Unit II

SHAFTS AND COUPLINGS

2.1 SHAFTS

2.1.1 Introduction

A shaft is a rotating machine element which is used to transmit power from one place to another. In order to transfer the power from one shaft to another, the various members such as pulleys, gears etc., are mounted on it. a shaft is used for the transmission of torque and bending moment. The various members are mounted on the shaft by means of keys or splines. The shafts are usually cylindrical, but may be square or cross-shaped in section. They are solid in cross-section but sometimes hollow shafts are also used.

An ***axle,*** though similar in shape to the shaft, is a stationary machine element and is used for the transmission of bending moment only. It simply acts as a support for some rotating body such as hoisting drum, a car wheel or a rope sheave.

A ***spindle*** is a short shaft that imparts motion either to a cutting tool (*e.g*. drill press spindles) or to a work piece (*e.g*. lathe spindles).

2.1.2 Material Used for Shafts

The material used for shafts should have the following properties :
1. It should have high strength.
2. It should have good machinability.
3. It should have low notch sensitivity factor.
4. It should have good heat treatment properties.
5. It should have high wear resistant properties.
The material used for ordinary shafts is carbon steel of grades 40 C 8, 45 C 8, 50 C 4 and 50 C 12.
The mechanical properties of these grades of carbon steel are given in the following table.

Table 2.1. Mechanical properties of steels used for shafts.

Indian standard designation	*Ultimate tensile strength, MPa*	*Yield strength, MPa*
40 C 8	560 - 670	320
45 C 8	610 - 700	350
50 C 4	640 - 760	370
50 C 12	700 Min.	390

When a shaft of high strength is required, then an alloy steel such as nickel, nickel-chromium or chrome-vanadium steel is used.

2.1.3 Manufacturing of Shafts

Shafts are generally manufactured by hot rolling and finished to size by cold drawing or turning and grinding. The cold rolled shafts are stronger than hot rolled shafts but with higher residual stresses. The residual stresses may cause distortion of the shaft when it is machined, especially when slots or keyways are cut. Shafts of larger diameter are usually forged and turned to size in a lathe.

2.1.4 Types of Shafts

The following two types of shafts are important from the subject point of view

1. *Transmission shafts*. These shafts transmit power between the source and the machines absorbing power. The counter shafts, line shafts, over head shafts and all factory shafts are transmission shafts. Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to bending in addition to twisting.

2. *Machine shafts*. These shafts form an integral part of the machine itself. The crank shaft is an example of machine shaft.

2.1.5 Stresses in Shafts

The following stresses are induced in the shafts :

1. Shear stresses due to the transmission of torque (*i.e.* due to torsional load).

2. Bending stresses (tensile or compressive) due to the forces acting upon machine elements like gears, pulleys etc. as well as due to the weight of the shaft itself.

3. Stresses due to combined torsional and bending loads.

2.1.6 Maximum Permissible Working Stresses for Transmission Shafts

According to American Society of Mechanical Engineers (ASME) code for the design of transmission shafts, the maximum permissible working stresses in tension or compression may be taken as

(***a***) 112 MPa for shafts without allowance for keyways.

(***b***) 84 MPa for shafts with allowance for keyways.

For shafts purchased under definite physical specifications, the permissible tensile stress (σ_t) may be taken as 60 per cent of the elastic limit in tension (σ_{el}), but not more than 36 per cent of the ultimate tensile strength (σ_u). In other words, the permissible tensile stress,

$$\sigma_t = 0.6\ \sigma_{el} \text{ or } 0.36\ \sigma_u, \text{ whichever is less.}$$

The maximum permissible shear stress may be taken as

(***a***) 56 MPa for shafts without allowance for key ways.

(***b***) 42 MPa for shafts with allowance for keyways.

For shafts purchased under definite physical specifications, the permissible shear stress (τ) may be taken as 30 per cent of the elastic limit in tension (σ_{el}) but not more than 18 per cent of the ultimate tensile strength (σ_u). In other words, the permissible shear stress,

$$\tau = 0.3\ \sigma_{el} \text{ or } 0.18\ \sigma_u, \text{ whichever is less.}$$

2.1.7 Design of Shafts

The shafts may be designed on the basis of

1. Strength, and **2.** Rigidity and stiffness.

In designing shafts on the basis of strength, the following cases may be considered :

(***a***) Shafts subjected to twisting moment or torque only,

(***b***) Shafts subjected to bending moment only,

(***c***) Shafts subjected to combined twisting and bending moments, and

(***d***) Shafts subjected to axial loads in addition to combined torsional and bending loads.

We shall now discuss the above cases, in detail, in the following pages.

2.1.8 Shafts Subjected to Twisting Moment Only

When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft may be obtained by using the torsion

$$\frac{T}{J} = \frac{\tau}{r} \qquad \text{.....(i)}$$

Where T = Twisting moment (or torque) acting upon the shaft,
J = Polar moment of inertia of the shaft about the axis of rotation,
$\tau =$ Torsional shear stress, and
r = Distance from neutral axis to the outer most fibre
= d / 2; where d is the diameter of the shaft.

We know that for round solid shaft, polar moment of inertia,

$$J = \frac{\pi}{32} \times d^4$$

The equation (*i*) may now be written as

$$\frac{T}{\frac{\pi}{32} \times d^4} = \frac{\tau}{\frac{d}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \times d^3 \qquad ...(ii)$$

From this equation, we may determine the diameter of round solid shaft (d).

We also know that for hollow shaft, polar moment of inertia,

$$J = \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]$$

where *do* and *di* = Outside and inside diameter of the shaft, and r = *do* / 2.
Substituting these values in equation ***(i)***, we have

$$\frac{T}{\frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]} = \frac{\tau}{\frac{d_o}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] \qquad ...(iii)$$

Let k = Ratio of inside diameter and outside diameter of the shaft
$= d_i / d_o$

Now the equation (*iii*) may be written as

$$T = \frac{\pi}{16} \times \tau \times \frac{(d_o)^4}{d_o}\left[1 - \left(\frac{d_i}{d_o}\right)^4\right] = \frac{\pi}{16} \times \tau\, (d_o)^3\, (1 - k^4) \qquad ...(iv)$$

From the equations ***(iii)*** or ***(iv)***, the outside and inside diameter of a hollow shaft may be determined. It may be noted that

1. The hollow shafts are usually used in marine work. These shafts are stronger per kg of material and they may be forged on a mandrel, thus making the material more homogeneous than would be possible for a solid shaft. When a hollow shaft is to be made equal in strength to a solid shaft, the twisting moment of both the shafts must be same. In other words, for the same material of both the shafts,

$$T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o} \right] = \frac{\pi}{16} \times \tau \times d^3$$

$$\therefore \quad \frac{(d_o)^4 - (d_i)^4}{d_o} = d^3 \quad \text{or} \quad (d_o)^3 (1 - k^4) = d^3$$

2. The twisting moment (T) may be obtained by using the following relation :
We know that the power transmitted (in watts) by the shaft,

$$P = \frac{2\pi N \times T}{60} \quad \text{or} \quad T = \frac{P \times 60}{2\pi N}$$

where T = Twisting moment in N-m, and
N = Speed of the shaft in r.p.m.

3. In case of belt drives, the twisting moment (T) is given by

$$T = (T1 - T2)\, R$$

where $T1$ and $T2$ = Tensions in the tight side and slack side of the belt respectively, and
R = Radius of the pulley.

Example 2.1. *A line shaft rotating at 200 r.p.m. is to transmit 20 kW. The shaft may be assumed to be made of mild steel with an allowable shear stress of 42 MPa. Determine the diameter of the shaft, neglecting the bending moment on the shaft.*

Given Data :
N = 200 r.p.m.
P = 20 kW = 20 × 10^3 W
τ= 42 MPa = 42 N/mm^2

To Find:
diameter of the shaft

Solution:
Let d = Diameter of the shaft.
We know that torque transmitted by the shaft,

$$T = \frac{P \: X \: 60}{2\pi N}$$

$$= \frac{20 \: X \: 10^3 \: X \: 60}{2 \: \pi N}$$

= 955 N-m = 955 × 10^3 N-mm

We also know that torque transmitted by the shaft (T),

$$955 \text{ X } 10^3 = \frac{\pi}{16} \: X \: \tau \: X \: d^3$$

$$= \frac{\pi}{16} \: X \: 42 \: X \: d^3$$

= 8.25 d^3

d^3= 955 × 10^3 / 8.25 = 115 733 or d = 48.7 say 50 mm **Ans.**

Example 2.2. *Find the diameter of a solid steel shaft to transmit 20 kW at 200 r.p.m. The ultimate shear stress for the steel may be taken as 360 MPa and a factor of safety as 8. If a hollow shaft is to be used in place of the solid shaft, find the inside and outside diameter when the ratio of inside to outside diameters is 0.5.*

Given Data :
P = 20 kW = 20 × 10^3 W
N = 200 r.p.m.
τ_u = 360 MPa = 360 N/mm^2
$F.S.$ = 8 ; $k = di / do$ = 0.5

To Find
inside and outside diameter

Solution

We know that the allowable shear stress

$$\tau = \frac{\tau_u}{F.S} = \frac{360}{8} = 45 \text{ N/mm}^2$$

Diameter of the solid shaft

Let d = Diameter of the solid shaft.

We know that torque transmitted by the shaft,

$$T = \frac{P \ X \ 60}{2 \ \pi \ N} = \frac{20 \ X \ 10^3 \ X \ 60}{2 \ \pi \ N} = 955 \text{ N-m} = 955 \text{ x } 10^3 \text{ N-mm}$$

We also know that torque transmitted by the solid shaft (T),

$$955 \text{ X } 10^3 = \frac{\pi}{16} \ X \ \tau \ X \ d^3 = \frac{\pi}{16} \ X \ \tau \ X \ d^3 = 8.84 \ d^3$$

d^3 = 955 × 10^3 / 8.84 = 108 032 or d = 47.6 say 50 mm **Ans.**

Diameter of hollow shaft

Let di = Inside diameter, and
do = Outside diameter.
We know that the torque transmitted by the hollow shaft (T),

$$955 \text{ x } 10^3 = \frac{\pi}{16} \ X \ \tau \ (d_o)^3 \ (1 - k^4)$$

$$= \frac{\pi}{16} \ X \ 45 \ (d_o)^3 \ (1 - (0.5)^4)$$

$$= 8.3 \ (do)^3$$

$(do)^3$ = 955 × 10^3 / 8.3 = 115 060 or do = 48.6 say 50 mm **Ans.**
and di = 0.5 do = 0.5 × 50 = 25 mm **Ans.**

2.1.9 Shafts Subjected to Bending Moment Only

When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation. We know that

$$\frac{M}{I} = \frac{\sigma_b}{y} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

where M = Bending moment,
I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation,
σ_b = Bending stress, and
y = Distance from neutral axis to the outer-most fibre.

We know that for a round solid shaft, moment of inertia,

$$I = \frac{\pi}{64} \times d^4 \quad \text{and} \quad y = \frac{d}{2}$$

Substituting these values in equation ***(i)***, we have

$$\frac{M}{\frac{\pi}{64} \times d^4} = \frac{\sigma_b}{\frac{d}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b \times d^3$$

From this equation, diameter of the solid shaft (d) may be obtained.
We also know that for a hollow shaft, moment of inertia,

$$I = \frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right] = \frac{\pi}{64}(d_o)^4 (1 - k^4) \quad \ldots(\text{where } k = d_i / d_o)$$

and $y = d_o / 2$

Again substituting these values in equation ***(i)***, we have

$$\frac{M}{\frac{\pi}{64}(d_o)^4 (1 - k^4)} = \frac{\sigma_b}{\frac{d_o}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b (d_o)^3 (1 - k^4)$$

From this equation, the outside diameter of the shaft (*do*) may be obtained.

Example 2.3. *A pair of wheels of a railway wagon carries a load of 50 kN on each axle box, acting at a distance of 100 mm outside the wheel base. The gauge of the rails is 1.4 m. Find the diameter of the axle between the wheels, if the stress is not to exceed 100 MPa.*

Given Data:

$W = 50$ kN $= 50 \times 10^3$ N

$L = 100$ mm ;

$x = 1.4$ m

$\sigma_b = 100$ MPa $= 100$ N/mm^2

To Find: *diameter of the axle between the wheels*

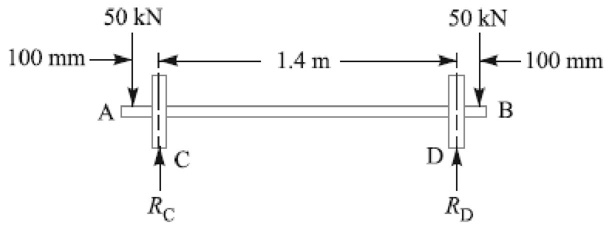


Fig. 1.1.

The axle with wheels is shown in Fig. 1.1.
A little consideration will show that the maximum bending moment acts on the wheels at *C* and *D*. Therefore maximum bending moment,

$$M = W.L = 50 \times 10^3 \times 100 = 5 \times 10^6 \text{ N-mm}$$

Let d = Diameter of the axle.

We know that the maximum bending moment (M)

$$5 \text{ X } 10^6 = \frac{\pi}{32} X\, \sigma_b\, X\, d^3 = \frac{\pi}{32} X\, 100\, X\, d^3 = 9.32d^3$$

$d^3 = 5 \times 10^6 / 9.82 = 0.51 \times 10^6$ or $d = 79.8$

say 80mm **Ans.**

2.1.10 Shafts Subjected to Combined Twisting Moment and Bending Moment

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously. Various theories have been suggested to account for the elastic failure of the materials when they are subjected to various types of combined stresses. The following two theories are important from the subject point of view :

1. Maximum shear stress theory or Guest's theory. It is used for ductile materials such as mild steel.
2. Maximum normal stress theory or Rankine's theory. It is used for brittle materials such as cast iron.

Let τ = Shear stress induced due to twisting moment, and

σ_b = Bending stress (tensile or compressive) induced due to bending moment.

According to maximum shear stress theory, the maximum shear stress in the shaft,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

Substituting the values of τ and σ_b from Art. 14.9 and Art. 14.10, we have

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3}\left[\sqrt{M^2 + T^2}\right]$$

or $$\frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2 + T^2} \qquad \text{...}(i)$$

The expression $\sqrt{M^2 + T^2}$ is known as ***equivalent twisting moment*** and is denoted by *Te*. The equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces the same shear stress (τ) as the actual twisting moment. By limiting the maximum shear stress (τmax) equal to the allowable shear stress (τ) for the material, the equation ***(i)*** may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau \times d^3 \qquad ...(ii)$$

From this expression, diameter of the shaft (d) may be evaluated.

Now according to maximum normal stress theory, the maximum normal stress in the shaft,

$$\sigma_{b(max)} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2} \qquad ...(iii)$$

$$= \frac{1}{2} \times \frac{32M}{\pi d^3} + \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{32}{\pi d^3}\left[\frac{1}{2}(M + \sqrt{M^2 + T^2})\right]$$

or $$\frac{\pi}{32} \times \sigma_{b\,(max)} \times d^3 = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] \qquad ...(iv)$$

The expression $\frac{1}{2}\left[(M + \sqrt{M^2 + T^2})\right]$ is known as ***equivalent bending moment*** and is denoted by M_e. The equivalent bending moment may be defined as **that moment which when acting alone produces the same tensile or compressive stress (σ_b) as the actual bending moment.** By limiting the maximum normal stress [$\sigma_{b(max)}$] equal to the allowable bending stress (σ_b), then the equation (*iv*) may be written as

$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \frac{\pi}{32} \times \sigma_b \times d^3 \qquad ...(v)$$

Example 2.4. *A shaft supported at the ends in ball bearings carries a straight tooth spur gear at its mid span and is to transmit 7.5 kW at 300 r.p.m. The pitch circle diameter of the gear is 150 mm. The distances between the centre line of bearings and gear are 100 mm each. If the shaft is made of steel and the allowable shear stress is 45 MPa, determine the diameter of the shaft. Show in a sketch how the gear will be mounted on the shaft; also indicate the ends where the bearings will be mounted? The pressure angle of the gear may be taken as 20°.*

Given Data :
P = 7.5 kW = 7500 W
N = 300 r.p.m.
D = 150 mm = 0.15 m
L = 200 mm = 0.2 m
τ = 45 MPa = 45 N/mm^2
α = 20°

To Find:
1. *diameter of the shaft*
2. *sketch how the gear will be mounted on the shaft*

Solution:
Fig. 2.2 shows a shaft with a gear mounted on the bearings.

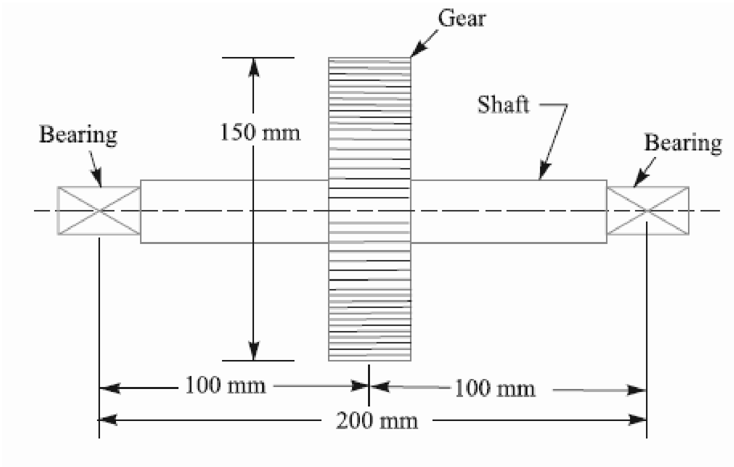


Fig. 2.2

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{7500 \times 60}{2\pi \times 300} = 238.7 \text{ N-m}$$

$\therefore$ Tangential force on the gear,

$$F_t = \frac{2T}{D} = \frac{2 \times 238.7}{0.15} = 3182.7 \text{ N}$$

and the normal load acting on the tooth of the gear,

$$W = \frac{F_t}{\cos\alpha} = \frac{3182.7}{\cos 20^\circ} = \frac{3182.7}{0.9397} = 3387 \text{ N}$$

Since the gear is mounted at the middle of the shaft, therefore maximum bending moment at the centre of the gear,

$$M = \frac{W.L}{4} = \frac{3387 \times 0.2}{4} = 169.4 \text{ N-m}$$

Let d = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(169.4)^2 + (238.7)^2} = 292.7 \text{ N-m}$$
$$= 292.7 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$292.7 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 45 \times d^3 = 8.84\ d^3$$

$\therefore$ $d^3 = 292.7 \times 10^3 / 8.84 = 33 \times 10^3$ or $d = 32$ say 35 mm **Ans.**

Example 2.5. *A line shaft is driven by means of a motor placed vertically below it. The pulley on the line shaft is 1.5 metre in diameter and has belt tensions 5.4 kN and 1.8 kN on the tight side and slack side of the belt respectively. Both these tensions may be assumed to be vertical. If the pulley be overhang from the shaft, the distance of the centre line of the pulley from the centre line of the bearing being 400 mm, find the diameter of the shaft. Assuming maximum allowable shear stress of 42 MPa.*

Given Data:

$D = 1.5$ m or $R = 0.75$ m;
$T_1 = 5.4$ kN = 5400 N
$T_2 = 1.8$ kN = 1800 N
$L = 400$ mm
$\tau = 42$ MPa = 42 N/mm^2

To Find:

diameter of the shaft

Solution .

A line shaft with a pulley is shown in Fig 2.3.

We know that torque transmitted by the shaft,

$T = (T_1 - T_2)\, R = (5400 - 1800)0.75 = 2700$ N-m $= 2700 \times 10^3$ N-mm

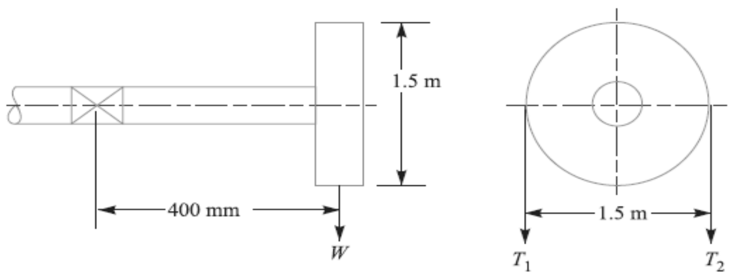


Fig. 2.3

Neglecting the weight of shaft, total vertical load acting on the pulley,

$$W = T_1 + T_2 = 5400 + 1800 = 7200 \text{ N}$$

$$\therefore \text{Bending moment, } M = W \times L = 7200 \times 400 = 2880 \times 10^3 \text{ N-mm}$$

Let d = Diameter of the shaft in mm.

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(2880 \times 10^3)^2 + (2700 \times 10^3)^2}$$
$$= 3950 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$3950 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

$d^3 = 3950 \times 10^3/8.25 = 479 \times 10^3$ or $d = 78$ say 80 mm **Ans.**

Example 2.6. *A shaft is supported by two bearings placed 1 m apart. A 600 mm diameter pulley is mounted at a distance of 300 mm to the right of left hand bearing and this drives a pulley directly below it with the help of belt having maximum tension of 2.25 kN. Another pulley 400 mm diameter is placed 200 mm to the left of right hand bearing and is driven with the help of electric motor and belt, which is placed horizontally to the right. The angle of contact for both the pulleys is 180° and μ = 0.24. Determine the suitable diameter for a solid shaft, allowing working stress of 63 MPa in tension and 42 MPa in shear for the material of shaft. Assume that the torque on one pulley is equal to that on the other pulley*

Given Data :

AB = 1 m
D_C = 600 mm or R_C = 300 mm = 0.3 m
AC = 300 mm = 0.3 m
T_1 = 2.25 kN = 2250 N
D_D = 400 mm or R_D = 200 mm = 0.2 m
BD = 200 mm = 0.2 m
θ= 180° = π rad
μ = 0.24
σ_b = 63 MPa = 63 N/mm^2
τ = 42 MPa = 42 N/mm^2

To Find :

suitable diameter for a solid shaft

Solution.

The space diagram of the shaft is shown in Fig. 2.4 (*a*).
Let T_1 = Tension in the tight side of the belt on pulley C = 2250 N...(Given)

T_2 = Tension in the slack side of the belt on pulley C.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.24 \times \pi = 0.754$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{0.754}{2.3} = 0.3278 \quad \text{or} \quad \frac{T_1}{T_2} = 2.127 \quad \text{...(Taking antilog of 0.3278)}$$

and
$$T_2 = \frac{T_1}{2.127} = \frac{2250}{2.127} = 1058 \text{ N}$$

∴ Vertical load acting on the shaft at C,
$W_C = T_1 + T_2$ = 2250 + 1058 = 3308 N
and vertical load on the shaft at D
= 0
The vertical load diagram is shown in Fig. 2.4 (*c*).
We know that torque acting on the pulley C,

$T = (T_1 – T_2) R_C$ = (2250 – 1058) 0.3 = 357.6 N-m
The torque diagram is shown in Fig. 2.4 (*b*).

Let T_3 = Tension in the tight side of the belt on pulley D, and
T_4 = Tension in the slack side of the belt on pulley D.

Since the torque on both the pulleys (*i.e.* C and D) is same, therefore

$$(T_3 - T_4) R_D = T = 357.6 \text{ N-m} \quad \text{or} \quad T_3 - T_4 = \frac{357.6}{R_D} = \frac{357.6}{0.2} = 1788 \text{ N} \quad \text{...}(i)$$

We know that
$$= \frac{T_3}{T_4} = \frac{T_1}{T_2} = 2.127 \quad \text{or} \quad T_3 = 2.127\, T_4 \quad \text{...}(ii)$$

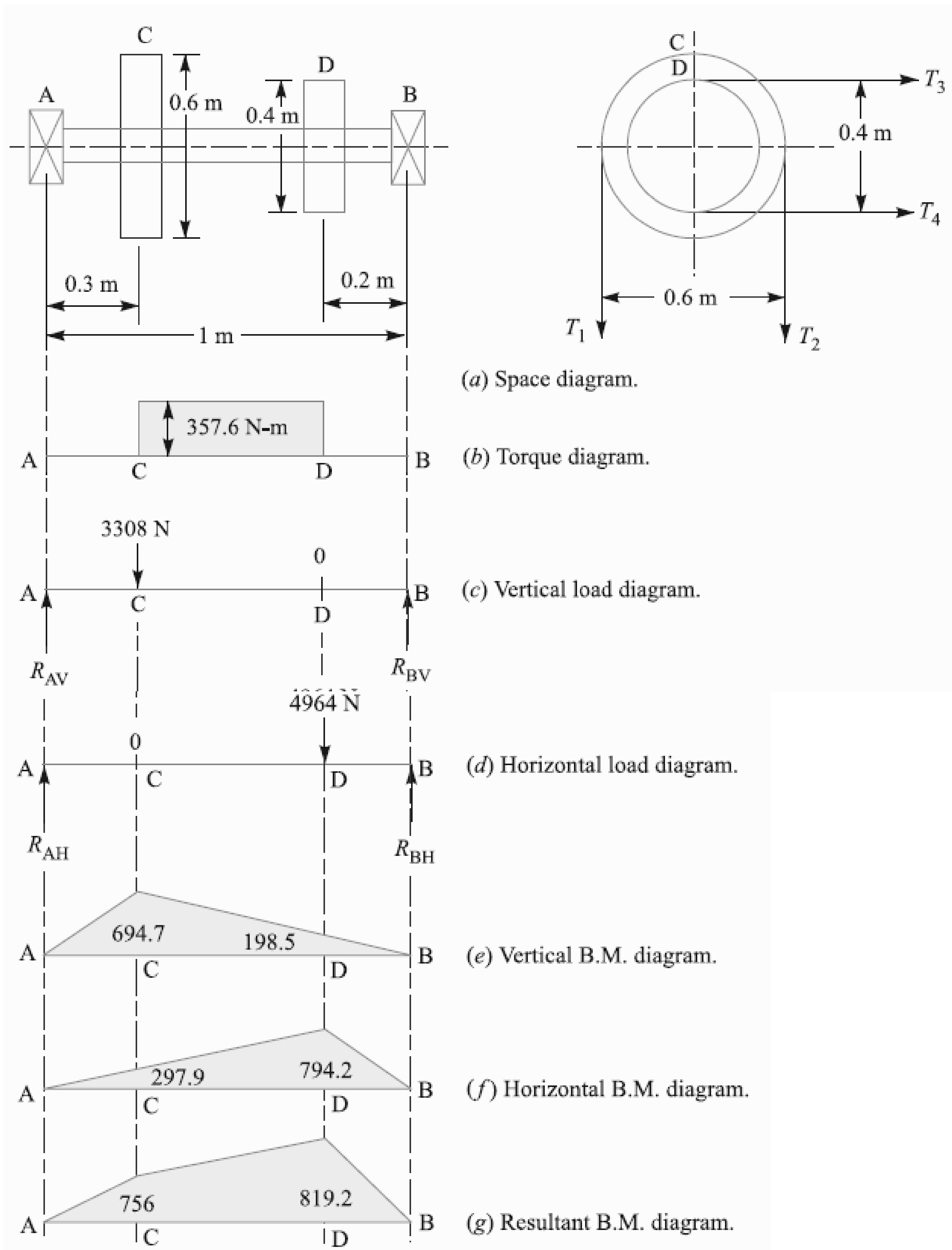

C
D
A
B
0.6 m
0.4 m
0.3 m
0.2 m
1 m
T1
T2
T3
T4
0.4 m
0.6 m
(a) Space diagram.
357.6 N-m
(b) Torque diagram.
3308 N
0
RAV
RBV
(c) Vertical load diagram.
4964 N
0
RAH
RBH
(d) Horizontal load diagram.
694.7
198.5
(e) Vertical B.M. diagram.
297.9
794.2
(f) Horizontal B.M. diagram.
756
819.2
(g) Resultant B.M. diagram.

Fig. 2.4

From equations (*i*) and (*ii*), we find that

$T3 = 3376$ N, and $T4 = 1588$ N

∴ Horizontal load acting on the shaft at *D*,

$WD = T3 + T4 = 3376 + 1588 = 4964$ N

and horizontal load on the shaft at $C = 0$

The horizontal load diagram is shown in Fig. 2.4 (*d*).

Now let us find the maximum bending moment for vertical and horizontal loading First of all, considering the vertical loading at *C*. Let *R*AV and *R*BV be the reactions at the bearings *A* and *B* respectively.

We know that

$RAV + RBV = 3308$ N

Taking moments about *A*,

$RBV \times 1 = 3308 \times 0.3$ or $RBV = 992.4$ N

and $RAV = 3308 – 992.4 = 2315.6$ N

We know that B.M. at *A* and *B*,

$MAV = MBV = 0$

B.M. at *C*, $MCV = RAV \times 0.3 = 2315.6 \times 0.3 = 694.7$ N-m

B.M. at *D*, $MDV = RBV \times 0.2 = 992.4 \times 0.2 = 198.5$ N-m

The bending moment diagram for vertical loading in shown in Fig. 2.4 (*e*).

Now considering horizontal loading at *D*. Let *R*AH and *R*BH be the reactions at the bearings *A* and *B* respectively.

We know that

$RAH + RBH = 4964$ N

Taking moments about *A*,

$RBH \times 1 = 4964 \times 0.8$ or $RBH = 3971$ N

and $RAH = 4964 – 3971 = 993$ N

We know that B.M. at *A* and *B*,

$MAH = MBH = 0$

B.M. at *C*, $MCH = RAH \times 0.3 = 993 \times 0.3 = 297.9$ N-m

B.M. at *D*, $MDH = RBH \times 0.2 = 3971 \times 0.2 = 794.2$ N-m

The bending moment diagram for horizontal loading is shown in Fig. 2.4 (*f*).

Resultant B.M. at *C*,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(694.7)^2 + (297.9)^2} = 756 \text{ N-m}$$

and resultant B.M. at *D*,

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(198.5)^2 + (794.2)^2} = 819.2 \text{ N-m}$$

The resultant bending moment diagram is shown in Fig. 2.4 (*g*).

We see that bending moment is maximum at *D*.

∴ Maximum bending moment,

$$M = M_D = 819.2 \text{ N-m}$$

Let *d* = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(819.2)^2 + (357.6)^2} = 894 \text{ N-m}$$
$$= 894 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$894 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

$$\therefore \quad d^3 = 894 \times 10^3 / 8.25 = 108 \times 10^3 \text{ or } d = 47.6 \text{ mm}$$

Again we know that equivalent bending moment,

$$M_e = \tfrac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \tfrac{1}{2}(M + T_e)$$
$$= \tfrac{1}{2}(819.2 + 894) = 856.6 \text{ N-m} = 856.6 \times 10^3 \text{ N-mm}$$

We also know that equivalent bending moment (*Me*),

$$856.6 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 63 \times d^3 = 6.2\, d^3$$

$$\therefore \quad d^3 = 856.6 \times 10^3 / 6.2 = 138.2 \times 10^3 \text{ or } d = 51.7 \text{ mm}$$

Taking larger of the two values,
we have

$$d = 51.7 \text{ say } 55 \text{ mm } \textbf{Ans.}$$

Example 2.7. *A shaft is supported on bearings A and B, 800 mm between centres. A 20° straight tooth spur gear having 600 mm pitch diameter, is located 200 mm to the right of the left hand bearing A, and a 700 mm diameter pulley is mounted 250 mm towards the left of bearing B. The gear is driven by a pinion with a downward tangential force while the pulley drives a horizontal belt having 180° angle of wrap. The pulley also serves as a flywheel and weighs 2000 N. The maximum belt tension is 3000 N and the tension ratio is 3 : 1. Determine the maximum bending moment and the necessary shaft diameter if the allowable shear stress of the material is 40 MPa.*

Given Data :

$AB = 800$ mm

$\alpha_C = 20°$

$D_C = 600$ mm or $R_C = 300$ mm

$AC = 200$ mm ;

$D_D = 700$ mm or $R_D = 350$ mm

$DB = 250$ mm

$\theta = 180° = \pi$ rad

$W = 2000$ N

$T_1 = 3000$ N

$T_1/T_2 = 3$

$\tau = 40$ MPa $= 40$ N/mm^2

To Find:

1. *maximum bending moment*
2. *shaft diameter*

Solution.
The space diagram of the shaft is shown in Fig. 2.5 (*a*).

We know that the torque acting on the shaft at *D*,

$$T = (T_1 - T_2)\,R_D = T_1\left(1 - \frac{T_2}{T_1}\right) R_D$$

$$= 3000\left(1 - \frac{1}{3}\right) 350 = 700 \times 10^3 \text{ N-mm} \qquad \ldots(\because\ T_1/T_2 = 3)$$

The torque diagram is shown in Fig. 2.5 (*b*).
Assuming that the torque at *D* is equal to the torque at *C*, therefore the tangential force acting on the gear *C*,

$$F_{tc} = \frac{T}{R_C} = \frac{700 \times 10^3}{300} = 2333 \text{ N}$$

and the normal load acting on the tooth of gear *C*,

$$W_C = \frac{F_{tc}}{\cos \alpha_C} = \frac{2333}{\cos 20°} = \frac{2333}{0.9397} = 2483 \text{ N}$$

The normal load acts at 20° to the vertical as shown in Fig. 2.6.

Resolving the normal load vertically and horizontally, we get

Vertical component of W_C *i.e.* the vertical load acting on the shaft at *C*,

$$W_{CV} = W_C \cos 20°$$

$$= 2483 \times 0.9397 = 2333 \text{ N}$$

and horizontal component of *W*C *i.e*. the horizontal load acting on the shaft at *C*,

$$W_{CH} = W_C \sin 20°$$

$$=2483 \times 0.342 = 849 \text{ N}$$

Since $T_1 / T_2 = 3$ and $T_1 = 3000$ N, therefore

$$T2 = T1 / 3 = 3000 / 3 = 1000 \text{ N}$$

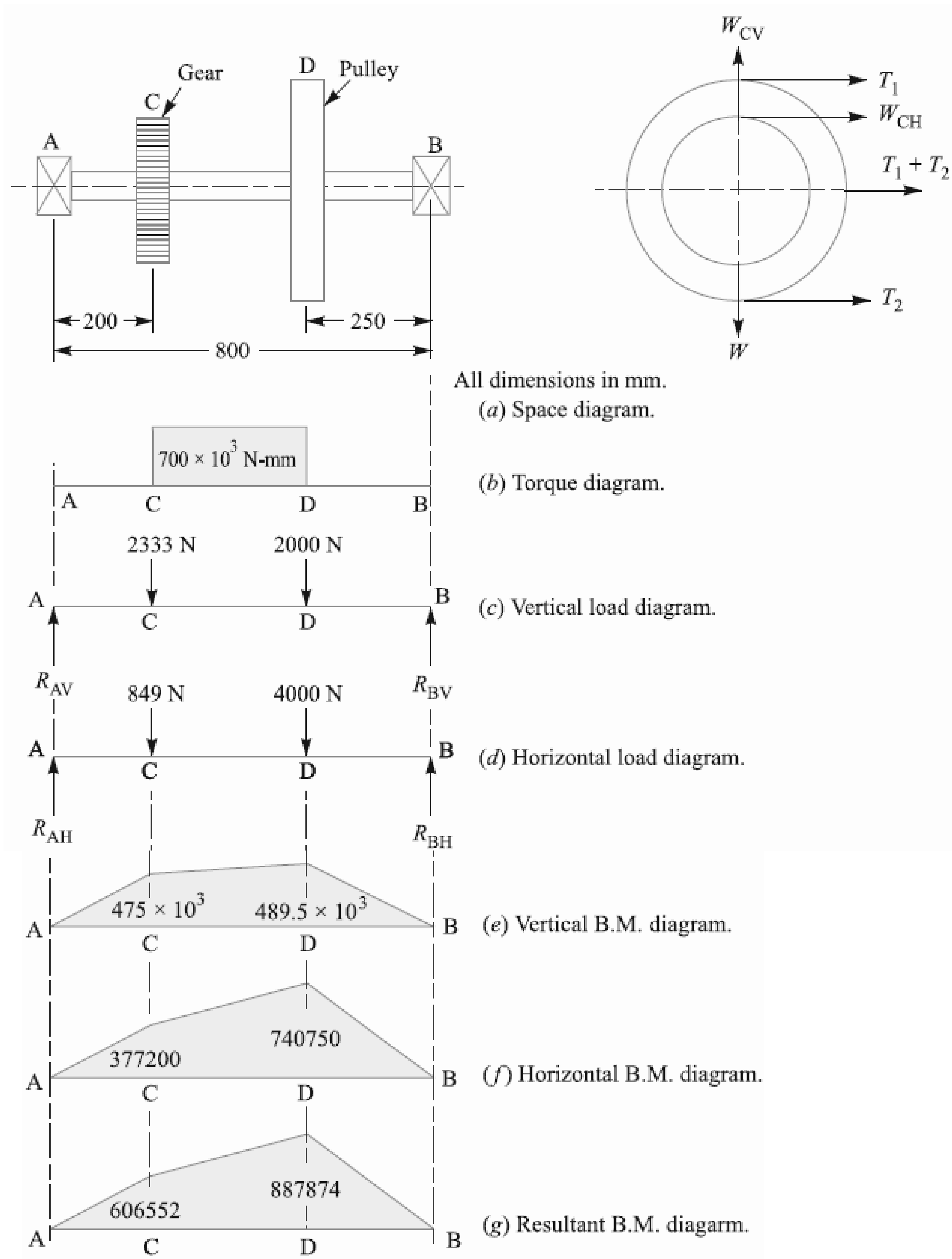
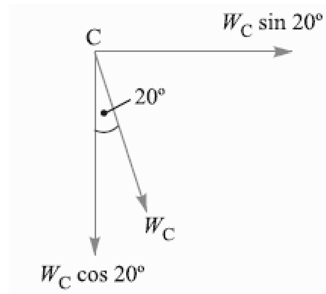

Gear
C
D
Pulley
A
B
200
250
800
W_CV
T_1
W_CH
T_1 + T_2
T_2
W
All dimensions in mm.
(a) Space diagram.
700 × 10^3 N-mm
(b) Torque diagram.
2333 N
2000 N
(c) Vertical load diagram.
R_AV
R_BV
849 N
4000 N
(d) Horizontal load diagram.
R_AH
R_BH
475 × 10^3
489.5 × 10^3
(e) Vertical B.M. diagram.
377200
740750
(f) Horizontal B.M. diagram.
606552
887874
(g) Resultant B.M. diagarm.

Fig. 2.5

∴ Horizontal load acting on the shaft at *D*,



$$W_{DH} = T_1 + T_2 = 3000 + 1000 = 4000 \text{ N}$$

and vertical load acting on the shaft at *D*,

$$W_{DV} = W = 2000 \text{ N}$$

Fig. 2.6

The vertical and horizontal load diagram at *C* and *D* is shown in Fig. 2.5 (*c*) and (*d*) respectively.

Now let us find the maximum bending moment for vertical and horizontal loading.

First of all considering the vertical loading at *C* and *D*. Let R_{AV} and R_{BV} be the reactions at the bearings *A* and *B* respectively. We know that

$$R_{AV} + R_{BV} = 2333 + 2000 = 4333 \text{ N}$$

Taking moments about *A*, we get

$$R_{BV} \times 800 = 2000\,(800 - 250) + 2333 \times 200$$

$$= 1\,566\,600$$

$$\therefore R_{BV} = 1\,566\,600 / 800 = 1958 \text{ N}$$

And $RAV = 4333 - 1958 = 2375$ N

We know that B.M. at *A* and *B*,

$$MAV = MBV = 0$$

B.M. at *C*, $MCV = RAV \times 200 = 2375 \times 200 = 475 \times 10^3$ N-mm

B.M. at D, $MDV = RBV \times 250 = 1958 \times 250 = 489.5 \times 10^3$ N-mm

The bending moment diagram for vertical loading is shown in Fig. 2.5 (*e*).

Now consider the horizontal loading at *C* and *D*. Let R_{AH} and R_{BH} be the reactions at the bearings *A* and *B* respectively. We know that

$$R_{AH} + R_{BH} = 849 + 4000 = 4849 \text{ N}$$

Taking moments about *A*, we get

$$R_{BH} \times 800 = 4000\,(800 - 250) + 849 \times 200 = 2\,369\,800$$

∴ $R_{BH} = 2\,369\,800 / 800 = 2963$ N

and $R_{AH} = 4849 - 2963 = 1886$ N

We know that B.M. at *A* and *B*,

$$M_{AH} = M_{BH} = 0$$

B.M. at *C*, $M_{CH} = R_{AH} \times 200 = 1886 \times 200 = 377\ 200$ N-mm

B.M. at *D*, $M_{DH} = R_{BH} \times 250 = 2963 \times 250 = 740\ 750$ N-mm

The bending moment diagram for horizontal loading is shown in Fig. 2.5 (*f*).

We know that resultant B.M. at *C*,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(475 \times 10^3)^2 + (377\,200)^2}$$
$$= 606\ 552 \text{ N-mm}$$

and resultant B.M. at *D*,

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(489.5 \times 10^3)^2 + (740\ 750)^2}$$
$$= 887\ 874 \text{ N-mm}$$

Maximum bending moment

The resultant B.M. diagram is shown in Fig. 2.5 (*g*). We see that the bending moment is maximum at *D*, therefore

Maximum B.M., $M = M$D = 887 874 N-mm **Ans.**

Diameter of the shaft

Let d = Diameter of the shaft.

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(887\ 874)^2 + (700 \times 10^3)^2} = 1131 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1131 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$\therefore \quad d^3 = 1131 \times 10^3 / 7.86 = 144 \times 10^3 \quad \text{or} \quad d = 52.4 \text{ say } 55 \text{ mm Ans.}$$

2.1.11 Shafts Subjected to Fluctuating Loads

In the previous articles we have assumed that the shaft is subjected to constant torque and bending moment. But in actual practice, the shafts are subjected to fluctuating torque and bending moments. In order to design such shafts like line shafts and counter shafts, the combined shock and fatigue factors must be taken into account for the computed twisting moment (T) and bending moment (M). Thus for a shaft subjected to combined bending and torsion, the equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t + T)^2}$$

and equivalent bending moment,

$$M_e = \tfrac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right]$$

where K_m = Combined shock and fatigue factor for bending, and

K_t = Combined shock and fatigue factor for torsion.

The following table shows the recommended values for K_m and K_t.

Table 2.2. Recommended values for *Km* and *Kt*.

Nature of load	K_m	K_t
1. Stationary shafts		
(*a*) Gradually applied load	1.0	1.0
(*b*) Suddenly applied load	1.5 to 2.0	1.5 to 2.0
2. Rotating shafts		
(*a*) Gradually applied or steady load	1.5	1.0
(*b*) Suddenly applied load with minor shocks only	1.5 to 2.0	1.5 to 2.0
(*c*) Suddenly applied load with heavy shocks	2.0 to 3.0	1.5 to 3.0

Example 2.8. *Design a shaft to transmit power from an electric motor to a lathe head stock through a pulley by means of a belt drive. The pulley weighs 200 N and is located at 300 mm from the centre of the bearing. The diameter of the pulley is 200 mm and the maximum power transmitted is 1 kW at 120 r.p.m. The angle of lap of the belt is 180° and coefficient of friction between the belt and the pulley is 0.3. The shock and fatigue factors for bending and twisting are 1.5 and 2.0 respectively.The allowable shear stress in the shaft may be taken as 35 MPa.*

Given Data :
W = 200 N
L = 300 mm
D = 200 mm or R = 100 mm
P = 1 kW = 1000 W
N = 120 r.p.m.
θ = 180° = π rad
μ = 0.3
K_m = 1.5
K_t = 2
τ = 35 MPa = 35 N/mm^2

To Find:

Design a shaft

Solution:

The shaft with pulley is shown in Fig. 14.9.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{1000 \times 60}{2\pi \times 120} = 79.6 \text{ N-m} = 79.6 \times 10^3 \text{ N-mm}$$

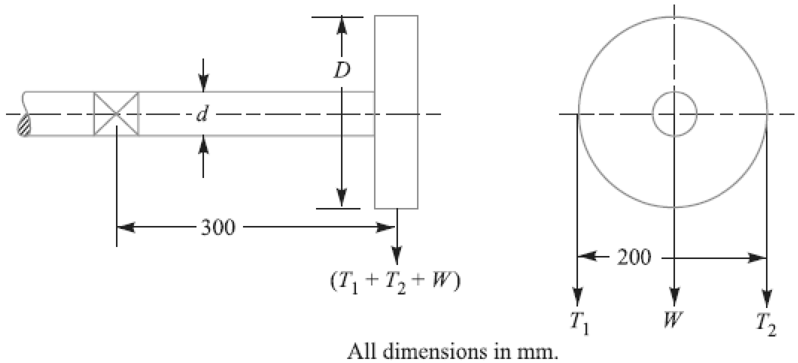


Fig. 2.7

Let $T1$ and $T2$ = Tensions in the tight side and slack side of the belt respectively in newtons.

∴ Torque transmitted (T),

$$79.6 \times 103 = (T1 - T2)\, R = (T1 - T2)\, 100$$

$$T1 - T2 = 79.6 \times 103 / 100 = 796 \text{ N} \qquad \text{..........}(\boldsymbol{i})$$

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3\,\pi = 0.9426$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{0.9426}{2.3} = 0.4098 \text{ or } \frac{T_1}{T_2} = 2.57 \qquad ...(ii)$$

...(Taking antilog of 0.4098)

From equations **(*i*)** and **(*ii*)**, we get,

$$T1 = 1303 \text{ N, and } T2 = 507 \text{ N}$$

We know that the total vertical load acting on the pulley,

$$W_T = T1 + T2 + W = 1303 + 507 + 200 = 2010 \text{ N}$$

∴ Bending moment acting on the shaft,

$$M = W_T \times L = 2010 \times 300 = 603 \times 103 \text{ N-mm}$$

Let d = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t + T)^2}$$

$$= \sqrt{(1.5 \times 603 \times 10^3)^2 + (2 \times 79.6 \times 10^3)^2} = 918 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$918 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 35 \times d^3 = 6.87\, d^3$$

$$\therefore \quad d^3 = 918 \times 10^3 / 6.87 = 133.6 \times 10^3 \text{ or } d = 51.1 \text{ say } 55 \text{ mm Ans.}$$

Example 2.9. *A horizontal nickel steel shaft rests on two bearings, A at the left and B at the right end and carries two gears C and D located at distances of 250 mm and 400 mm respectively from the centre line of the left and right bearings. The pitch diameter of the gear C is 600 mm and that of gear D is 200 mm. The distance between the centre line of the bearings is 2400 mm. The shaft transmits 20 kW at 120 r.p.m. The power is delivered to the shaft at gear C and is taken out at gear D in such a manner that the tooth pressure F_{tC} of the gear C and F_{tD} of the gear D act vertically downwards.*

Find the diameter of the shaft, if the working stress is 100 MPa in tension and 56 MPa in shear. The gears C and D weighs 950 N and 350 N respectively. The combined shock and fatigue factors for bending and torsion may be taken as 1.5 and 1.2 respectively.

Given Data :

AC = 250 mm
BD = 400 mm
D_C = 600 mm or R_C = 300 mm
D_D = 200 mm or R_D = 100 mm
AB = 2400 mm
P = 20 kW = 20 × 10^3 W
N = 120 r.p.m
σ_t = 100 MPa = 100 N/mm^2
τ = 56 MPa = 56 N/mm^2
W_C = 950 N
W_D = 350 N
K_m = 1.5
K_t = 1.2

To Find:
diameter of the shaft

Solution:

The shaft supported in bearings and carrying gears is shown in Fig. 2.8.

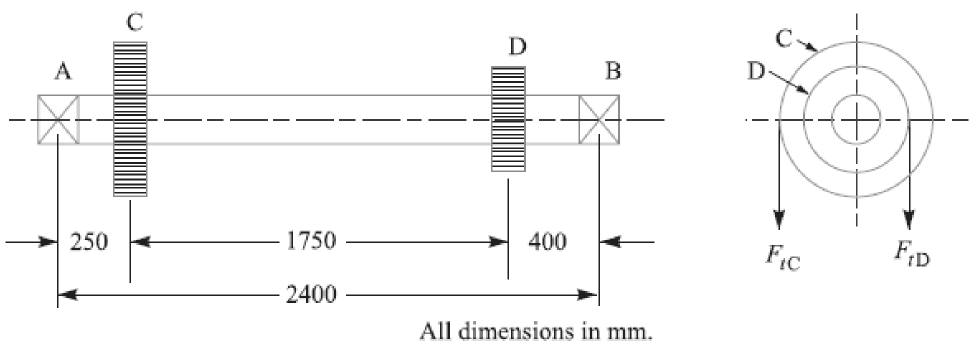


Fig. 2.8

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 120} = 1590 \text{ N-m} = 1590 \times 10^3 \text{ N-mm}$$

Since the torque acting at gears *C* and *D* is same as that of the shaft, therefore the tangential force acting at gear *C*,

$$F_{tC} = \frac{T}{R_C} = \frac{1590 \times 10^3}{300} = 5300 \text{ N}$$

and total load acting downwards on the shaft at *C*

$= F_{tC} + W_C = 5300 + 950 = 6250$ N

Similarly tangential force acting at gear *D*,

$$F_{tD} = \frac{T}{R_D} = \frac{1590 \times 10^3}{100} = 15\,900 \text{ N}$$

and total load acting downwards on the shaft at *D*

$= F_{tD} + W_D = 15\,900 + 350 = 16\,250$ N

Now assuming the shaft as a simply supported beam as shown in Fig. 2.9, the maximum bending moment may be obtained as discussed below :

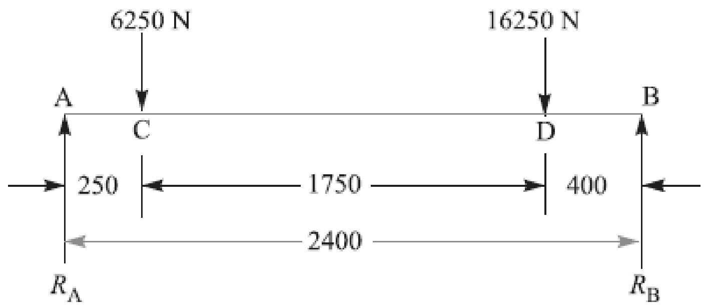


Fig. 2.9

Let R_A and R_B = Reactions at *A* and *B* respectively.

∴ $R_A + R_B$ = Total load acting downwards at *C* and *D*

$= 6250 + 16\,250 = 22\,500$ N

Now taking moments about *A*,

$R_B \times 2400 = 16\,250 \times 2000 + 6250 \times 250 = 34\,062.5 \times 103$

∴ $R_B = 34\,062.5 \times 103 / 2400 = 14\,190$ N

and $R_A = 22\,500 – 14\,190 = 8310$ N

A little consideration will show that the maximum bending moment will be either at *C* or *D*. We know that bending moment at *C*,

$M_C = R_A \times 250 = 8310 \times 250 = 2077.5 \times 103$ N-mm

Bending moment at *D*,

$M_D = R_B \times 400 = 14\,190 \times 400 = 5676 \times 103$ N-mm

∴ Maximum bending moment transmitted by the shaft,

$M = M_D = 5676 \times 103$ N-mm

Let *d* = Diameter of the shaft.

We know that the equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2}$$

$$= \sqrt{(1.5 \times 5676 \times 10^3)^2 + (1.2 \times 1590 \times 10^3)^2}$$

$$= 8725 \times 10^3 \text{ N-mm}$$

We also know that the equivalent twisting moment (T_e),

$$8725 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 56 \times d^3 = 11\ d^3$$

$$\therefore \quad d^3 = 8725 \times 10^3 / 11 = 793 \times 10^3 \text{ or } d = 92.5 \text{ mm}$$

Again we know that the equivalent bending moment,

$$M_e = \tfrac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right] = \tfrac{1}{2}\,(K_m \times M + T_e)$$

$$= \tfrac{1}{2}\left[1.5 \times 5676 \times 10^3 + 8725 \times 10^3\right] = 8620 \times 10^3 \text{ N-mm}$$

We also know that the equivalent bending moment (M_e),

$$8620 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 100 \times d^3 = 9.82\ d^3 \quad \text{...(Taking } \sigma_b = \sigma_t)$$

$$\therefore \quad d^3 = 8620 \times 10^3 / 9.82 = 878 \times 10^3 \text{ or } d = 95.7 \text{ mm}$$

Taking the larger of the two values, we have

$$d = 95.7 \text{ say } 100 \text{ mm Ans.}$$

2.1.12 Shafts Subjected to Axial Load in addition to Combined Torsion and Bending Loads

When the shaft is subjected to an axial load (F) in addition to torsion and bending loads as in propeller shafts of ships and shafts for driving worm gears, then the stress due to axial load must be added to the bending stress (σ_b). We know that bending equation is

$$\frac{M}{I} = \frac{\sigma_b}{y} \quad \text{or} \quad \sigma_b = \frac{M \cdot y}{I} = \frac{M \times d/2}{\frac{\pi}{64} \times d^4} = \frac{32M}{\pi d^3}$$

and stress due to axial load

$$= \frac{F}{\frac{\pi}{4} \times d^2} = \frac{4F}{\pi d^2} \quad \text{...(For round solid shaft)}$$

$$= \frac{F}{\frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right]} = \frac{4F}{\pi\left[(d_o)^2 - (d_i)^2\right]} \quad \text{...(For hollow shaft)}$$

$$= \frac{F}{\pi\,(d_o)^2\,(1 - k^2)} \quad \text{... } (\because k = d_i/d_o)$$

$\therefore$ Resultant stress (tensile or compressive) for solid shaft,

$$\sigma_1 = \frac{32M}{\pi d^3} + \frac{4F}{\pi d^2} = \frac{32}{\pi d^3}\left(M + \frac{F \times d}{8}\right) \quad \text{...}(i)$$

$$= \frac{32M_1}{\pi d^3} \quad \text{...}\left(\text{Substituting } M_1 = M + \frac{F \times d}{8}\right)$$

In case of a hollow shaft, the resultant stress,

$$\sigma_1 = \frac{32M}{\pi\,(d_o)^3\,(1 - k^4)} + \frac{4F}{\pi\,(d_o)^2\,(1 - k^2)}$$

$$= \frac{32}{\pi\,(d_o)^3\,(1 - k^4)}\left[M + \frac{F\,d_o\,(1 + k^2)}{8}\right] = \frac{32M_1}{\pi(d_o)^3\,(1 - k^4)}$$

$$\ldots\left[\text{Substituting for hollow shaft, } M_1 = M + \frac{F\,d_o\,(1+k^2)}{8}\right]$$

In case of long shafts (slender shafts) subjected to compressive loads, a factor known as ***column factor*** (α) must be introduced to take the column effect into account.

$\therefore$ Stress due to the compressive load,

$$\sigma_c = \frac{\alpha \times 4F}{\pi d^2} \qquad \text{...(For round solid shaft)}$$

$$= \frac{\alpha \times 4F}{\pi (d_o)^2\,(1-k^2)} \qquad \text{...(For hollow shaft)}$$

The value of column factor (α) for compressive loads* may be obtained from the following relation :

$$\text{Column factor,} \quad \alpha = \frac{1}{1 - 0.0044\,(L/K)}$$

This expression is used when the slenderness ratio (L / K) is less than 115. When the slenderness ratio (L / K) is more than 115, then the value of column factor may be obtained from the following relation :

$$\text{**Column factor,} \quad \alpha = \frac{\sigma_y\,(L/K)^2}{C\pi^2 E}$$

where L = Length of shaft between the bearings,

K = Least radius of gyration,

σ_y = Compressive yield point stress of shaft material, and

C = Coefficient in Euler's formula depending upon the end conditions.

The following are the different values of C depending upon the end conditions.

C =1, for hinged ends,

= 2.25, for fixed ends,

=1.6, for ends that are partly restrained as in bearings.

Example 2.10. *A hollow shaft of 0.5 m outside diameter and 0.3 m inside diameter is used to drive a propeller of a marine vessel. The shaft is mounted on bearings 6 metre apart and it transmits 5600 kW at 150 r.p.m. The maximum axial propeller thrust is 500 kN and the shaft weighs 70 kN. Determine :*

1. The maximum shear stress developed in the shaft, and

2. The angular twist between the bearings.

Given Data:

d_o = 0.5 m
d_i = 0.3 m
P = 5600 kW = 5600 × 10^3 W
L = 6 m
N = 150 r.p.m.

F = 500 kN = 500 × 10^3 N
W = 70 kN = 70 × 10^3 N

To Find:

maximum shear stress

angular twist between the bearings.

Solution:

Maximum shear stress developed in the shaft

Let τ = Maximum shear stress developed in the shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{5600 \times 10^3 \times 60}{2\pi \times 150} = 356\ 460 \text{ N-m}$$

and the maximum bending moment,

$$M = \frac{W \times L}{8} = \frac{70 \times 10^3 \times 6}{8} = 52\ 500 \text{ N-m}$$

Now let us find out the column factor α. We know that least radius of gyration,

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right]}{\frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right]}}$$

$$= \sqrt{\frac{[(d_o)^2 + (d_i)^2]\,[(d_o)^2 - (d_i)^2]}{16\,[(d_o)^2 - (d_i)^2]}}$$

$$= \frac{1}{4}\sqrt{(d_o)^2 + (d_i)^2} = \frac{1}{4}\sqrt{(0.5)^2 + (0.3)^2} = 0.1458 \text{ m}$$

$\therefore$ Slenderness ratio,

$$L / K = 6 / 0.1458 = 41.15$$

and column factor,

$$\alpha = \frac{1}{1 - 0.0044\left(\frac{L}{K}\right)} \qquad \ldots\left(\because \frac{L}{K} < 115\right)$$

$$= \frac{1}{1 - 0.0044 \times 41.15} = \frac{1}{1 - 0.18} = 1.22$$

Assuming that the load is applied gradually, therefore from Table 14.2, we find that

$$K_m = 1.5 \text{ and } K_t = 1.0$$

Also $k = d_i / d_o = 0.3 / 0.5 = 0.6$

We know that the equivalent twisting moment for a hollow shaft,

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha F d_o\,(1 + k^2)}{8}\right]^2 + (K_t \times T)^2}$$

$$= \sqrt{\left[1.5 \times 52\ 500 + \frac{1.22 \times 500 \times 10^3 \times 0.5\ (1 + 0.6^2)}{8}\right]^2 + (1 \times 356\ 460)^2}$$

$$= \sqrt{(78\ 750 + 51\ 850)^2 + (356\ 460)^2} = 380 \times 10^3 \text{ N-m}$$

We also know that the equivalent twisting moment for a hollow shaft (T_e),

$$380 \times 10^3 = \frac{\pi}{16} \times \tau\ (d_o)^3\ (1 - k^4) = \frac{\pi}{16} \times \tau\ (0.5)^3\ [1 - (0.6)^4] = 0.02\ \tau$$

$$\therefore \quad \tau = 380 \times 10^3 / 0.02 = 19 \times 10^6 \text{ N/m}^2 = 19 \text{ MPa } \textbf{Ans.}$$

2. *Angular twist between the bearings*

Let θ = Angular twist between the bearings in radians.

We know that the polar moment of inertia for a hollow shaft,

$$J = \frac{\pi}{32}[(d_o)^4 - (d_i)^4] = \frac{\pi}{32}[(0.5)^4 - (0.3)^4] = 0.005\ 34 \text{ m}^4$$

From the torsion equation,

$$\frac{T}{J} = \frac{G \times \theta}{L}, \text{ we have}$$

$$\theta = \frac{T \times L}{G \times J} = \frac{356\ 460 \times 6}{84 \times 10^9 \times 0.00\ 534} = 0.0048 \text{ rad}$$

... (Taking G = 84 GPa = 84×10^9 N/m²)

$$= 0.0048 \times \frac{180}{\pi} = 0.275° \textbf{ Ans.}$$

2.1.13 Design of Shafts on the basis of Rigidity

Sometimes the shafts are to be designed on the basis of rigidity. We shall consider the following two types of rigidity.

1. *Torsional rigidity.* The torsional rigidity is important in the case of camshaft of an I.C. engine where the timing of the valves would be effected. The permissible amount of twist should not exceed 0.25° per metre length of such shafts. For line shafts or transmission shafts, deflections 2.5 to 3 degree per metre length may be used as limiting value. The widely used deflection for the shafts is limited to 1 degree in a length equal to twenty times the diameter of the shaft. The torsional deflection may be obtained by using the torsion equation,

$$\frac{T}{J} = \frac{G\,.\,\theta}{L} \quad \text{or} \quad \theta = \frac{T\,.\,L}{J\,.\,G}$$

where θ = Torsional deflection or angle of twist in radians,

T = Twisting moment or torque on the shaft,

J = Polar moment of inertia of the cross-sectional area about the axis of rotation,

$= \frac{\pi}{32} \times d^4$ ...(For solid shaft)

$= \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]$ ...(For hollow shaft)

G = Modulus of rigidity for the shaft material, and

L = Length of the shaft.

2. *Lateral rigidity*. It is important in case of transmission shafting and shafts running at high speed, where small lateral deflection would cause huge out-of-balance forces. The lateral rigidity is also important for maintaining proper bearing clearances and for correct gear teeth alignment. If the shaft is of uniform cross-section, then the lateral deflection of a shaft may be obtained by using the deflection formulae as in Strength of Materials. But when the shaft is of variable cross-section, then the lateral deflection may be determined from the fundamental equation for the elastic curve of a beam,

Example 2.11. *A steel spindle transmits 4 kW at 800 r.p.m. The angular deflection should not exceed 0.25° per metre of the spindle. If the modulus of rigidity for the material of the spindle is 84 GPa, find the diameter of the spindle and the shear stress induced in the spindle.*

Given Data :

P = 4 kW = 4000 W

N = 800 r.p.m.

$\theta = 0.25° = 0.25 \text{ X } \frac{\pi}{180} = 0.0044$ rad

L = 1 m = 1000 mm

$G = 84 \text{ GPa} = 84 \times 10^9 \text{ N/m}^2 = 84 \times 10^3 \text{ N/mm}^2$

To Find:

1. *diameter of the spindle*
2. *shear stress induced in the spindle*

Diameter of the spindle

Let d = Diameter of the spindle in mm.

We know that the torque transmitted by the spindle,

$$T = \frac{P \times 60}{2\pi N} = \frac{4000 \times 60}{2\pi \times 800} = 47.74 \text{ N-m} = 47\ 740 \text{ N-mm}$$

We also know that $\dfrac{T}{J} = \dfrac{G \times \theta}{L}$ or $J = \dfrac{T \times l}{G \times \theta}$

or $$\frac{\pi}{32} \times d^4 = \frac{47\ 740 \times 1000}{84 \times 10^3 \times 0.0044} = 129\ 167$$

$\therefore$ $d^4 = 129\ 167 \times 32 / \pi = 1.3 \times 10^6$ or $d = 33.87$ say 35 mm **Ans.**

Shear stress induced in the spindle

Let τ = Shear stress induced in the spindle.

We know that the torque transmitted by the spindle (T),

$$47\ 740 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times \tau\ (35)^3 = 8420\ \tau$$

$\therefore$ $\tau = 47\ 740 / 8420 = 5.67 \text{ N/mm}^2 = 5.67$ MPa **Ans.**