EQUIVALENCE OF PUSHDOWN AUTOMATA AND CFG

If a grammar **G** is context-free, we can build an equivalent nondeterministic PDA which accepts the language that is produced by the context-free grammar **G**. A parser can be built for the grammar **G**.

Also, if **P** is a pushdown automaton, an equivalent context-free grammar G can be constructed where

L(G) = L(P)

In the next two topics, we will discuss how to convert from PDA to CFG and vice versa.

Algorithm to find PDA corresponding to a given CFG

Input − A CFG, G = (V, T, P, S)

Output − Equivalent PDA, P = (Q, ∑, S, δ, q_0, I, F)

Step 1 − Convert the productions of the CFG into GNF.

Step 2 − The PDA will have only one state {q}.

Step 3 − The start symbol of CFG will be the start symbol in the PDA.

Step 4 − All non-terminals of the CFG will be the stack symbols of the PDA and all the terminals of the CFG will be the input symbols of the PDA.

Step 5 − For each production in the form **A → aX** where a is terminal and **A, X** are combination of terminal and non-terminals, make a transition **δ (q, a, A)**.

Problem

Construct a PDA from the following CFG.

G = ({S, X}, {a, b}, P, S)

where the productions are −

S → XS | ε , A → aXb | Ab | ab

Solution

Let the equivalent PDA,

P = ({q}, {a, b}, {a, b, X, S}, δ, q, S)

where δ −

$\delta(q, \varepsilon, S) = \{(q, XS), (q, \varepsilon)\}$

$\delta(q, \varepsilon, X) = \{(q, aXb), (q, Xb), (q, ab)\}$

$\delta(q, a, a) = \{(q, \varepsilon)\}$

$\delta(q, 1, 1) = \{(q, \varepsilon)\}$

Algorithm to find CFG corresponding to a given PDA

Input − A CFG, G = (V, T, P, S)

Output − Equivalent PDA, P = (Q, ∑, S, δ, q_0, I, F) such that the non- terminals of the grammar G will be $\{X_{wx} \mid w,x \in Q\}$ and the start state will be $A_{q0,F}$.

Step 1 − For every w, x, y, z ∈ Q, m ∈ S and a, b ∈ ∑, if δ (w, a, ε) contains (y, m) and (z, b, m) contains (x, ε), add the production rule $X_{wx} \rightarrow a X_{yz}b$ in grammar G.

Step 2 − For every w, x, y, z ∈ Q, add the production rule $X_{wx} \rightarrow X_{wy}X_{yx}$ in grammar G.

Step 3 − For w ∈ Q, add the production rule $X_{ww} \rightarrow \varepsilon$ in grammar G.