Spearman's Rank Correlation

Rank Coefficient of correlation is calculated under three conditions

I. When Actual ranks are given the formula is

$$R = \rho = \frac{1-6\sum D^2}{n(n^2-1)},$$

where ρ = Rank correlation coefficient

D = Difference of rank between paired item in two series.

n = Total number of observation.

Calculation Method

Where actual ranks are given computation can be done by following these steps-

• Take the difference of the two ranks (R1-R2) this difference is denoted by D.

• Calculate $\sum D2$ by squaring the differences and take their total.

• Apply the formula

$$R = \rho = \frac{1-6\sum D^2}{n(n^2-1)}$$

Problem 1

The ranks of the ten students in two subjects A and B are given in the table below:

Ranks In Subject A	Ranks in Subject B
3	4
5	6
4	3
8	9
9	10
7	7
1	2
2	1
6	5
10	8

Find out correlation coefficient by using spearman's rank correlation formula.

Soln:

Step 1: Take the difference of the two ranks and square it-

Ranks In Subject A R₁	Ranks in Subject B R₂	D^2 (R₁-R₂)
3	4	1
5	6	1
4	3	1
8	9	1
9	10	1
7	7	0
1	2	1
2	1	1
6	5	1
10	8	4

Step 2:

Take a total of d^2

i.e., $\sum D^2 = 12$

Step 3:

Make the computation using spearman's Rank coefficient formula

$$\rho = \frac{1 - 6\sum D^2}{n(n^2 - 1)}$$

$$\rho = \frac{1 - 6*12}{10(10^2 - 1)}$$

$$= 1 - 72/990$$

$$\rho = 0.927$$

II When ranks are not given

In case ranks are not given we need to assign the ranks to the available observations.

We can start giving ranks by giving first or last to highest or lowest but we must follow the same rule for all other available variables.

Problem :2

Calculate the rank correlation coefficient for the following given marks of two Subjects A and B.

Marks of Subject A	Marks of Subject B
92	86
89	83
87	91
86	77
83	68
77	85
71	52
63	82
53	37
50	57

Soln:

Calculation of rank correlation coefficient

Marks of Subject A	R_1	Marks of Subject B	R_2	$(R_1-R_2)^2 = D^2$
92	10	86	9	1
89	9	83	7	4
87	8	91	10	4
86	7	77	5	4
83	6	68	4	4
77	5	85	8	9
71	4	52	2	4
63	3	82	6	9
53	2	37	1	1
50	1	57	3	4
N= 10				$\sum D^2$=44

$$\rho = \frac{1-6\sum D^2}{n(n^2-1)}$$

$$\rho = \frac{1-6*44}{10(10^2-1)}$$

$$= \frac{1-264}{990}$$

$$\rho = 0.733$$

III When Ranks are equal

In some cases two or more data can be equal hence equal ranks are given, in those cases we assign average ranks to each of them for example if two individuals are equal at fourth place then each given the average rank (4+5)/2 = 4.5

Similarly if three are ranked equal at fourth place, they are given an average rank of (4+5+6)/ 3 = 5.

In case of equal ranks formula for calculating Spearman's coefficient of correlation is

$$R = 1 - \frac{6\left\{\sum D^2 + \frac{1}{12}(m1^3 - m1) + \frac{1}{12}(m2^3 - m2)\ldots\ldots\right\}}{N^3 - N}$$

where m is number of items whose ranks are common.

Problem:3

Marks in the two subjects X and Y of eight applicants are shown below. Calculate rank coefficient of correlation

Applicants	Subject X	Subject Y
A	15	40
B	20	30
C	28	50
D	12	30
E	40	20
F	60	10

Soln:

Applicants	Marks in Subject X	Rank Assigned	Marks in Subject Y	Rank Assigned	$(R_1-R_2)^2 = D^2$
A	15	2	40	6	16
B	20	3.5	30	4	0.25
C	28	5	50	7	4
D	12	1	30	4	9
E	40	6	20	2	16
F	60	7	10	1	36
G	20	3.5	30	4	0.25
H	80	8	60	8	0
N= 8					$\sum D^2 = 81.5$

$$R = 1 - \frac{6\left\{\sum D^2 + \frac{1}{12}(m1^3 - m1) + \frac{1}{12}(m2^3 - m2)\ldots\ldots\ldots\right\}}{N^3 - N}$$

In Subject X 20 is repeated two times hence $m_1 = 2$ and in subject Y 30 is repeated three times hence $m_2 = 3$, putting these values in the formula

$$R = 1 - \frac{6\left\{81.5 + \frac{1}{12}(2^3 - 2) + \frac{1}{12}(3^3 - 3)\right\}}{8^3 - 8}$$

$$= 1 - \frac{6(81.5 + 0.5 + 2)}{504}$$

$$= 1 - \frac{6 \, X \, 84}{504}$$

$$= 0$$

Since R= 0 it means there is no correlation between marks obtained in two subjects.