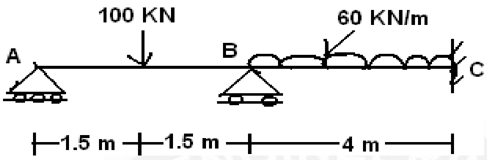
4.3. ANALYSIS THE CONTINUOUS BEAM BY FLEXIBILITY METHOD.

4.3.1. NUMERICAL PROBLEMS ON CONTINUOUS BEAMS;

PROBLEM NO:01

Analysis the continuous beam shown in fig,by using Flexibility method.



Solution:

- **Static indeterminacy:**

  Degree of redundancy = (1 + 1 + 3) – 3 = 2

  Release at B and C by apply hinge

- **Fixed End Moments:**

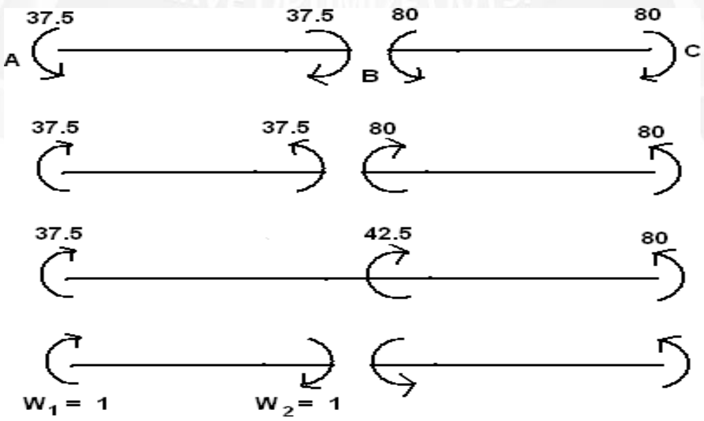
  MFAB = -Wl/8 = -100X3/8 = -37.5 kNm

  MFBA = Wl/8 = 100X3/8 = 37.5 kNm

  MFBC = $-Wl^2/12 = -60x4^2/12$ = -80 kNm

  MFCB = $Wl^2/12 = 60x4^2/12$ = 80 kNm

- **Equivalent Joint Loads:**



- **Flexibility Co-efficent Matrix (B):**

  $B = B_W . B_X$

$$B_W = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{and}\; B_X = \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **Flexibility Matrix (F):**

$$F = \frac{L}{6EI}\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

$$F = \frac{1}{EI}\begin{bmatrix} 1 & -0.5 & 0 & 0 \\ -0.5 & 1 & 0 & 0 \\ 0 & 0 & 1.33 & -0.67 \\ 0 & 0 & -0.67 & 1.33 \end{bmatrix}$$

$$\mathbf{F_X = B_X^T \,.\, F \,.\, B_X}$$

$$= \frac{1}{EI}\begin{bmatrix} 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -0.5 & 0 & 0 \\ -0.5 & 1 & 0 & 0 \\ 0 & 0 & 1.33 & -0.67 \\ 0 & 0 & -0.67 & 1.33 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$F_x = \frac{1}{EI}\begin{bmatrix} 2.33 & -0.67 \\ -0.67 & 1.33 \end{bmatrix}$$

$$F_x^{-1} = EI\begin{bmatrix} 0.502 & 0.253 \\ 0.253 & 0.879 \end{bmatrix}$$

$$\mathbf{F_W = B_X^T \,.\, F \,.\, B_W}$$

$$= \frac{1}{EI}\begin{bmatrix} 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -0.5 & 0 & 0 \\ -0.5 & 1 & 0 & 0 \\ 0 & 0 & 1.33 & -0.67 \\ 0 & 0 & -0.67 & 1.33 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$F_W = \frac{1}{EI}\begin{bmatrix} 0.5 & -1 \\ 0 & 0 \end{bmatrix}$$

- **Displacement Matrix (X):**

  $X = - F_X^{-1} . F_W . W$

$$= -\frac{EI}{EI}\begin{bmatrix} 0.502 & 0.253 \\ 0.253 & 0.879 \end{bmatrix}\begin{bmatrix} 0.5 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 37.5 \\ 42.5 \end{bmatrix}$$

$$= -\begin{bmatrix} 0.251 & -0.502 \\ 0.127 & -0.253 \end{bmatrix}\begin{bmatrix} 37.5 \\ 42.5 \end{bmatrix}$$

$$= -\begin{bmatrix} -11.923 \\ -\ 5.99 \end{bmatrix}$$

$$X = \begin{bmatrix} 11.923 \\ 5.99 \end{bmatrix}$$

- **Internal Force (P):**

  $P = [\ ]_X$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 37.5 \\ 42.5 \\ 11.923 \\ 5.99 \end{bmatrix}$$

$$P = \begin{bmatrix} 37.5 \\ 30.58 \\ 11.923 \\ 5.99 \end{bmatrix}$$

- **Final Moments (M):**

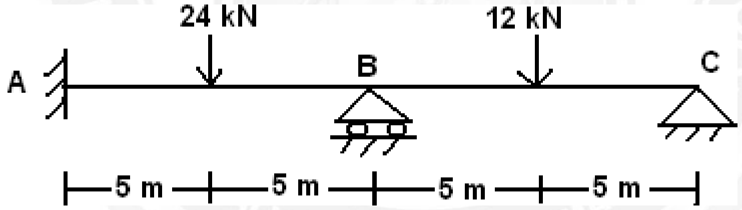
M = μ + P

$$= \begin{bmatrix} -37.5 \\ 37.5 \\ -80 \\ 80 \end{bmatrix} + \begin{bmatrix} 37.5 \\ 30.58 \\ 11.923 \\ 5.99 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 \\ 68.08 \\ -68.08 \\ 95.99 \end{bmatrix}$$

PROBLEM NO:02

Analysis the continuous beam ABC shown in fig,by using Flexibility method.And sketch the bending moment diagram.



Solution:

- **Static indeterminacy:**

  Degree of redundancy = (3 + 1 + 1) – 3 = 2

  Release at A and B by apply hinge

- **Fixed End Moments:**

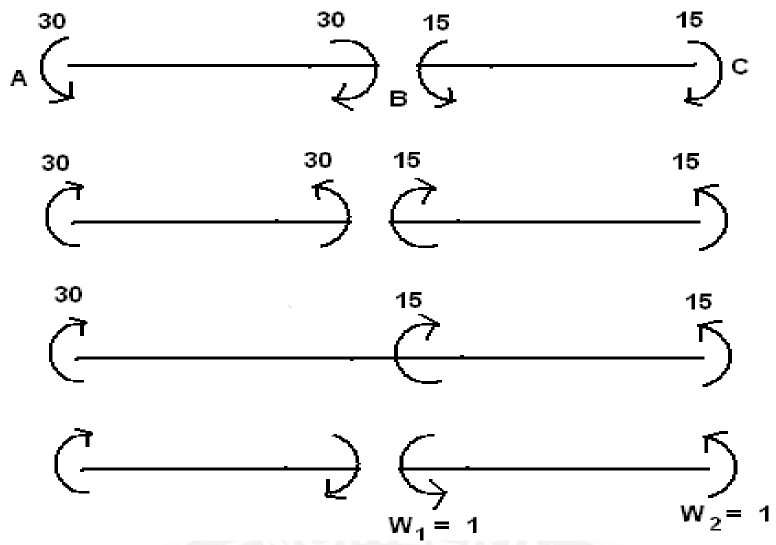
  MFAB = -Wl/8 = -24X10/8 = -30 kNm

  MFBA = Wl/8 = 24X10/8 /8 = 30 kNm

  MFAB = -Wl/8 = -12X10/8 = -15 kNm

  MFBA = Wl/8 = 12X10/8 /8 = 15 kNm

- **Equivalent Joint Loads:**



- **Flexibility Co-efficent Matrix (B):**

$B = B_W \cdot B_X$

$$B = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

- **Flexibility Matrix (F):**

$$F = \frac{L}{6EI}\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

$$F = \frac{1}{EI}\begin{bmatrix} 3.33 & -1.67 & 0 & 0 \\ -1.67 & 3.33 & 0 & 0 \\ 0 & 0 & 3.33 & -1.67 \\ 0 & 0 & -1.67 & 3.33 \end{bmatrix}$$

$F_X = B_X^T \cdot F \cdot B_X$

$$= \frac{1}{EI}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 3.33 & -1.67 & 0 & 0 \\ -1.67 & 3.33 & 0 & 0 \\ 0 & 0 & 3.33 & -1.67 \\ 0 & 0 & -1.67 & 3.33 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$F_x = \frac{1}{EI}\begin{bmatrix} 3.33 & 1.67 \\ 1.67 & 6.66 \end{bmatrix}$$

$$F_x^{-1} = EI\begin{bmatrix} 0.3435 & -0.086 \\ -0.086 & 0.1717 \end{bmatrix}$$

$$\mathbf{F_W = B_X^T \,.\, F \,.\, B_W}$$

$$= \frac{1}{EI}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 3.33 & -1.67 & 0 & 0 \\ -1.67 & 3.33 & 0 & 0 \\ 0 & 0 & 3.33 & -1.67 \\ 0 & 0 & -1.67 & 3.33 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$F_W = \frac{1}{EI}\begin{bmatrix} 0 & 0 \\ 3.33 & -1.67 \end{bmatrix}$$

- **Displacement Matrix (X):**

  $\mathbf{X = - F_X^{-1} \,.\, F_W \,.\, W}$

$$= -\frac{EI}{EI}\begin{bmatrix} 0.3435 & -0.086 \\ -0.086 & 0.1717 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 3.33 & -1.67 \end{bmatrix}\begin{bmatrix} -15 \\ -15 \end{bmatrix}$$

$$= -\begin{bmatrix} -0.286 & 0.144 \\ 0.144 & -0.286 \end{bmatrix}\begin{bmatrix} -15 \\ -15 \end{bmatrix}$$

$$= -\begin{bmatrix} 2.13 \\ -4.29 \end{bmatrix}$$

$$X = \begin{bmatrix} -2.13 \\ 4.29 \end{bmatrix}$$

- **Internal Force (P):**

$$\mathbf{P = [\ ]}_{\boldsymbol{X}}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} -15 \\ -15 \\ -2.13 \\ 4.29 \end{bmatrix}$$

$$P = \begin{bmatrix} -2.13 \\ -4.29 \\ -10.71 \\ -15 \end{bmatrix}$$

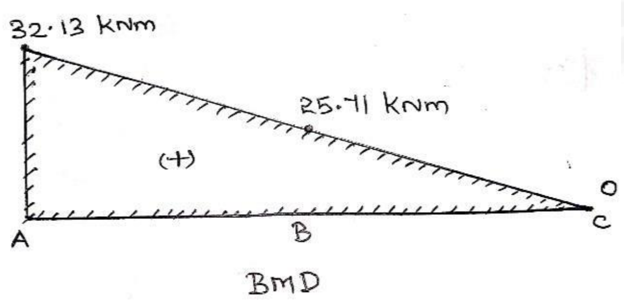
- **Final Moments (M):**

$$\mathbf{M = \mu + P}$$

$$= \begin{bmatrix} -30 \\ 30 \\ -15 \\ 15 \end{bmatrix} + \begin{bmatrix} -2.13 \\ -4.29 \\ -10.71 \\ -15 \end{bmatrix}$$

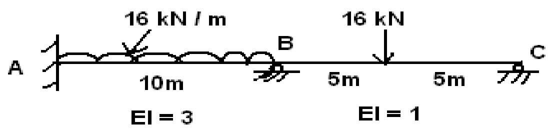
$$M = \begin{bmatrix} -32.13 \\ 25.71 \\ -25.71 \\ 0 \end{bmatrix}$$

- **Bending Moment Diagram:**



PROBLEM NO:03

Analysis the continuous beam shown in fig,by using Flexibility method.



Solution:

- **Static indeterminacy:**

  Degree of redundancy = (3 + 1 + 1) – 3 = 2

  Release at A and B by apply hinge

- **Fixed End Moments:**

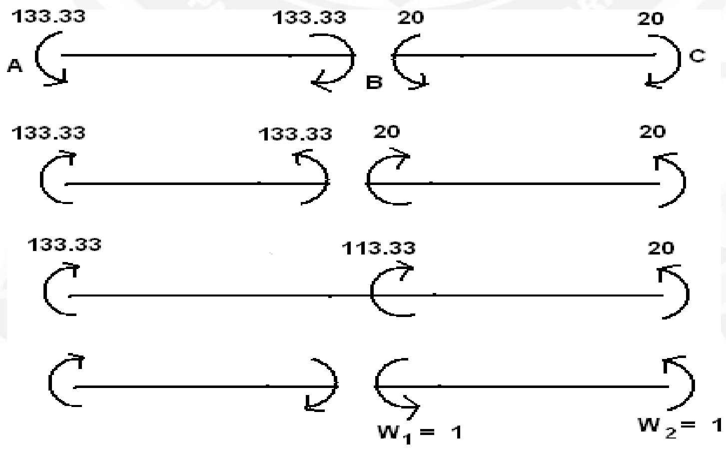
  MFAB = $-Wl^2/12 = -16x10^2/12 = -133.33$ kNm

  MFBA = $Wl^2/12 = 16x10^2/12 = 133.33$ kNm

  MFBC = -Wl/8 = -16X10/8 = -20 kNm

  MFCB = Wl/8 = 16X10/8 = 20 kNm

- **Equivalent Joint Loads:**



- **Flexibility Co-efficent Matrix (B):**

  $$\mathbf{B = B_W \cdot B_X}$$

$$B = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

- **Flexibility Matrix (F):**

$$F = \frac{L}{6EI}\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

$$F = \begin{bmatrix} 1.11 & -0.56 & 0 & 0 \\ -0.56 & 1.11 & 0 & 0 \\ 0 & 0 & 3.33 & -1.67 \\ 0 & 0 & -1.67 & 3.33 \end{bmatrix}$$

$F_X = B_X^T \cdot F \cdot B_X$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1.11 & -0.56 & 0 & 0 \\ -0.56 & 1.11 & 0 & 0 \\ 0 & 0 & 3.33 & -1.67 \\ 0 & 0 & -1.67 & 3.33 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$F_x = \begin{bmatrix} 1.11 & 0.56 \\ 0.56 & 4.44 \end{bmatrix}$$

$$F_x^{-1} = \begin{bmatrix} 0.962 & -0.121 \\ -0.121 & 0.241 \end{bmatrix}$$

$F_W = B_X^T \cdot F \cdot B_W$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1.11 & -0.56 & 0 & 0 \\ -0.56 & 1.11 & 0 & 0 \\ 0 & 0 & 3.33 & -1.67 \\ 0 & 0 & -1.67 & 3.33 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$F_W = \begin{bmatrix} 0 & 0 \\ 3.33 & -1.67 \end{bmatrix}$$

- **Displacement Matrix (X):**

  $X = - F_X^{-1} . F_W . W$

$$= -\begin{bmatrix} 0.962 & -0.121 \\ -0.121 & 0.241 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 3.33 & -1.67 \end{bmatrix}\begin{bmatrix} -113.33 \\ -20 \end{bmatrix}$$

$$= -\begin{bmatrix} 41.62 \\ -82.90 \end{bmatrix}$$

$$X = \begin{bmatrix} -41.62 \\ 82.90 \end{bmatrix}$$

- **Internal Force (P):**

$$\mathbf{P = [\;]_{\mathit{X}}}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} -113.33 \\ -20 \\ -41.62 \\ 82.90 \end{bmatrix}$$

$$P = \begin{bmatrix} -41.62 \\ -82.90 \\ -30.43 \\ -20 \end{bmatrix}$$

- **Final Moments (M):**

$$\mathbf{M = \mu + P}$$

$$= \begin{bmatrix} -133.33 \\ 133.33 \\ -20 \\ 20 \end{bmatrix} + \begin{bmatrix} -41.62 \\ -82.90 \\ -30.43 \\ -20 \end{bmatrix}$$

$$M = \begin{bmatrix} -174.95 \\ 50.43 \\ -50.43 \\ 0 \end{bmatrix}$$