# ROHINI college of engineering \& TECHNOLOGY DEPARTMENT OF MATHEMATICS 

## LEAST COST METHOD

## INTRODUCTION

The Least Cost Method is another method used to obtain the initial feasible solution for the transportation problem. Here, the allocation begins with the cell which has the minimum cost. The lower cost cells are chosen over the higher-cost cell with the objective to have the least cost of transportation. The Least Cost Method is considered to produce more optimal results than the North-west Corner because it considers the shipping cost while making the allocation, whereas the North-West corner method only considers the availability and supply requirement and allocation begin with the extreme left corner, irrespective of the shipping cost. The procedure is given below:

Step 1: Balance the problem i.e. $\sum$ Supply $=\sum$ Demand
Step 2: Select the lowest cost from the entire matrix and allocate the minimum of supply or demand.
i.e. Identifying the lowest cell value in this entire matrix.

Step 3: Remove the row or column whose supply or demand is fulfilled and prepare a new matrix
Step 4: Repeat the procedure until all the allocations are over
i.e. Repeat the same procedure of allocation of the smallest value in the new generated matrix
Step 5: After all the allocations are over, write the allocations and calculate the transportation cost
i.e. Once all allocations are over, prepare the table with all allocations marked and calculate the transportation cost

## Problem 1:

A mobile phone manufacturing company has three branches located in three different regions, say Jaipur, Udaipur and Mumbai. The company has to transport mobile phones to three destinations, say Kanpur, Pune and Delhi. The availability from Jaipur, Udaipur and

Mumbai is 40, 60 and 70 units respectively. The demand at Kanpur, Pune and Delhiare 70, 40 and 60 respectively. The transportation cost is shown in the matrix below. Use the Least Cost method to find a basic feasible solution (BFS).


## Solution

Step 1: Balance the problem : $\Sigma$ Supply $=\Sigma$ Demand
$\rightarrow$ The given transportation problem is balanced.


Step 2: Select the lowest cost from the entire matrix and allocate the minimum of supply or demand.

Identifying the lowest cell value in this entire matrix. Here, in this matrix we have 1 (For cell: Jaipur-Delhi) as the lowest value. So, moving with that cell, and allocating the minimum of demand or supply, i.e. allocating 40 here (as supply value is 40 , whereas demand is of 60). Subtracting allocated value (i.e. 40) from corresponding supply and demand.

Destinations

|  |  | Kanpur | Pune | Delhi | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Jaipur | 4 | 5 | 1(40) | H00 |
|  | Udaipur | 3 | 4 | 3 | 60 |
| sources | Mumbai | 6 | 2 | 8 | 70 |
|  | Demand | 70 | 40 | ${ }_{20}$ |  |

Step 3:
Remove the row or column whose supply or demand is fulfilled and prepare a new matrix


Step 4: Repeat the procedure until all the allocations are over
Repeat the same procedure of allocation of the smallest value in the new generated matrix and check out demand or supply based on the smallest value (of demand or supply) as shown below, until all allocations are over.


|  | Destinations |  |  |
| :---: | :---: | :---: | :---: |
|  | Kanpur |  | Delhi | Supply

##  <br> Demand 70 30 <br> 0

Step 5: After all the allocations are over, write the allocations and calculate the transportation cost

Once all allocations are over, prepare the table with all allocations marked and calculate the transportation cost as follows:


Therefore, Transportation Cost $=(1 * 40)+(3 * 40)+(3 * 20)+(6 * 30)+(2 * 40)=$ Rs. 480.

Problem : 2
Find Solution using Least Cost method

| Source | Do | E | F | Supply |
| :---: | :---: | :---: | :---: | :---: |
| A | 5 | 8 | 4 | 50 |
| B | 6 | 6 | 3 | 40 |
| C | 3 | 9 | 6 | 60 |
| Demand | 20 | 95 | 35 | 150 |

## Solution

Balance the problem : $\Sigma$ Supply $=\Sigma$ Demand
$\rightarrow$ The given transportation problem is balanced.

In the given matrix, the supply of each source A, B, C is given Viz. 50units, 40 units, and 60 units respectively. The weekly demand for three retailers D, E, F i.e. 20 units, 95 units and 35 units is given respectively. The shipping cost is given for all the routes.

The minimum transportation cost can be obtained by following the steps given below:

The minimum cost in the matrix is Rs 3, but there is a tie in the cell BF , and CD , now the question arises in which cell we shall allocate. Generally, the cost where maximum quantity can be assigned should be chosen to obtain the better initial solution. Therefore, 35 units shall be assigned to the cell BF. With this, the demand for retailer F gets fulfilled, and only 5 units are left with the source B. Again the minimum cost in the matrix is Rs 3 . Therefore, 20 units shall be assigned to the cell CD. With this, the demand of retailer D gets fulfilled. Only 40 units are left with the source C.

The next minimum cost is Rs 4 , but however, the demand for $F$ is completed, we will move to the next minimum cost which is 5 . Again, the demand of $D$ is completed. The next
minimum cost is 6 , and there is a tie between three cells. But however, no units can be assigned to the cells BD and CF as the demand for both the retailers D and F are saturated. So, we shall assign 5 units to Cell BE. With this, the supply of source B gets saturated.

The next minimum cost is 8 , assign 50 units to the cell AE. The supply of source A gets saturated.

The next minimum cost is Rs 9 ; we shall assign 40 units to the cell CE. With his both the demand and supply of all the sources and origins gets saturated.

| Source | Do | E | F | Supply |
| :---: | :---: | :---: | :---: | :---: |
| A | 5 | 8 | 4 | 50 |
| B | 6 | 6 | 3 | 45 |
| C | 320 | 9 | 6 | 60 |
| Demand | 20 | 95 | 35 | 150 |

The total cost can be calculated by multiplying the assigned quantity with the concerned cost of the cell. Therefore,

Total Cost $=50 * 8+5 * 6+35 * 3+20 * 3+40 * 9=$ Rs 955.

Problem : 3 Find Solution using Least Cost method

|  | D1 | D2 | D3 | D4 | Supply |
| :--- | :--- | :--- | :--- | :--- | :--- |
| S1 | 19 | 30 | 50 | 10 | 7 |
| S2 | 70 | 30 | 40 | 60 | 9 |
| S3 | 40 | 8 | 70 | 20 | 18 |
| Demand | 5 | 8 | 7 | 14 |  |

## Solution:

Problem Table is

|  | $D 1$ | $D 2$ | $D 3$ | $D 4$ | Supply |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $S 1$ | 19 | 30 | 50 | 10 | 7 |
| $S 2$ | 70 | 30 | 40 | 60 | 9 |
| $S 3$ | 40 | 8 | 70 | 20 | 18 |
| Demand | 5 | 8 | 7 | 14 |  |

## $\Sigma$ Supply $=\Sigma$ Demand

$\rightarrow$ The given transportation problem is balanced.
The smallest transportation cost is 8 in cell $S 3 D 2$.
The allocation to this cell is $\min (18,8)=8$.
This satisfies the entire demand of $D 2$ and leaves $18-8=10$ units with $S 3$

|  | $D 1$ | $D 2$ | $D 3$ | $D 4$ | Supply |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $S 1$ | 19 | 30 | 50 | 10 | 7 |
| $S 2$ | 70 | 30 | 40 | 60 | 9 |
| $S 3$ | 40 | $8(8)$ | 70 | 20 | 10 |
| Demand | 5 | 0 | 7 | 14 |  |

The smallest transportation cost is 10 in cell $S 1 D 4$
The allocation to this cell is $\min (7,14)=7$.
This exhausts the capacity of $S 1$ and leaves $14-7=7$ units with $D 4$

|  | $D 1$ | $D 2$ | $D 3$ | $D 4$ | Supply |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $S 1$ | 19 | 30 | 50 | $10(7)$ | 0 |
| $S 2$ | 70 | 30 | 40 | 60 | 9 |
| $S 3$ | 40 | $8(8)$ | 70 | 20 | 10 |
| Demand | 5 | 0 | 7 | 7 |  |

The smallest transportation cost is 20 in cell S3D4
The allocation to this cell is $\min (10,7)=7$.
This satisfies the entire demand of $D 4$ and leaves $10-7=3$ units with $S 3$

|  | $D 1$ | $D 2$ | $D 3$ | $D 4$ | Supply |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $S 1$ | 19 | 30 | 50 | $10(7)$ | 0 |
| $S 2$ | 70 | 30 | 40 | 60 | 9 |
| $S 3$ | 40 | $8(8)$ | 70 | $20(7)$ | 3 |
| Demand | 5 | 0 | 7 | 0 |  |

The smallest transportation cost is 40 in cell $S 2 D 3$
The allocation to this cell is $\min (9,7)=7$.
This satisfies the entire demand of $D 3$ and leaves $9-7=2$ units with $S 2$

|  | $D 1$ | $D 2$ | $D 3$ | $D 4$ | Supply |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $S 1$ | 19 | 30 | 50 | $10(7)$ | 0 |
| $S 2$ | 70 | 30 | $40(7)$ | 60 | 2 |
| $S 3$ | 40 | $8(8)$ | 70 | $20(7)$ | 3 |
| Demand | 5 | 0 | 0 | 0 |  |

The smallest transportation cost is 40 in cell $S 3 D 1$
The allocation to this cell is $\min (3,5)=3$.
This exhausts the capacity of $S 3$ and leaves 5-3 $=2$ units with $D 1$

|  | $D 1$ | $D 2$ | $D 3$ | $D 4$ | Supply |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $S 1$ | 19 | 30 | 50 | $10(7)$ | 0 |
| $S 2$ | 70 | 30 | $40(7)$ | 60 | 2 |
| $S 3$ | $40(3)$ | $8(8)$ | 70 | $20(7)$ | 0 |
| Demand | 2 | 0 | 0 | 0 |  |

The smallest transportation cost is 70 in cell $S 2 D 1$
The allocation to this cell is $\min (2,2)=2$.

|  | $D 1$ | $D 2$ | $D 3$ | $D 4$ | Supply |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $S 1$ | 19 | 30 | 50 | $10(7)$ | 0 |
| $S 2$ | $70(2)$ | 30 | $40(7)$ | 60 | 0 |
| $S 3$ | $40(3)$ | $8(8)$ | 70 | $20(7)$ | 0 |
| Demand | 0 | 0 | 0 | 0 |  |

Initial feasible solution is

|  | $D 1$ | $D 2$ | $D 3$ | $D 4$ | Supply |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $S 1$ | 19 | 30 | 50 | $10(7)$ | 7 |
| $S 2$ | $70(2)$ | 30 | $40(7)$ | 60 | 9 |
| $S 3$ | $40(3)$ | $8(8)$ | 70 | $20(7)$ | 18 |
| Demand | 5 | 8 | 7 | 14 |  |

The minimum total transportation cost $=10 \times 7+70 \times 2+40 \times 7+40 \times 3+8 \times 8+20 \times 7=814$ Here, the number of allocated cells $=6$ is equal to $m+n-1=3+4-1=6$
$\therefore$ This solution is non-degenerate.

Problem : 4 Find Solution using Least Cost method

|  | D1 | D2 | D3 | D4 | Supply |
| :--- | :--- | :--- | :--- | :--- | :--- |
| S1 | 11 | 13 | 17 | 14 | 250 |
| S2 | 16 | 18 | 14 | 10 | 300 |
| S3 | 21 | 24 | 13 | 10 | 400 |
| Demand | 200 | 225 | 275 | 250 |  |

## Solution:

Problem Table is

|  | $D 1$ | $D 2$ | $D 3$ | $D 4$ | Supply |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $S 1$ | 11 | 13 | 17 | 14 | 250 |
| $S 2$ | 16 | 18 | 14 | 10 | 300 |
| $S 3$ | 21 | 24 | 13 | 10 | 400 |
| Demand | 200 | 225 | 275 | 250 |  |

$\Sigma$ Supply $=\Sigma$ Demand, $\rightarrow$ The given transportation problem is balanced.
The smallest transportation cost is 10 in cell $S 3 D 4$
The allocation to this cell is $\min (400,250)=250$.
This satisfies the entire demand of $D 4$ and leaves $400-250=150$ units with $S 3$

|  | $D 1$ | $D 2$ | $D 3$ | $D 4$ | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $S 1$ | 11 | 13 | 17 | 14 | 250 |
| $S 2$ | 16 | 18 | 14 | 10 | 300 |
| $S 3$ | 21 | 24 | 13 | $10(250)$ | 150 |
| Demand | 200 | 225 | 275 | 0 |  |

The smallest transportation cost is 11 in cell $S 1 D 1$
The allocation to this cell is $\min (250,200)=200$.
This satisfies the entire demand of $D 1$ and leaves 250-200 $=50$ units with $S 1$

|  | $D 1$ | $D 2$ | $D 3$ | $D 4$ | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $S 1$ | $11(\mathbf{2 0 0})$ | 13 | 17 | 14 | 50 |
| $S 2$ | 16 | 18 | 14 | 10 | 300 |
| $S 3$ | 21 | 24 | 13 | $10(\mathbf{2 5 0})$ | 150 |
| Demand | 0 | 225 | 275 | 0 |  |

The smallest transportation cost is 13 in cell $S 3 D 3$
The allocation to this cell is $\min (150,275)=150$.
This exhausts the capacity of $S 3$ and leaves 275-150 = 125 units with D3

|  | $D 1$ | $D 2$ | $D 3$ | $D 4$ | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $S 1$ | $11(\mathbf{2 0 0})$ | 13 | 17 | 14 | 50 |
| $S 2$ | 16 | 18 | 14 | 10 | 300 |
| $S 3$ | 21 | 24 | $13(\mathbf{1 5 0})$ | $10(250)$ | 0 |
| Demand | 0 | 225 | 125 | 0 |  |

The smallest transportation cost is 13 in cell $S 1 D 2$
The allocation to this cell is $\min (50,225)=\mathbf{5 0}$.
This exhausts the capacity of $S 1$ and leaves 225-50 $=175$ units with $D 2$

|  | $D 1$ | $D 2$ | $D 3$ | $D 4$ | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $S 1$ | $11(\mathbf{2 0 0})$ | $13(50)$ | 17 | 14 | 0 |
| $S 2$ | 16 | 18 | 14 | 10 | 300 |
| $S 3$ | 21 | 24 | $13(\mathbf{1 5 0})$ | $10(250)$ | 0 |
| Demand | 0 | 175 | 125 | 0 |  |

The smallest transportation cost is 14 in cell $S 2 D 3$
The allocation to this cell is $\min (300,125)=\mathbf{1 2 5}$.
This satisfies the entire demand of $D 3$ and leaves 300-125 = 175 units with $S 2$

|  | $D 1$ | $D 2$ | $D 3$ | $D 4$ | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $S 1$ | $11(\mathbf{2 0 0})$ | $13(\mathbf{5 0})$ | 17 | 14 | 0 |
| $S 2$ | 16 | 18 | $14(\mathbf{1 2 5})$ | 10 | 175 |
| $S 3$ | 21 | 24 | $13(\mathbf{1 5 0})$ | $10(250)$ | 0 |


| Demand | 0 | 175 | 0 | 0 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |

The smallest transportation cost is 18 in cell $S 2 D 2$
The allocation to this cell is $\min (175,175)=175$.

|  | $D 1$ | $D 2$ | $D 3$ | $D 4$ | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $S 1$ | $11(\mathbf{2 0 0})$ | $13(50)$ | 17 | 14 | 0 |
| $S 2$ | 16 | $18(\mathbf{1 7 5})$ | $14(\mathbf{1 2 5})$ | 10 | 0 |
| $S 3$ | 21 | 24 | $13(\mathbf{1 5 0})$ | $10(250)$ | 0 |
| Demand | 0 | 0 | 0 | 0 |  |

Initial feasible solution is

|  | $D 1$ | $D 2$ | $D 3$ | $D 4$ | Supply |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $S 1$ | 11 (200) | $13(\mathbf{5 0 )}$ | 17 | 14 | 250 |
| $S 2$ | 16 | $18(\mathbf{1 7 5 )}$ | $14(\mathbf{1 2 5 )}$ | 10 | 300 |
| $S 3$ | 21 | 24 | $13(\mathbf{1 5 0 )}$ | $10(\mathbf{2 5 0})$ | 400 |
| Demand | 200 | 225 | 275 | 250 |  |

The minimum total transportation
cost $=11 \times 200+13 \times 50+18 \times 175+14 \times 125+13 \times 150+10 \times 250=12200$
Here, the number of allocated cells $=6$ is equal to $m+n-1=3+4-1=6$
$\therefore$ This solution is non-degenerate
Problem 5 : Find Solution using Least Cost method

|  | D1 | D2 | D3 | Supply |
| :--- | :--- | :--- | :--- | :--- |
| S1 | 4 | 8 | 8 | 76 |
| S2 | 16 | 24 | 16 | 82 |
| S3 | 8 | 16 | 24 | 77 |
| Demand | 72 | 102 | 41 |  |

## Solution:

Problem Table is

|  | $D 1$ | $D 2$ | $D 3$ | Supply |
| :--- | :--- | :--- | :--- | :--- |
| $S 1$ | 4 | 8 | 8 | 76 |
| $S 2$ | 16 | 24 | 16 | 82 |
| $S 3$ | 8 | 16 | 24 | 77 |
| Demand | 72 | 102 | 41 |  |

Here Total Demand $=215$ is less than Total Supply $=235$. So We add a dummy demand (D4) constraint with 0 unit cost and with allocation 20.
Now, The modified table is

|  | $D 1$ | $D 2$ | $D 3$ | $D 4$ | Supply |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $S 1$ | 4 | 8 | 8 | 0 | 76 |
| $S 2$ | 16 | 24 | 16 | 0 | 82 |
| $S 3$ | 8 | 16 | 24 | 0 | 77 |
| Demand | 72 | 102 | 41 | 20 |  |

The smallest transportation cost is 0 in cell S1Ddummy
The allocation to this cell is $\min (76,20)=\mathbf{2 0}$.
This satisfies the entire demand of Ddummy and leaves 76-20 $=56$ units with $S 1$

|  | $D 1$ | $D 2$ | $D 3$ | $D 4$ | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $S 1$ | 4 | 8 | 8 | $0(\mathbf{2 0})$ | 56 |
| $S 2$ | 16 | 24 | 16 | 0 | 82 |
| $S 3$ | 8 | 16 | 24 | 0 | 77 |
| Demand | 72 | 102 | 41 | 0 |  |

The smallest transportation cost is 4 in cell $S 1 D 1$
The allocation to this cell is $\min (56,72)=56$.
This exhausts the capacity of $S 1$ and leaves $72-56=16$ units with $D 1$

|  | $D 1$ | $D 2$ | $D 3$ | $D 4$ | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $S 1$ | $4(56)$ | 8 | 8 | $0(20)$ | 0 |
| $S 2$ | 16 | 24 | 16 | 0 | 82 |
| $S 3$ | 8 | 16 | 24 | 0 | 77 |
| Demand | 16 | 102 | 41 | 0 |  |

The smallest transportation cost is 8 in cell $S 3 D 1$
The allocation to this cell is $\min (77,16)=\mathbf{1 6}$.
This satisfies the entire demand of $D 1$ and leaves 77-16 $=61$ units with $S 3$

|  | $D 1$ | $D 2$ | $D 3$ | $D 4$ | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $S 1$ | $4(56)$ | 8 | 8 | $0(20)$ | 0 |
| $S 2$ | 16 | 24 | 16 | 0 | 82 |
| $S 3$ | $8(16)$ | 16 | 24 | 0 | 61 |
| Demand | 0 | 102 | 41 | 0 |  |

The smallest transportation cost is 16 in cell $S 3 D 2$
The allocation to this cell is $\min (61,102)=61$.
This exhausts the capacity of $S 3$ and leaves $102-61=41$ units with $D 2$

|  | $D 1$ | $D 2$ | $D 3$ | $D 4$ | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $S 1$ | $4(56)$ | 8 | 8 | $0(20)$ | 0 |
| $S 2$ | 16 | 24 | 16 | 0 | 82 |
| $S 3$ | $8(\mathbf{1 6})$ | $16(61)$ | 24 | 0 | 0 |
| Demand | 0 | 41 | 41 | 0 |  |

The smallest transportation cost is 16 in cell $S 2 D 3$
The allocation to this cell is $\min (82,41)=41$.
This satisfies the entire demand of $D 3$ and leaves $82-41=41$ units with $S 2$

|  | $D 1$ | $D 2$ | $D 3$ | $D 4$ | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $S 1$ | $4(56)$ | 8 | 8 | $0(20)$ | 0 |
| $S 2$ | 16 | 24 | $16(41)$ | 0 | 41 |
| $S 3$ | $8(16)$ | $16(61)$ | 24 | 0 | 0 |
| Demand | 0 | 41 | 0 | 0 |  |

The smallest transportation cost is 24 in cell $S 2 D 2$
The allocation to this cell is $\min (41,41)=41$.

|  | $D 1$ | $D 2$ | $D 3$ | $D 4$ | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $S 1$ | $4(56)$ | 8 | 8 | $0(\mathbf{2 0})$ | 0 |
| $S 2$ | 16 | $24(41)$ | $16(41)$ | 0 | 0 |
| $S 3$ | $8(\mathbf{1 6 )}$ | $16(61)$ | 24 | 0 | 0 |
| Demand | 0 | 0 | 0 | 0 |  |

Initial feasible solution is

|  | $D 1$ | $D 2$ | $D 3$ | Ddummy | Supply |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $S 1$ | $4(56)$ | 8 | 8 | $0(\mathbf{2 0})$ | 76 |
| $S 2$ | 16 | $24(\mathbf{4 1})$ | $16(41)$ | 0 | 82 |
| $S 3$ | $8(\mathbf{1 6 )}$ | $16(\mathbf{6 1})$ | 24 | 0 | 77 |
| Demand | 72 | 102 | 41 | 20 |  |

The minimum total transportation cost $=4 \times 56+0 \times 20+24 \times 41+16 \times 41+8 \times 16+16 \times 61=2968$ Here, the number of allocated cells $=6$ is equal to $m+n-1=3+4-1=6$ $\therefore$ This solution is non-degenerate

