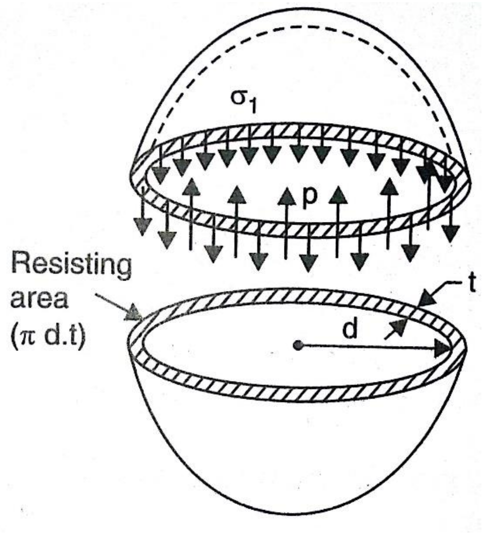
## 5.10. THIN SPHERICAL SHELLS

A thin spherical shell of internal diameter d and thickness t and subjected to an internal fluid pressure p. The fluid inside the shell has a tendency to split the shell into two hemisphere along x axis

The force P which has a tendency to split the shell




$$= p \times \frac{\pi}{4} \times d^2$$

The area resisting this force=π.d.L

∴ Hoop or circumferential stress

induced in the material of the shell is given by

$$\sigma_1 = \frac{\text{Force P}}{\text{Area resisting the force P}}$$

$$= \frac{p \times \frac{\pi}{4} \times d^2}{\pi.d.t} = \frac{pd}{4t}$$

The stress $\sigma_1$ is tensile in nature The fluid inside the shell is also having tendency to split the shell into two hemispheres along y-y axis. Then it can be shown that the tensile hoop stress will also be equal to $\frac{pd}{4t}$. Let the stress is $\sigma_2$

∴ $\sigma_2 = \frac{pd}{4t}$ The stress $\sigma_2$ will be right angles to $\sigma_1$

**Problem 5.19:** A vessel in the shape of a spherical shell of 1.20m internal diameter and 12mm shell thickness is subjected to pressure of 1.6N/mm2. Determine the stress induced in the material of the shell.

**Given data:**

Internal diameter $d = 1.2m = 1.2\times10^3$ mm

shell thickness $t = 12mm$

Fluid pressure $p = 1.6\ N/mm^2$

**To find:**

Stress induced in the shell ($\sigma_1$)=?

**Solution:**

The stress induced in the material of spherical shell is given by

$$\sigma_1 = \frac{pd}{4t} = \frac{1.6\times1.2\times10^3}{4\times12} = 40\ N/mm^2$$

**Result:**

Stress induced in the shell ($\sigma_1$)= 40 $N/mm^2$

**Problem 5.20:** A spherical vessel 1.5m diameter is subjected to an internal pressure of $2N/mm^2$, find the thickness of the plate required if maximum stress is not to exceed $150N/mm^2$ and joint efficiency is 75%

**Given data:**

Diameter of the shell $d = 1.5m = 1.5\times10^3$ mm

Fluid pressure $p = 2\ N/mm^2$

Stress in material $\sigma_1 = 150\ N/mm^2$

Joint efficiency $\eta = 75\% = 0.75$

**To find:**

The thickness of the plate =?

**Solution:**

The stress induced is given by

$$\sigma_1 = \frac{pd}{4t}$$

⋙ $$t = \frac{pd}{4\times\sigma_1} = \frac{2\times1.5\times10^3}{4\times150} = 6.67\ mm$$

**Result:** The thickness of the plate = 6.67mm

## 5.11 CHANGE IN DIMENSIONS OF A THIN SPHERICAL SHELL DUE TO AN INTERNAL PRESSURE

In previous article, we have seen that the stresses at any point are equal to $\frac{pd}{4t}$to like. There is no shear stress at any point in the shell.

Maximum shear stress = $\frac{\sigma_1 - \sigma_2}{2} = \frac{\frac{pd}{4t} - \frac{pd}{4t}}{2} = 0$

These stresses $\sigma_1$ and$\sigma_2$ are acting at right angles to each other.

∴ Strain in any one direction is given by

$$e_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_1}{E}$$

$$= \frac{\sigma_1}{E}(1 - \mu)$$

$$= \frac{pd}{4tE}(1 - \mu)$$

We know that strain in any direction is also $= \frac{\delta d}{d}$

∴ $$\frac{\delta d}{d} = \frac{pd}{4tE}(1 - \mu)$$

Then change in diameter $\delta d = \frac{pd}{4tE}(1 - \mu)d$

**Volumetric strain**

The ratio of change of volume to the original volume is known as volumetric strain. If v=original volume and dv=change in volume. Then volumetric strain = $\frac{dV}{V}$

Let V=Original volume = $\frac{\pi}{6}\times d^3$

Taking the differential of the above equations we get

$$dV = \frac{\pi}{6} \times 3d^2 \times d(d)$$

$$\frac{dV}{V} = \frac{\frac{\pi}{6}\times 3d^2 \times d(d)}{\frac{\pi}{6}\times d\Lambda 3}$$

$$= 3\times\frac{d(d)}{d}$$

But from change in diameter equation we have

$$\frac{\delta d}{d} \text{ or } \frac{d(d)}{d} = \frac{pd}{4tE}(1 - \mu)$$

Substituting this value in $\frac{dv}{v}$ we get,

$$\frac{dV}{V} = \frac{3pd}{4tE}(1-\mu)$$

**Problem 5.21:** A spherical shell of internal diameter 0.9m and of thickness 10mm is subjected to an internal pressure of 1.4 N/mm$^2$. Determine the increases in diameter and increases in volume. Take E=2×10$^5$ N/mm$^2$ and μ=1/3

**Given data:**

| | |
|---|---|
| Internal diameter | d = 0.9m = 0.9×10$^3$mm |
| Thickness of shell | t = 10mm |
| Fluid pressure | p =1.4 N/mm$^2$ |

**To find:**

Increases in diameter and increases in volume $dd$, dV =?

**Solution:**

wkt $$\delta d = \frac{pd}{4tE}(1-\mu)d$$

$$= \frac{1.4\times(0.9\times10^{3})}{4\times10\times2\times10^{5}}\left[1-\frac{1}{3}\right]\times 0.9\times10^{3}$$

= 0.0945mm

Now volumetric strain

$$\frac{dV}{V} = \frac{3pd}{4tE}(1-\mu)$$

$$dV = \frac{3pd}{4tE}(1-\mu)V \qquad \left(\because V = \frac{\pi}{6}\times d^3\right)$$

$$= \frac{3\times1.4\times(0.9\times10^{3})}{4\times10\times2\times10^{5}}\left(1-\frac{1}{3}\right)\times\frac{\pi}{6}\times(0.9\times10^3)^3$$

=12028.5mm3

**Result:**

Increases in diameter $dd$ = 0.0945mm

Increases in volume dV = 12028.5mm$^3$

## 5.12 ROTATIONAL STRESSES IN THIN CYLINDER

Thin cylinder rotating at an angular velocity about the axis

Let r=mean radius of the cylinder

t=thickness of the cylinder

ω=angular speed of the cylinder

ρ=density of the material of the cylinder

Due to rotational of cylinder centrifugal force will be acting on the walls of the cylinder. This centrifugal force will produce a circumferential stress σ. For a thin cylinder, this hoop stresses σ may be assumed constant.

Consider a small element ABCD of the rotating cylinder. Let this element makes an angle δθ at the centre. Consider unit length of this element perpendicular to the plane of paper

The forces acting on the element are;

(i)Centrifugal force ($mv^2/r$ or $m\omega^2 r$) acting radially outwards. Here m is the mass of the element per unit length.

m = mass of element

= ρ ×volume of element

= ρ ×(area of element)×unit length

= ρ ×[(r×δθ)×t]×1

= ρ ×r×δθ×t

∴ Centifigual force = $m\omega^2 r$

= $(\rho\, r\delta\theta t)\omega^2 r$

= ρ r2 × ω2 × δθ × t

(ii)Tensile force due to hoop stress (σ) on the face AB. This force is equal to (σ×t×1) and acts as perpendicular to face AB

(iii)Tensile force due to hoop stress (σ) on the face CD. This force is equal t0 (σ×t×1) and acts perpendicular to face CD.

The horizontal component $\sigma\times t\times\cos\frac{\delta\theta}{2}$ on the face AB and CD are equal and opposite. The radial component $\sigma\times t\times\sin\frac{\delta\theta}{2}$ are acting towards centre and they will be added.

Resolving the forces radially for equilibrium, we get

Centrifugal force $= \sigma t \times \sin\frac{\delta\theta}{2} + \sigma t \times \sin\frac{\delta\theta}{2}$

or $\rho\, r^2 \times \omega^2 \times \delta\theta \times t = 2 \times \sigma \times t \times \sin\frac{\delta\theta}{2}$

$= 2 \times \sigma \times t \times \delta\theta/2$

or $\rho\, r^2 \times \omega^2 = \sigma$

or $\sigma = \rho\, r^2 \times \omega^2$

**Problem 5.22:** A rim type flywheel is rotating at a speed of 2400 rpm If the mean diameter of the flywheel is 750mm and density of the material of the wheel is 8000kg/m$^3$, then find the hoop stress produced in the rim due to rotation. If E=200GN/m$^2$ then what will be the changes in diameter of the flywheel due to rotation

**Given data:**

Speed $N = 2400$rpm

Mean diameter $d = 750$ mm $= 0.75$ m

Density $\rho = 8000$kg/m$^3$

$E = 200$GN/m$^2$ $= 200 \times 10^9$ N/m$^2$

**To find:**

Hoop stress produced in the rim due to rotation =?

**Solution:**

Wkt, angular speed $\omega = 2\pi N/60 = 2 \times \pi \times 2400/60$

$\omega = 80\pi$ rad/sec

$r = 750/2 = 0.375$m

Using hoop stress produced in the rim due to rotation equation we get,

$\sigma = \rho \times r^2 \times \omega^2$

$= 8000 \times 0.3752 \times (80\pi)^2$

$= 71.0485 \times 10^6$ N/m$^2$

$= 71.0485$MN/m$^2$

Change in diameter calculation

Due to hoop stress circumferential strain

$e = \frac{\sigma}{E}$ and $e = \frac{\delta d}{d}$

equating the above equation, we get

$$\frac{\delta d}{d} = \frac{\sigma}{E}$$

⋙ Change in diameter $\delta d = \frac{\sigma \times d}{E}$

$$\delta d = \frac{71.0485 \times 106 \ \times 0.75}{200 \times 10^9}$$

$$= 0.002664m$$

$$= 0.2664mm$$

**Result:**

Hoop stress produced in the rim due to rotation = **71.0485MN/m²**

Change in diameter $\delta d$ = **0.2664mm**

**Problem 5.23:** Find the speed of rotation of a wheel of diameter 750mm if the hoop stress is not to exceed 120MN/m². The wheel has a thin rim and density of the wheel is 7200kg/m³

**Given data:**

Diameter $d = 750mm$

$r = 750/2 = 375mm = 0.375m$

Max. hoop stress $\sigma = 120MN/m^2 = 120\times10^6\ N/m^2$

$\rho = 7200kg/m^3$

**To find:**

**S**peed of rotation of a wheel (N) =?

**Solution:**

Using hoop stress produced in the rim due to rotation equation we get,

$$\sigma = \rho \times r^2 \times \omega^2$$

$$120\times10^6 = 7200 \times 0.375^2 \times \omega^2$$

$$\omega = \sqrt{\frac{120\times10^6}{7200\times0.375^2}}$$

$$= 344.26 \text{ rad/s}$$

But $\omega = 2\pi N/60$

$$N = 60\times344.26/2\pi$$

$$N = 3287.4 \text{ rpm}$$

**Result:**

Speed of rotation of a wheel (N) = **3287.4 rpm**

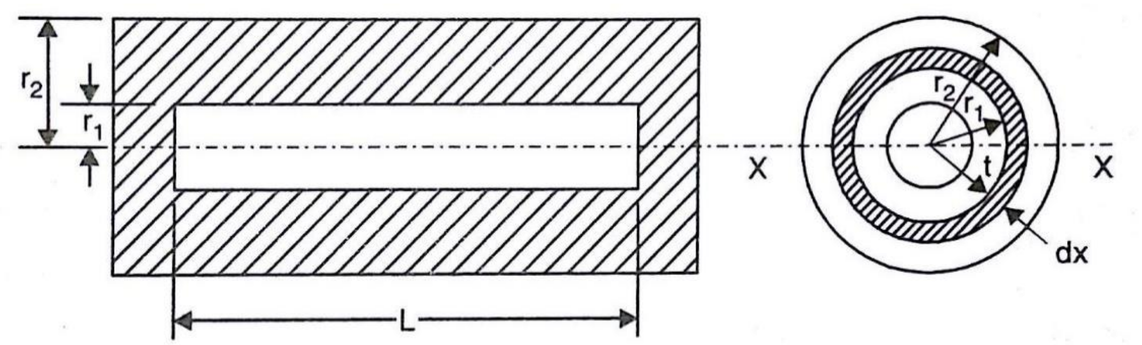
## 5.13. INTRODUCTION OF THICK CYLINDER

In the last chapter, we have mentioned that if the ratio of thickness to internal diameter of a cylindrical shell is less than about 1/20,the cylinder shell is know as thin cylinders. For them it may be assumed with reasonable accuracy that the hoop and longitudinal stresses are constant over the thickness and the radial stress is small and can be neglected. If the ratio of thickness to internal diameter is more than 1/20,then cylinder shell is know as thick cylinders.

The hoop stress in case of a thick cylinder will not be uniform across the thickness. Actually the hoop stress will vary from a maximum value at the inner circumference to a minimum value at the outer circumference.

## 5.14. STRESSES IN A THICK CYLINDERICAL SHELL

Fig. shows a thick cylinder subjected to an internal fluid pressure.

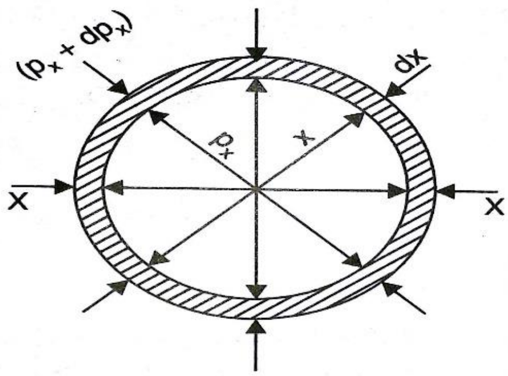



Let $r_2$ = External radius of the cylinder,

$r_1$ = Internal radius of the cylinder, and

L = Length of cylinder.

Consider an elementary ring of the cylinder of radius x and thickness dx as shown in the figure




Let $p_x$ = Radial pressure on the inner surface of the ring

$p_x$ + d$p_x$ = Radial pressure on the outer surface of the ring

$\sigma_x$ = Hoop stress induced in the ring.

Take a longitudinal section x-x and consider the equilibrium of half of the ring of figure.