

for example, assume a simple RC network, fig. 3.14b, whose transfer function is a first-order one. i.e.

$$\frac{V_o}{V_i} = \frac{1}{1 + RCs}$$

When the capacitor is initially charged to 5V the expression for the voltage across the capacitor is given by,

$$V_o = 5 + (20 - 5(1 - e^{-t/\tau}) \text{ volts}$$

The variation of V_o with respect to time is given in fig. 3.15.

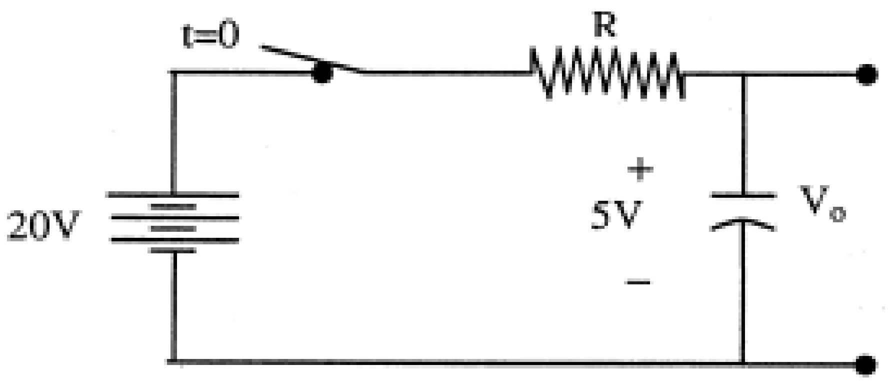


Fig 3.15a

First-order system with non-zero initial condition

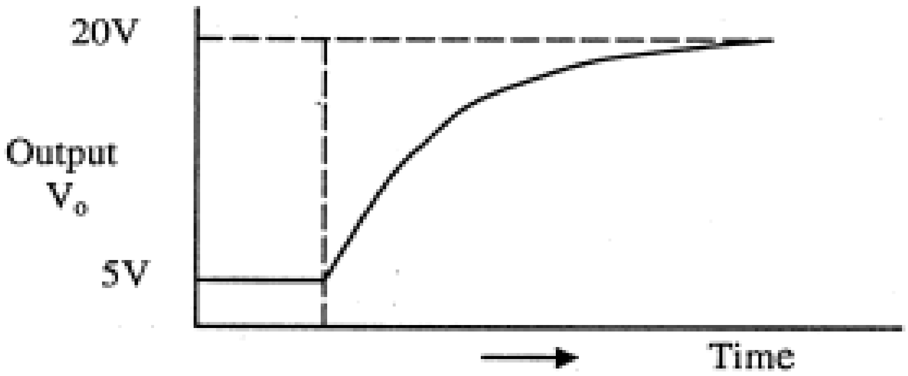


Fig 3.15b

Step response of I-order system with non-zero initial condition

Experimental Determination of Time-Constant

The time-constant of the first-order transducer can be determined experimentally from the step-response in three ways. The transducer is selected to a step change in input and its output is recorded. To do this, the following steps are taken the transducer is initially relaxed. An unit-step input, is applied. Simultaneously a stop watch is started. For every second the output value is noted. A graph of the output V_s time is plotted.from the output curve the following procedure is adopted to determine the time constant.

First method

The equation for the step output is given by

$$y(t) = K(1 - e^{-t/\tau})$$

assuming zero initial conditions.

When $t = \tau$, $y(t) = K(1 - e^{-1}) = 0.632\,K$

Hence the time-constant is the time taken for the output of the transducer to reach 63.2% of the final value for a step-input. The time taken for the output to reach 63.2% of the final value is determined by the graph which gives the time constant of the system.

Second method

The output, $y(t) = K(1 - e^{-t/\tau})$

The slope of the output curve, (ref. fig 3.16)

$$\frac{dy(t)}{dt} = -Ke^{-t/\tau}\left(-\frac{1}{\tau}\right) = \frac{K}{\tau}e^{-t/\tau}$$

$$\frac{dy(t)}{dt} = \frac{K}{\tau}e^{-0} = \frac{K}{\tau}$$

If the transducer output increases with the same slope it has at t = 0, then the final value of the output will be reached in t. This fact can also be used to determine the time constant from the step-response. A tangent to the experimentally obtained output curve at t = 0 is drawn. This line cuts the final value of the output at τ.

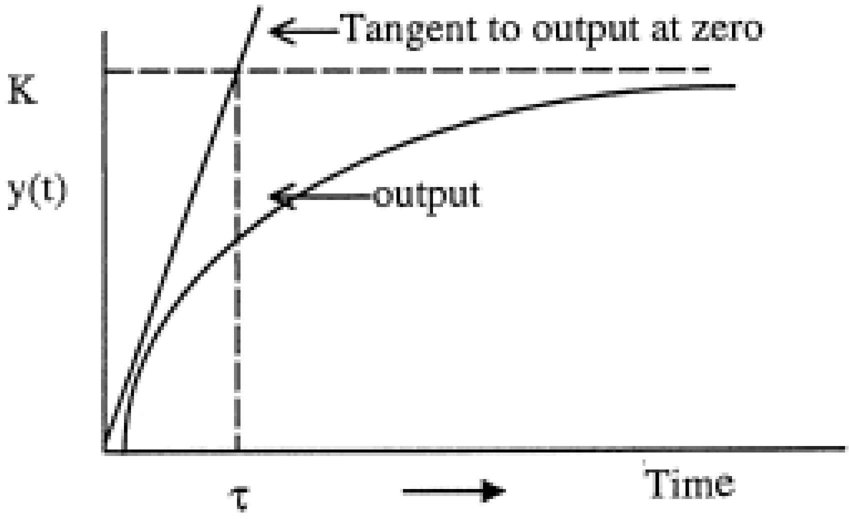


Fig 3.16
Time constant by initial slope method

Third method

When the complete plot of the step-response is not available, the time-constant can be determined if the output at two different instances are known. (ref. fig 3.16)

Assume $y(t) = y_1$ at $t = t_1$

and $y(t) = y_2$ at $t = t_2$

now $y_1 = K(1 - e^{-t_1/\tau})$

$$y_2 = K(1 - e^{-t_2/\tau})$$

$$\frac{y_1}{y_2} = \frac{1 - e^{-t_1/\tau}}{1 - e^{-t_2/\tau}}$$

If t_2 is chosen as $2t_1$, where t_1 can be arbitrarily chosen, then

$$\frac{y_1}{y_2} = \frac{1 - e^{-t_1/\tau}}{1 - e^{-2t_1/\tau}} = \frac{1}{1 + e^{-t_1/\tau}}$$

$$y_1(1 + e^{-t_1/\tau}) = y_2$$

$$e^{-t_1/\tau} = \frac{y_2 - y_1}{y_1} = \left(\frac{y_2}{y_1} - 1\right)$$

$$\tau = \frac{t_1}{\ln\left(\frac{y_1}{y_2 - y_1}\right)} \qquad (3.23)$$

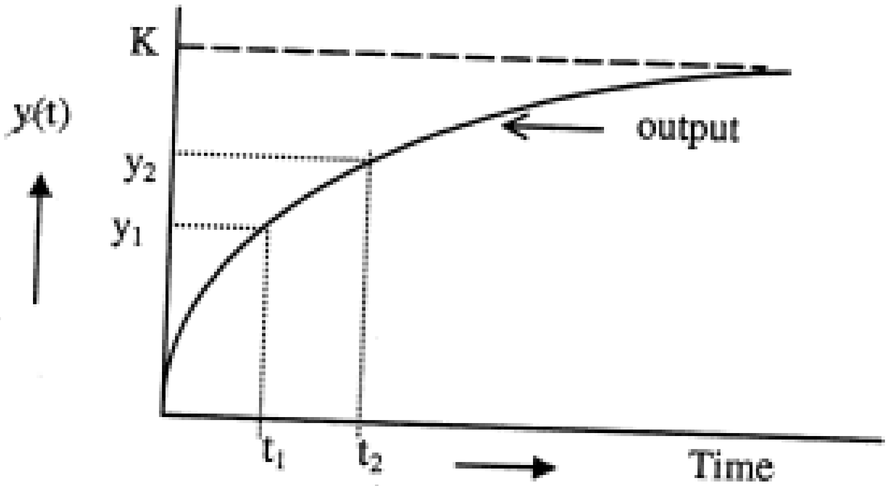


Fig 3.17
Time constant by two point method

3.6.4 Ramp Response Of First-Order Transducer

Consider a first-order transducer subjected to a ramp-input given by

$$u(t) = R\,t \quad \text{for} \quad t \geq 0$$
$$= 0 \quad \text{for} \quad t < 0$$

where R is the slope of the ramp.

$$u(s) = \frac{R}{s^2}$$

$$y(s) = \frac{K}{1 + s\tau} \cdot \frac{R}{s^2}$$

$$y(t) = L^{-1} \frac{KR}{s^2(1 + s\tau)}$$

$$= KR(\tau e^{-t/\tau} + t - \tau) \qquad (3.24)$$

for an unit ramp, R=1 and $y(t) = (\tau e^{-t/\tau} + t - \tau)$
The plot of y(t) Vs time is shown in fig. 3.18.

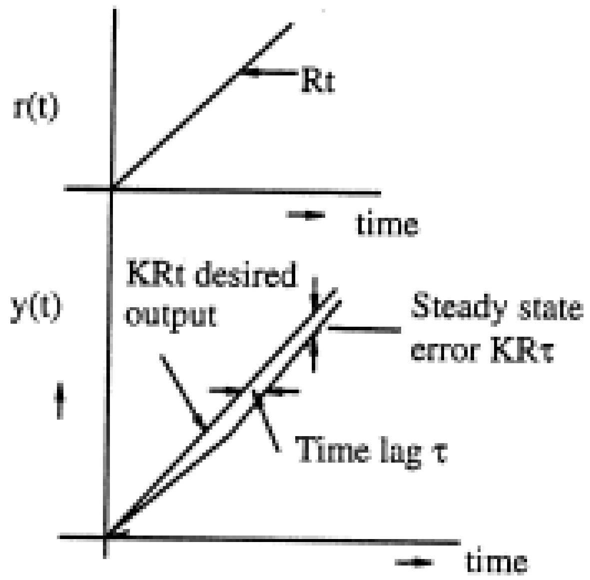


Fig 3.18
Ramp response of first-order transducer

By examining the equation (3.24), it is observed that the first-order system always has a measurement error for a ramp input which is given by

$$e(t) = K\,u(t) - y(t)$$

where K u(t) is the desired output and y(t) is the actual output.

$$e(t) = + KRt - KR\left(\tau e^{-t/\tau} + t - \tau\right)$$

$$e(t) = -KR\ \tau e^{-t/\tau} + KR\,\tau \qquad (3.25)$$

transient error steady-state error

The transient- error is the error which dies down as time becomes large. The steady-state error persists even if time becomes infinity. When the time-constant of the transducer is very small, the transient error disappears quickly and the magnitude of the steady-state error is also very less. When the transducer is subjected to a fast changing input (large R), the steady-state error also increases as the transducer finds it difficult to follow rapid change in input. During steady-state, the horizontal displacement between desired output and actual output curves is observed to be τ. This can be interpreted as that the transducer is reading the output which was there τ seconds ago.

3.7.1 Examples of Second Order Instrument

A vibration galvanometer is an example of second-order instrument, which is shown in fig. 3.24. This consists of a suspended coil placed in a magnetic field between a pole pair.

When a current i flows through the coil, the coil is subjected to a deflecting torque equal to Gi where G is the displacement constant of the galvanometer. This results in a deflection of the coil by an angle θ. The three opposing torque to this motion are due to moment of inertia, damping effect and elasticity of suspension, which are given by

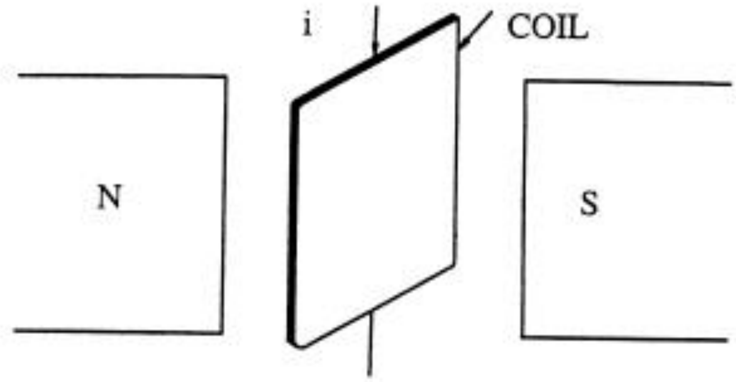


Fig 3.24
Vibration galvanometer

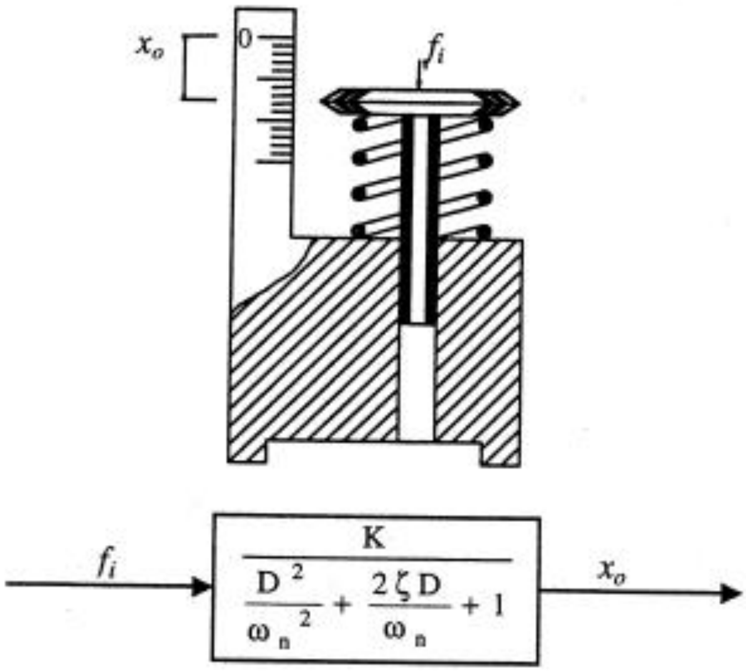


Fig 3.25
Second-order instrument - Spring scale system.

$$a\frac{d^2\theta}{dt^2},\ b\frac{d\theta}{dt}\quad \text{and}\quad c\,\theta$$

respectively, where a, b and c are constants. According to Newton's laws of motion, the deflecting torque is equal to the opposing torque at every instant of time. Hence

$$a\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + c\,\theta = Gi\,(t) \qquad (3.33)$$

The Laplace transfer function of the galvanometer dynamics is

$$\frac{\theta(s)}{I(s)} = \frac{G}{as^2 + bs + c} \text{ which is a second order system.} \qquad (3.34)$$

Spring scale system is shown in fig. 3.25 is another example of a second order system. The applied force is opposed by the mass, damping force and the spring force manometer, a thermometer in a thermowell are some of the other examples of second-order transducers.

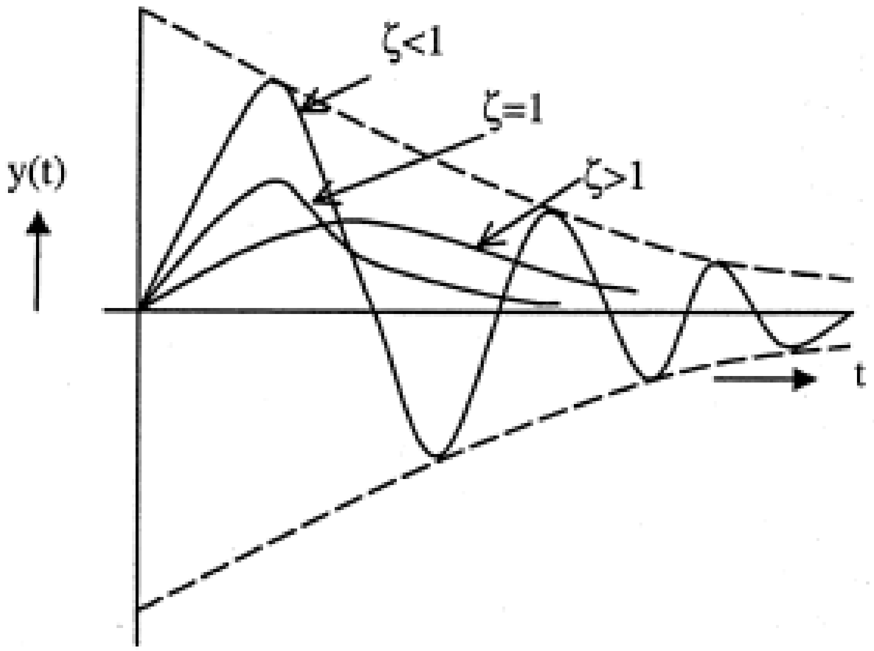


Fig 3.26
Impulse-response of second-order transducer

3.7.2 Impulse-Response of Second-Order Transducers

When a second-order transducer is subjected to an unit impulse, the laplace transform of the output is given by

$$Y(s) = \frac{K}{\frac{s^2}{\omega_n^2} + \frac{2\zeta}{\omega_n} s + 1} R(s) \quad (3.35)$$

$$= \frac{K}{\frac{s^2}{\omega_n^2} + \frac{2\zeta}{\omega_n} s + 1} \quad \because R(s) = 1 \quad (3.36)$$

$$y(t) = L^{-1} Y(s)$$

$$= L^{-1} \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$= L^{-1} \frac{K\omega_n^2}{(s + \zeta\omega_n)^2 + \omega_n^2(1-\zeta^2)}$$

(see the derivation for step response)

y(t) can be obtained from Laplace transform table as given below.

For under damped conditions ($\zeta < 1$),

$$y(t) = \frac{K\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(w_n \sqrt{1-\zeta^2}\, t) \quad (3.37)$$

For critically damped condition ($\zeta = 1$),

$$y(t) = K\omega_n^2 t e^{-\omega_n t} \quad (3.38)$$

For over damped conditions ($\zeta > 1$),

$$y(t) = \frac{K\omega_n}{2\sqrt{\zeta^2 - 1}} \left[e^{(-\zeta + \sqrt{\zeta^2-1})\omega_n t} - e^{(-\zeta - \sqrt{\zeta^2-1})\omega_n t} \right] \quad (3.39)$$

The response of the second-order transducer for unit impulse is shown in fig. 3.26 for different damping conditions.

3.7.3 Step-Response of Second-Order Transducer

When a second-order transducer is subjected to an unit step input,

$$R(s) = 1/s$$

The laplace transform of the output is given by

$$Y(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \cdot \frac{1}{s} \qquad (3.40)$$

This can be split into partial fractions as given below

$$Y(s) = K\left[\frac{1}{s} - \frac{s + 2\zeta\omega_n}{s^2 + 2\zeta\omega_n^s + \omega_n^2}\right] \qquad (3.41)$$

which can be written by adding and subtracting $\zeta^2\omega_n^2$ to the denominator of the second term as

$$= K\left[\frac{1}{s} - \frac{s + \zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_n^2(1 - \zeta^2)} - \frac{\zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_n^2(1 - \zeta^2)}\right]$$

By using the laplace transform table, for $\zeta < 1$,

$$y(t) = K\left[1 - e^{-\zeta\omega_n t}\cos\omega_n(\sqrt{1-\zeta^2})\,t - \frac{\zeta}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\omega_n(\sqrt{1-\zeta^2})\,t\right] \qquad (3.42)$$

This can be reduced to the following form

$$y(t) = K\left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_n\sqrt{1-\zeta^2}\,t + \phi)\right] \qquad (3.43a)$$

where $\phi = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$ (3.43b)

In a similar way, for $\zeta = 1$, y(t) can be obtained as

$$y(t) = K[1 - (1 + \omega_n t)e^{-\omega_n t}] \qquad (3.44)$$

For $\zeta > 1$,

$$y(t) = K\left[1 + \frac{\zeta - \sqrt{\zeta^2 - 1}}{2\sqrt{\zeta^2 - 1}} e^{(-\zeta - \sqrt{\zeta^2 - 1})\omega_n t} - \frac{\zeta + \sqrt{\zeta^2 - 1}}{2\sqrt{\zeta^2 - 1}} e^{(-\zeta + \sqrt{\zeta^2 - 1})\omega_n t}\right] \qquad (3.45)$$

The step response of second-order transducer for various values of damping ratios are shown in fig. 3.27.

Whenever a second-order transducer is suddenly connected to an input, it is equivalent to the application of step input. To have a quick indication of the measured values, the time taken for the transducer response to reach the steady-state value should be minimum. As the second-order system subjected to step-input takes infinite time (theoretically) to reach the steady-state value, it is customary to define settling time for such systems. The settling time is the time taken for the output to reach and stay within a specified percentage of steady-state value. For example, 5% settling time means, the time taken for the system output to reach and stay within 95% to 105% of the steady-state value.

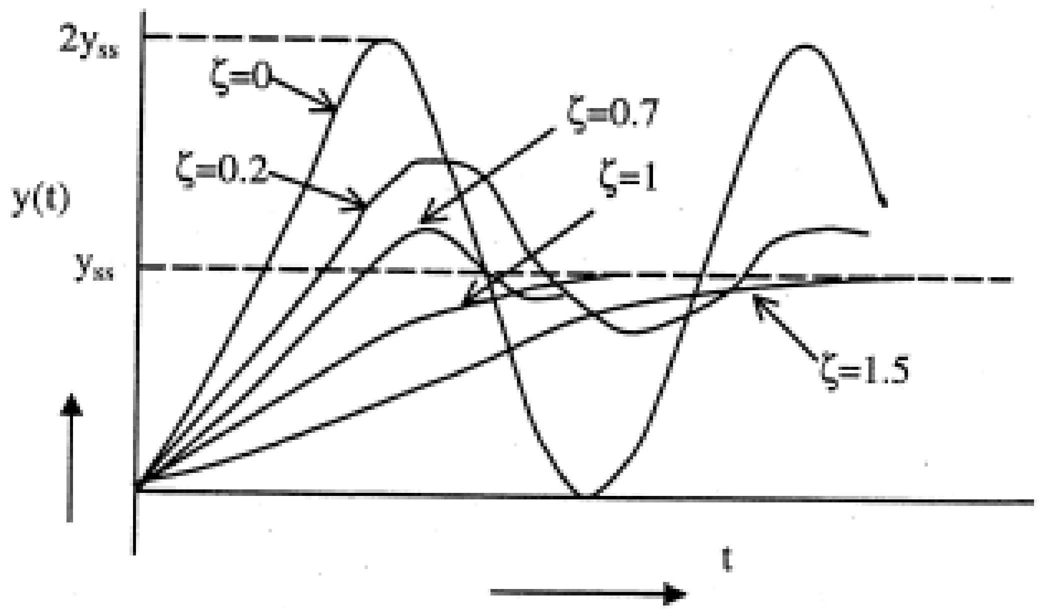


Fig 3.27
Step response of a II-order transducer for ζ = 0, 0.2, 0.7, 1, 1.5

When the second-order transducer is undamped ($\zeta = 0$), the settling time is infinity. As the damping ratio is increased, the settling time decreases and reaches an optimum value and again increases for over damped conditions. If we are interested in 10% settling time, then $\zeta = 0.6$ gives optimum value whereas $\zeta = 0.7$ to 0.8 gives the optimum value for 5% settling time. Many commercial transducer systems are designed to have a damping ratio of 0.6 to 0.7.

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