3.5. LMTD for Cross-flow Heat Exchangers

LMTD given by Eq (10.6) is strictly applicable to either parallel flow or counter flow exchangers. When we have multipass parallel flow or counter flow or cross flow exchangers, LMTD is first calculated for single pass counter flow exchanger and the mean temperature difference is obtained by multiplying the LMTD with a correction factor F which takes care of the actual flow arrangement of the exchanger. Or,

$$\dot{Q} = U\,A\,F\,(LMTD) \qquad (3.7)$$

The correction factor F for different flow arrangements are obtained from charts given in Fig. 3.10 (a, b, c, d).

3.6. Fouling Factors in Heat Exchangers

Heat exchanger walls are usually made of single materials. Sometimes the walls are bimettalic (steel with aluminium cladding) or coated with a plastic as a protection against corrosion, because, during normal operation surfaces are subjected to fouling by fluid impurities, rust formation, or other reactions between the fluid and the wall material. The deposition of a film or scale on the surface greatly increases the resistance to heat transfer between the hot and cold fluids. And, a scale coefficient of heat transfer h, is defined as:

$R_s = 1/h_s A$, °C/ W or K / W

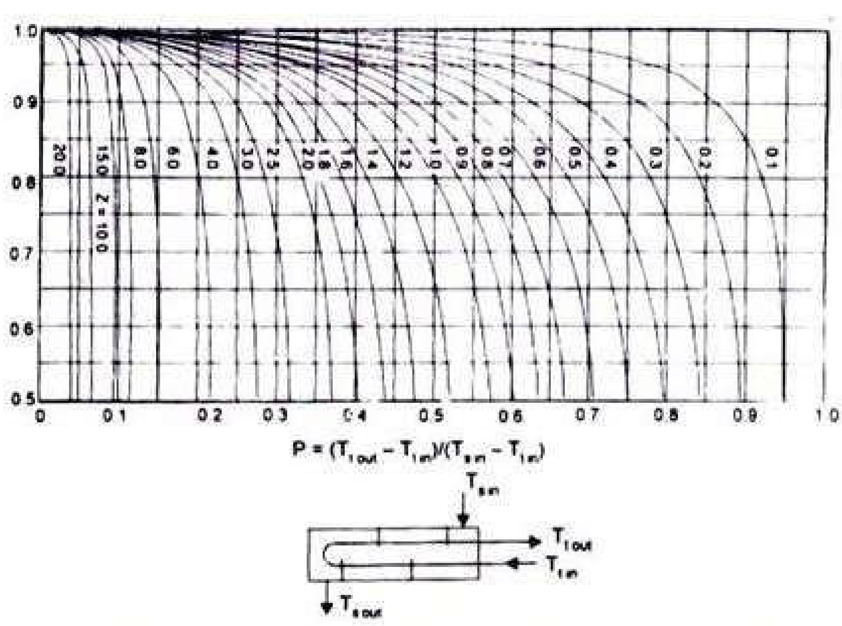


Fig 3.10(a) correctio factor to counter flow LMTD for heat exchanger with one shell pass andtwo, or a muliple of two,tube passes

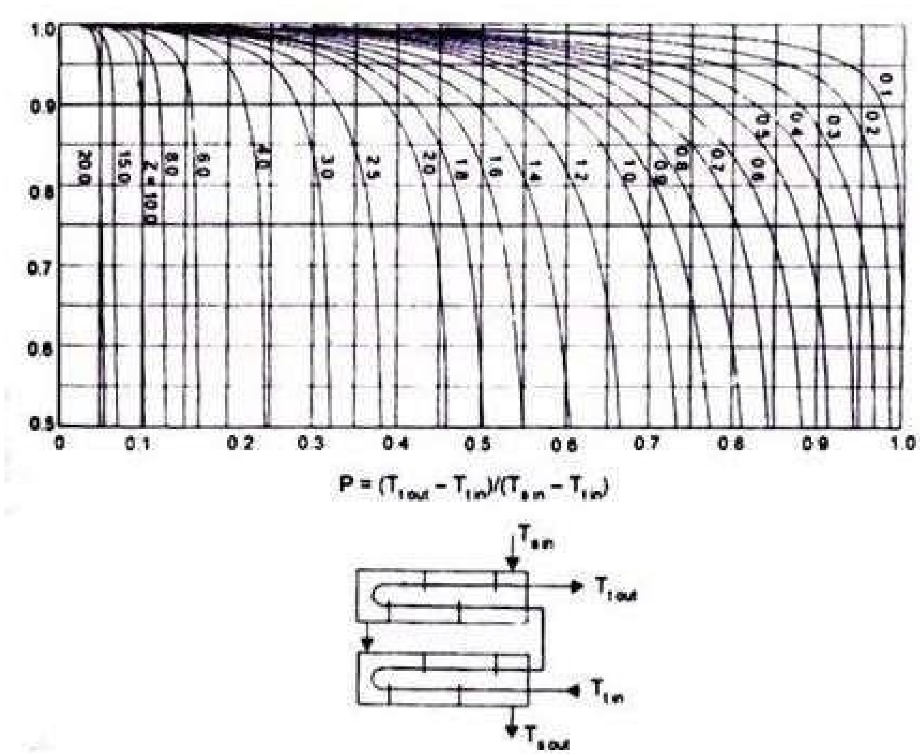


Fig 3.10 (b) Correction factor to counter flow LMTD for heat exchanger with two shell passes and a multiple of two tube passes

where A is the area of the surface before scaling began and l/h_s, is called 'Fouling Factor'. Its value depends upon the operating temperature, fluid velocity, and length of service of the heat exchanger. Table 10.1 gives the magnitude of l/h, recommended for inclusion in the overall heat transfer coefficient for calculating the required surface area of the exchanger

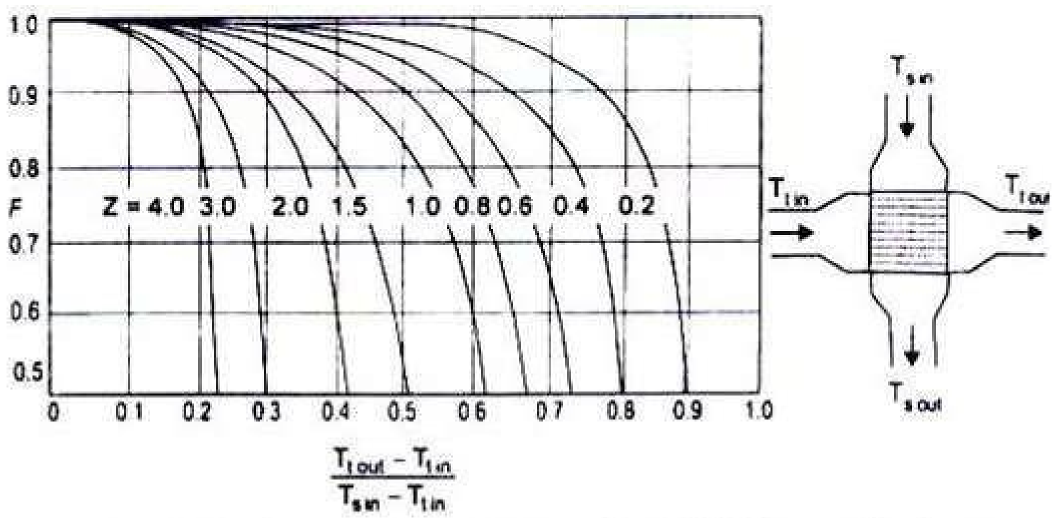


Fig3.10(c) Correction factor to counter flow LMTD for cross flow heat exchangers, fluid on shell side mixed, other fluid unmixed one tube pass..

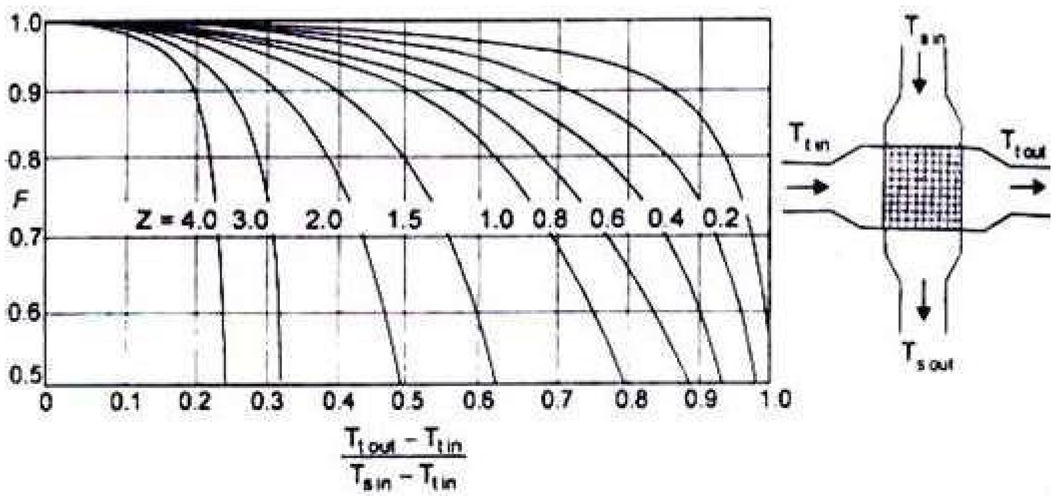


Fig. 3.10 (d) Correction factor to counter flow LMTD for cross flow heat exchangers, both fluids unmixed, one tube pass..

Table 3.1 Representative fouling factors ($1/h_s$)

Type of fluid	Fouling factor	Type of fluid	Fouling Factor
Sea water below 50°C	000009 m'K/W	Refrigerating liquid	0.0002 m'K/W

above 50°C	0.002		
Treated feed water	0.0002	Industrial air	0.0004
Fuel oil	0.0009	Steam, non-oil-bearing	0.00009
Quenching oil	0.0007	Alcohol vapours	0.00009

However, fouling factors must be obtained experimentally by determining the values of U for both clean and dirty conditions in the heat exchanger.

7. The Overall Heat Transfer Coefficient

The determination of the overall heat transfer coefficient is an essential, and often the most uncertain, part of any heat exchanger analysis. We have seen that if the two fluids are separated by a plane composite wall the overall heat transfer coefficient is given by:

$$1/U=(1/h_i)+(L_1/k_1)+(L_2/k_2)+(1/h_o) \qquad (3.8)$$

If the two fluids are separated by a cylindrical tube (inner radius r_i, outer radius r_0), the overall heat transfer coefficient is obtained as:

$$1/U_i=(1+h_i)+(r_i/k)\ln(r_o/r_i)+(r_i/r_o)(1/h_o) \qquad (3.9)$$

where h_i, and h_o are the convective heat transfer coefficients at the inside and outside surfaces and V, is the overall heat transfer coefficient based on the inside surface area. Similarly, for the outer surface area, we have:

$$1/U_o=(1/h_o)+(r_o/k)\ln(r_o+r_i)+(r_o+r_i)(1/h_i) \qquad (3.10)$$

and U_iA_i will be equal to U_oA_o; or, $U_ir_i=U_or_o$.

The effect of scale formation on the inside and outside surfaces of the tubes of a heat exchanger would be to introduce two additional thermal resistances to the heat flow path. If h_{si} and h_{so} are the two heat transfer coefficients due to scale formation on the inside and outside surface of the inner pipe, the rate of heat transfer is given by

$$Q=(T_i-T_o)/\left[(1/h_iA_i)+1/h_{si}A_i+\ln(r_o+r_i)/2\pi Lk+1/h_{so}A_o+(1/h_oA_o)\right] \qquad (3.11)$$

where T_i, and T_o are the temperature of the fluid at the inside and outside of the tube. Thus, the overall heat transfer coeffiCIent based on the inside and outside surface area of the tube would be:

$$1/U_i = 1/h_i + 1/h_{si} + (r_i/k)\ln(r_o/r_i) + (r_i/r_o)(1/h_{so}) + (r_i/r_o)(1/h_o); \qquad (3.12)$$

and

$$1/U_o = (r_o/r_i)(1/h_i) + (r_o/r_i)(1/h_{si}) + \ln(r_o/r_i)(r_0/k) + 1/h_{so} + 1/h_o$$

Example 3.1 In a parallel flow heat exchanger water flows through the inner pipe and is heated from 25°C to 75°C. Oil flowing through the annulus is cooled from 210°C to 110°C. It is desired to cool the oil to a lower temperature by increasing the length of the tube. Estimate the minimum temperature to which the oil can be cooled.

Solution: By making an energy balance, heat received by water must be equal to 4he heat given out by oil.

$$\dot{m}_w c_w (75-25) = \dot{m}_o c_o (210-110); \dot{C}_w/\dot{C}_o = 100/50 = 2.0$$

In a parallel flow heat exchanger, the minimum temperature to which oil can be cooled will be equal to the maximum temperature to which water can be heated,

Fig. 10.2: $(T_{ho} = T_{co})$

therefore, C_w (T –25) = C_o (210 –T);

(T –25)/(210 –T) = 1/2 = 0.5; or, T = 260/3 = 86.67°C.

or the same capacity rates the oil can be cooled to 25°C (equal to the water inlet temperature) in a counter-flow arrangement.

Example 3.2 Water at the rate of 1.5 kg/s IS heated from 30°C to 70°C by an oil (specific heat 1.95 kJ/kg C). Oil enters the exchanger at 120°C and leaves the exchanger at 80°C. If the overall heat transfer coefficient remains constant at 350 W /m^2°C, calculate the heat exchange area for (i) parallel-flow, (ii) counter-flow, and (iii) cross-flow arrangement.

Solution: Energy absorbed by water,

$$\dot{Q} = \dot{m}_w c_w (\Delta T) = 1.5 \times 4.182 \times 40 = 250.92 \text{ kW}$$

(i) Parallel flow: Fig. 10.9; $\Delta T_a = 120 - 30 = 90$; $\Delta T_b = 80 - 70 = 10$

LMTD = (90 – 10)/ln(90/10) = 36.4;

Area = Q/U (LMTD) = 250920 /(350 × 36.4) = 19.69 m^2.

(ii) Counter flow: Fig 10.9; ΔT_a. = 120 – 70 = 50, ΔT_b = 80 – 30 = 50

Since ΔT_a. = ΔT_b,. LMTD should be replaced by ΔT = 50

Area A = $\dot{Q}/U(\Delta T) = 250920/(350 \times 50) = 14.33\ m^2$

(iii) Cross flow: assuming both fluids unmixed - Fig. 10.10d

using the nomenclature of the figure and assuming that water flows through the tubes and oil flows through the shell,

$$P = (T_{to} - T_{ti})/(T_{si} - T_{ti}) = (70-30)/(120-30) = 0.444$$

$$Z = (T_{si} - T_{so})/(T_{to} - T_{ti}) = (120-80)/(70-30) = 1.0$$

and the correction factor, F = 0.93

$\dot{Q} = UAF(\Delta T)$; or Area A = 250920/(350 × 0.93 × 50) = 15.41 m^2.

Example 3.3 0.5 kg/s of exhaust gases flowing through a heat exchanger are cooled from 400°C to 120°C by water initially at 25°C. The specific heat capacities of exhaust gases and water are 1.15 and 4.19 kJ/kgK respectively, and the overall heat transfer coefficient from gases to water is 150 W/m^2K. If the cooling water flow rate is 0.7 kg/s, calculate the surface area when (i) parallel-flow (ii) cross-flow with exhaust gases flowing through tubes and water is mixed in the shell.

Solution: The heat given out by the exhaust gases is equal to the heat gained by water.

or, 0.5 × 1.15 × (400 – 120) = 0.7 × 4.19 × (T – 25)

Therefore, the temperature of water at exit, T = 79.89°C

For paralle1-flow: $\Delta T_a = 400 - 25 = 375$; $\Delta T_b = 120 - 79.89 = 40.11$

LMID = (375 – 40.11)/ln(375/40.11) = 149.82

$\dot{Q}$ = 0.5 × 1.15 × 280 = 161000 W;

Therefore Area A = 161000/(150 × 149.82) = 7.164 m^2

For cross-flow: $\dot{Q}$ = U A F (LMTD);

and LMTD is calculated for counter-flow system.

$$\Delta T_a = (400 - 79.89) = 320.11;\ \Delta T_b = 120 - 25 = 95$$

$$\text{LMTD} = (320.11 - 95)/\ln(320.11/95) = 185.3$$

Using the nomenclature of Fig 10.10c,

$$P = (120 - 400)/(25 - 400) = 0.747$$

$$Z = (25 - 79.89)/(120 - 400) = 0.196 \qquad\qquad \therefore\ F = 0.92$$

and the area $A = 161000/(150 \times 0.92 \times 185.3) = 6.296\ m^2$

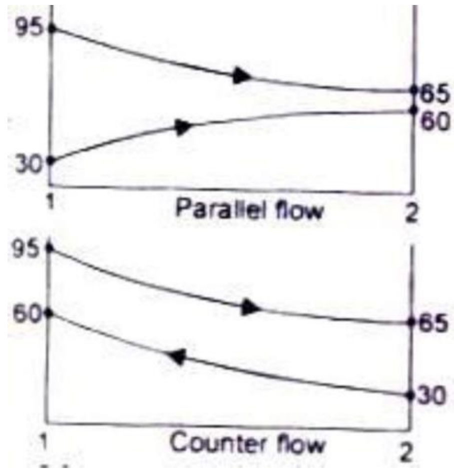
Example 3.4 In a certain double pipe heat exchanger hot water flows at a rate of 5000 kg/h and gets cooled from 95°C to 65°C. At the same time 5000 kg/h of cooling water enters the heat exchanger. The overall heat transfer coefficient is 2270 W/m^2K. Calculate the heat transfer area and the efficiency assuming two streams are in (i) parallel flow (ii) counter flow. Take C_p for water as 4.2 kJ/kgK, cooling water inlet temperature 30°C.

Solution: By making an energy balance:

Heat lost by hot water = 5000 ×4.2 ×(95 –65)

= heat gained by cold water = 5000 ×4.2 ×(T –30) 30

T = 60°C



(i) Parallel now

$\theta_1 = (95 - 30) = 65$

$\theta_2 = (65 - 60) = 5$

$\text{LMTD} = (65 - 5) / \ln(65/5) = 23.4$

Area, $A = \dot{Q} / (U \times LMTD) = \dfrac{500 \times 4.2 \times 10^3 \times 30}{3600 \times 2270 \times 23.4} = 3.295\ m^2$

(ii) Counter flow: $\theta_1 = (95 - 60) = 35$

$\theta_2 = (65 - 30) = 35$

LMTD = □T = 35

Area A = 500 × 4200 × 30/(3600 × 2270 × 35) = 2.2 m^2

$\in$, Efficiency.= Actual heat transferred/Maximum heat that could be transferred. Therefore, for parallel flow, $\in$ = (95 – 65)/(95 – 60) = 0.857

For counter flow, $\in$ = (95 - 65)/(95 – 30) = 0.461.

Counter now

Example 3.5 The flow rates of hot and cold water streams running through a double pipe heat exchanger (inside and outside diameter of the tube 80 mm and 100 mm) are 2 kg/s and 4 kg/so The hot fluid enters at 75°C and comes out at 45ºC. The cold fluid enters at 20°C. If the convective heat transfer at the inside and outside surface of the tube is 150 and 180 W /m^2K, thermal conductivity of the tube material 40 W/mK, calculate the area of the heat exchanger assuming counter flow.

Solution: Let T is the temperature of the cold water at outlet.

By making an energy balance, $\dot{Q} = \dot{m}_h c_h (T_{h1} - T_{h2}) = \dot{m}_c c_c (T_{c2} - T_{c1})$

since $c_h = c_c$, 4.2 kJ / kgK; $2 \times (75 - 45) = 4 \times (T - 20)$; $T = 35°C$

and $\dot{Q} = 252$ kW

for counter flow: $\theta_1 = (75 - 35) = 40$; $\theta_2 = (45 - 20) = 25$

LMTD = (40 – 25) / ln (40/25) = 31.91

overall heat transfer coefficient based in the inside surface of tube

$1/U = (1/h_i) + (r_i/k)\ln(r_o/r_i) + (r_o r_i)(1/h_o)$

$= 1/150 + (0.04/40)\ln(50/40) + (50/40)(1/180) = 0.0138$

and $U = 72.28$

area $A = \dot{Q}/(U \times LMTD) = 252 \times 10^3 / (72.28 \times 31.91) = 109.26\ m^2$

Example 3.6 Water flows through a copper tube (k = 350 W/mK, inner and outer diameter 2.0 cm and 2.5 cm respectively) of a double pipe heat exchanger. Oil flows through the annulus between this pipe and steel pipe. The convective heat transfer coefficient on the inside and outside of the copper tube are 5000 and 1500 W /m2K. The fouling factors on the water and oil sides are 0.0022 and 0.00092 K1W. Calculate the overall heat transfer coefficient with and without the fouling factor.

Solution: The scales formed on the inside and outside surface of the copper tube introduces two additional resistances in the heat flow path. Resistance due to inside convective heat transfer coefficient

$1/h_iA_i = 1/5000\ A_i$

Resistance due to scale formation on the inside = $1/h_sA_i = 0.0022$

Resistance due to conduction through the tube wall = $\ln\left(r_o/r_i\right)/2\pi Lk$

$= \ln\left(2.5/2.0\right)/2\pi\times L\times 350 = 1.014\times10^{-4}/L$

Resistance due to convective heat transfer on the outside

$1/h_oA_o = 1/1500A_o$

Resistance due to scale formation on the outside = $1/h_sA_o = 0.00092$

Since, $Q = \Delta T\sum R = U_iA_1\left(\Delta T\right) = \Delta T/\left(1/U_iA_i\right)$; we have

(a) With fouling factor:-

Overall heat transfer coefficient based on the inside pipe surface

$$U_i = 1/\left(1/5000 + \pi\times0.02\left(0.0022+0.00092\right)+0.02\pi\times1.014\times10^{-4}+8.33\times10^{-4}\right)$$

= 809.47 W/m^2K per metre length of pipe

(b) Without fouling factor

$$U_i = 1/\left(1/5000 + 0.02\pi\times1.014\times10^{-4}+8.33\times10^{-4}\right)$$

= 962.12 W/m^2K per m of pipe length.

The heat transfer rate will reduce by (962.12–809.47)1962.12 = 15.9 percent when fouling factor is considered.

Example 3.7 In a surface condenser, dry and saturated steam at 50°C enters at the rate of 1 kg/so The circulating water enters the tube, (25 nun inside diameter, 28 mm outside diameter, k = 300 W/mK) at a velocity of 2 m/s. If the convective heat transfer coefficient on the outside surface of the tube is 5500 W/m^2K, the inlet and outlet temperatures of water are 25°C and 35°C respectively, calculate the required surface area.

Solution: For calculating the convective heat transfer coefficient on the inside surface of the tube, we calculate the Reynolds number on the basis of properties of water at the mean temperature of 30^{o}C. The properties are:

μ = 0.001 Pa-s, □ = 1000 kg/m^3, k = 0.6 W/mK, h_{fg} at 50^{o}C = 2375 kJ/kg

Re = □VD/□ = 10^3 × 2 × 0.025/0.001 = 50,000, a turbulent flow. Pr = 7.0.

The heat transfer coefficient at the inside surface can be calculated by:

Nu = 0.023 $Re^{0.8}$ 8 $Pr^{0.3}$ = 0.023 $(50000)^{0.8}$ $(7)^{0.3}$ = 236.828

and h_i = 236.828 × 0.6/0.025 = 5684 W/m^{2}K.

The overall heat transfer coefficient based on the outer diameter,

U = 1/(0.028/(0.025 × 5684) + 1/5500 + 0.014 ln(28/25)/300)

= 2603.14 W/m^{2}K

□T_a. = (50 - 25) = 25; □T_b = (50 - 35) = 15;

□□□□LMTD = (25 -15)/ln(25/15) = 19.576.

Assuming one shell pass and one tube pass, Q = UA (LMTD)

or A = 2375 × 10^3/(2603.14 x 19.576) = 46.6 m^2

Mass of Circulating water = Q/(c_p □T) = 2375/(4.182 × 10) = 56.79 kg/s

also, m_w = □ × area × V × n, where n is the number of tubes.

n = 56.79 × 4/(2 × □ × 0.025 × 0.025 × 1000) = 58 tubes

Surface area, 46.6 = n × □ × d × L

and L = 46.6/(58 × □ × 0.025) = 10.23 m.

Hence more than one pass should be used.

Example 3.8 A heat exchanger is used to heat water from 20°C to 50°C when thin walled water tubes (inner diameter 25 mm, length IS m) are laid beneath a hot spring water pond, temperature 75°C. Water flows through the tubes with a velocity of 1 m/s. Estimate the required overall heat transfer coefficient and the convective heat transfer coefficient at the outer surface of the tube.

Solution: Water flow rate, $\dot{m}$ = □ × V × A = 10^3 × 1 × (□/4) $(0.025)^2$

= 0.49 kg/so

Heat transferred to water, Q = $\dot{m}c$ (□T) = 0.49 × 4200 × 30 = 61740 W.

Since the temperature of the water in the hot spring is constant,

$\theta_1 = (75 - 20) = 55;\ \theta_2 = (75 - 50) = 25$;

LMTD = (55 – 25) / ln(55/25) = 38

Overall heat transfer coefficient, U = Q/ (A × LMTD)

= 61740/(38 × □ × 0.025 × 15) = 1378.94 W/m^2K.

The properties of water at the mean temperature (20 + 50)/2 = 35°C are:

$\mu = 0.001$ Pa – s, $k = 0.6$ W/ mk and Pr = 7.0

Reynolds number, $Re = \rho V d / \mu = 1000 \times 1.0 \times 0.25 / 0.001 = 25000$, turbulent flow.

Nu = 0.023 $(Re)^{0.8}$ $(Pr)^{0.33}$ = 0.023 $(25000)^{0.8}$ × $(7)^{0.33}$ = 144.2

and h_i = 144.2 × k/d = 144.2 × 0.6/0.025 = 3460.8 W/m^2K

Neglecting the resistance of the thin tube wall,

$1/U = 1/h_i + 1/h_o$; $\therefore\ 1/h_o = 1/1378.94 = 1/3460.8$

or, $h_o = 2292.3$ W / m^2K

Example 3.9 A hot fluid at 200°C enters a heat exchanger at a mass rate of 10000 kg/h. Its specific heat is 2000 J/kg K. It is to be cooled by another fluid entering at 25°C with a mass flow rate 2500 k g/h and specific heat 400 J/kgK. The overall heat transfer coefficient based on outside area of 20 m2 is 250 W/m^2K. Find the exit temperature of the hot fluid when the fluids are in parallel flow.

Solution: From Eq(10.3a), $-U\ dA(1/C_h + 1/C_c) = d(\Delta T)/\Delta T$

Upon integration,

$$-UA(1/C_h + 1/C_c) = \ln(\Delta T)\Big|_1^2 = \ln(T_{h_0} - T_{c_0})/(T_{h_i} - T_{c_i})$$

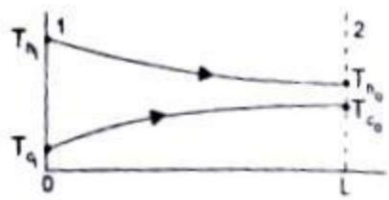
The values are: $U = 250\ W/m^2K$

$A = 20\ m^2$

$1/C_h = 3600/(10000 \times 2000) = 1.8 \times 10^{-4}$

$1/C_c = 3600/(2500 \times 400) = 3.6 \times 10^{-3}$

$-UA(1/C_h + 1/C_c) = -250 \times 20\ (1.8 \times 10^{-4} + 3.6 \times 10^{-3}) = -18.9$



; or, $T_{h_0} = T_{c_0}$

By making an energy balance,

$$10000 \times 2000\left(200 - T_{h_0}\right) = 2500 \times 400\left(T_{c_0} - 25\right)$$

$$= 2500 \times 400\left(T_{h_0} - 25\right) \text{ and } 21\ T_{h_0} = 20 \times 200 + 25$$

or, $T_{h_0} = 191.67^\circ C$

Example 3.10 Cold water at the rate of 4 kg/s is heated from 30°C to 50°C in a shell and tube heat exchanger with hot water entering at 95°C at a rate of 2 kg/s. The hot water flows through the shell. The cold water flows through tubes 2 cm inner diameter, velocity of flow 0.38 m/s. Calculate the number 0 f tube passes, the number 0 f tubes per pass if the maximum length of the tube is limited to 2.0 m and the overall heat transfer coefficient is 1420 W/m^2K.

Solution: Let T be the temperature of the hot water at exit. By making an energy balance: $4c\left(50 - 30\right) = 2c\left(95 - T\right); \quad \therefore T = 55^\circ C$

For a counter-flow arrangement:

$\Delta T_a = \left(95 - 50\right) = 45, \quad \Delta T_b = \left(55 - 30\right) = 25,$

$\therefore$ LMTD $= (45-25)/\ln(45/25) = 34$; $Q = mC(\Delta T) = 4 \times 4.182 \times 20 = 334.56$ kW

Since the cold water is flowing through the tubes, the number of tubes, n is given by

$\dot{m}$ $\times \rho \times$ Area × velocity; the cross-sectional area 3.142 × 10^{-2} m^2

4 = n × 1000 × 3.142 × 10^{-4} × 0.38; □ n = 33.5, or 34 (say)

Assuming one shell and two tube pass, we use Fig. 10.9(a).

$P(50-30)/(95-30) = 0.3$; $Z = (95-55)/(50-30) = 2.0$

Therefore, the correction factor, F = 0.88

Q = UAF LMTD; 34560 = 1420 × A × 0.88 × 34; or A = 7.875 m^2.

For 2 tube pass, the surface area of 34 tubes per pass = 2 L□d 34

L = 1.843 m

Thus we will have 1 shell pass, 2 tube; 34 tubes of 1.843 m in length.

Example 3.11 A double pipe heat exchanger is used to cool compressed air (pressure A bar, volume flow rate 5 mJ/mm at I bar and 15°C) from 160°C to 35°C. Air flows with a velocity of 5 m/s through thin walled tubes, 2 cm inner diameter. Cooling water flows through the annulus and its temperature rises from 25°C to 40°C. The convective heat transfer coefficient at the inside and outside tube surfaces are 125 W/m^2K and 2000 W/m^2K respectively. Calculate (i) mass of water flowing through the exchanger, and (ii) number of tubes and length of each tube.

Solution: Air is cooled from 1600C to 35°C while water is heated from 25°C to 400C and therefore this must be a counter flow arrangement.

Temperature difference at section 1 : $(T_{h_i} - T_{c_0}) = (160-40) = 120$

Temperature difference at section 2 : $(T_{h_0} - T_{c_i}) = (35-25) = 10$

LMTD = (120 -10)/ln 120/10) = 44.27

Mass of air flowing, $\dot{m}$ = □ × Volume = $(10^5 / 287 \times 288)(5/60) = 0.1$ kg/s

Heat given out by air = Heat taken in by water,

$\therefore\ 0.1\times1.005\times(160-35)=\dot{m}_w\times4.182\times(40-25)$; Or $\dot{m}_w = 0.20\ \text{kg/s}$

Density of air flowing through the tube, □ = p/RT. The mean temperature of air flowing through the tube is (160 + 35)/2 = 97.5 ºC = 370.5K

□□□□□□ = 4 × 10^5/(287 × 370.5) = 3.76 kg/m^3. If n is the number of tubes, from the conservation of mass, $\dot{m}=\rho AV$; 0.1 = 3.76 × (□/4) $(0.02)^2$ × 5 × n

□□□n = 16.9 ≡ 17 tubes; $\dot{Q}$ = UA (LMTD)

U = 1/(1/2000 + 1/125) = 117.65, Area for heat transfer A = □DLn

Q = UA(LMTD); 0.1 × 1005 × 125 = 117.65 × 3.142 × 0.02 × L × 17 × 44.27 and L = 2.26 m.

Example 3.12 A refrigerant (mass rate of flow 0.5 kg/s, S = 907 J/kgK k = 0.07 W/mK, □ = 3.45 × Г0 4 Pa-s) at 20°C flows through the annulus (inside diameter 3 c m) of a double pipe counter flow heat exchanger used to cool water (mass flow rate 0.05 kg/so k = 0.68 W/mK, □ = 2.83 × 10^{-4} Pa-s) at 98°C flowing through a thin walled copper tube of 2 cm inner diameter. If the length of the tube is 3m, estimate (i) the overall heat transfer coefficient, and (ii) the temperature of the fluid streams at exit.

Solution: Mass rate of flow, $\dot{m}=\rho AV = \rho(\pi/4)D^2V$;

$\rho VD = 4\dot{m}/\pi D$ and, Reynolds number, $Re=\rho VD/\mu = 4\dot{m}/\pi D\mu$

Water is flowing through the tube of diameter 2 cm,

$\therefore\ Re = 4\times0.05/\left(3.142\times0.02\times2.83\times10^{-4}\right) = 1.12\times10^4$, turbulent flow.

$$Nu = 0.023\ Re^{0.8}\,(Pr)^{0.33} = 0.023\left(1.12\times10^4\right)^{0.8}(1.8)^{0.33}$$

= 48.45; and $h_i = Nu\times k/D = 48.45\times0.68/0.02 = 1647.3\ \text{W/m}^2\text{K}$

Refrigerant is flowing through the annulus. The hydraulic diameter is

D_o - D_i, and the Reynolds number would be, $Re = 4m / \mu\pi(D_0 + D_i)$

$$Re = 4 \times 0.5 / \left(3.45 \times 10^{-4} \times 3.142 \times (0.02 + 0.03)\right)$$ = 3.69 × 10^4, a turbulent flow.

$$Nu = 0.023(Re)^{0.8} (Pr)^{0.33},$$

where $Pr = \mu\ c / k = 3.45 \times 10^{-4} \times 907 / 0.07 = 4.47$

$$= 0.023\left(3.69 \times 10^4\right)^{0.8} (4.47)^{0.33} = 169.8$$

$$\therefore\ h_o = nu \times k / (D_o - D_i) = 169.8 \times 0.07 / 0.01 = 1188.6\ \ W / m^2K$$

and, the overall heat transfer coefficient, U = 1/(1/1647.3 + 1/1188.6)

= 690.43 W/m^2K

For a counter flow heat exchanger, from Eq. (10.4), we have,

$$(1/C_c - 1/C_h)UA = \ln(\Delta T_0 / \Delta T_i) = \ln\left[\left(T_{h_0} - T_{c_i}\right)/\left(T_{h_i} - T_{c_0}\right)\right]$$

$$C_c = 0.5 \times 907 = 453.5;\ C_h = 0.05 \times 4182 = 209.1$$

$$1/C_c - 1/C_h UA = (1/453.5 - 1/209.1) \times 690.43 \times 3.142 \times 0.02 \times 3 = -0.335$$

$$\therefore\ \left(T_{h_0} - T_{c_i}\right)/\left(T_{h_i} - T_{c_0}\right) = \exp(-0.335) = 0.715$$

or, $\left(T_{h_0} + 20\right)/\left(98 - T_{c_0}\right) = 0.715$; By making an energy balance,

$$453.5\left(T_{c_0} + 20\right) = 209.1\left(98 - T_{h_0}\right)$$

which gives $T_{c_0} = 3.12^o\,C;\ T_{h_0} = 47.8^o\,C$