Convolution:
Two types
1.Linear Convolution
2.Circular Convolution
1.Linear Convolution
Formula:

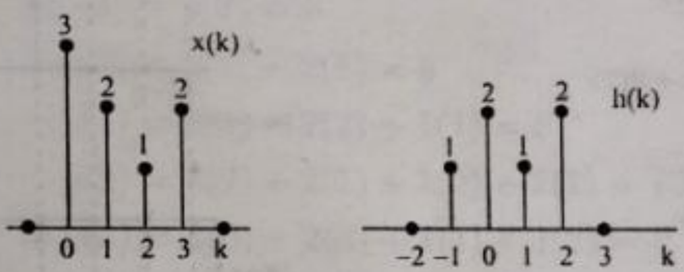
$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

Example 1.11 Determine the convolution sum of two sequences
$x(n) = \{3, 2, 1, 2\}$; $h(n) = \{1, \underset{\uparrow}{2}, 1, 2\}$

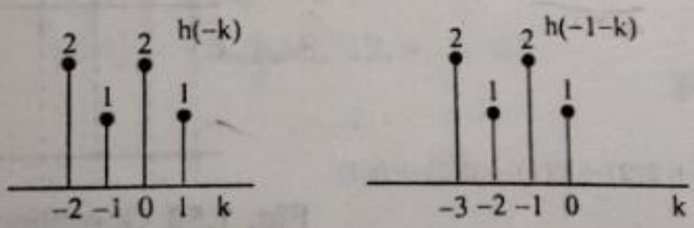
Solution

Step 1 The sequence $x(n)$ starts at $n = 0$ and $h(n)$ starts at $n_2 = -1$. Therefore the starting time for evaluating the output sequence $y(n)$ is $n = n_1 + n_2 = 0 + (-1) = -1$

Step 2 Express both sequences in terms of the index k.



Step 3 Fold $h(k)$ about $k = 0$ to obtain $h(-k)$



As starting time to evaluate $y(n)$ is -1, shift $h(k)$ by one unit to left to obtain $h(-1-k)$

$$y(-1) = \sum_{k=-\infty}^{\infty} x(k)h(-1-k)$$

Multiply the two sequences $x(k)$ and $h(-1-k)$ element by element and sum the products

$$\Rightarrow y(-1) = 0(2) + 0(1) + 0(2) + 3(1) + 2(0) + 1(0) + 2(0)$$
$$= 3$$

Increment the index by 1, shift the sequence to right to obtain $h(-k)$ and multiply the two sequences $x(k)$ and $h(-k)$ element by element and sum the products

$$y(0) = \sum_{k=-\infty}^{\infty} x(k)h(-k)$$
$$= 0(2) + 0(1) + 3(2) + 2(1) + 1(0) + 2(0) = 8$$

Similarly

$$y(1) = \sum_{k=-\infty}^{\infty} x(k)h(1-k)$$
$$= 0(2) + 3(1) + 2(2) + 1(1) + 2(0)$$
$$= 8$$

$$y(2) = \sum_{k=-\infty}^{\infty} x(k)h(2-k)$$
$$= 3(0) + 2(1) + 1(2) + 2(1)$$
$$= 12$$

$$y(3) = \sum_{k=-\infty}^{\infty} x(k)h(3-k)$$
$$= 3(0) + 2(2) + 1(1) + 2(2)$$
$$= 9$$

$$y(4) = \sum_{k=-\infty}^{\infty} x(k)h(4-k)$$
$$= 3(0) + 2(0) + 1(2) + 2(1) + 0(2)$$
$$= 4$$

$$y(5) = \sum_{k=-\infty}^{\infty} x(k)h(5-k)$$
$$= 3(0) + 2(0) + 1(0) + 2(2) + 0(1) + 0(2) + 0(1)$$
$$= 4$$

y(n)= {3,8,8,12,9,4,4}

2.Circular Convolution

Perform the circular convolution of the following sequences x(n)={1,1,2,1}, h (n)={1,2,3,4} using DFT and IDFT method

We know $X_3(k) = X_1(k)X_2(k)$

$$X_1(k) = \sum_{n=0}^{N-1} x_1(n)e^{-j2\pi kn/N} \quad k = 0, 1, \ldots N-1$$

Given $x_1(n) = \{1, 1, 2, 1\}$ and $N = 4$

$$X_1(0) = \sum_{n=0}^{3} x_1(n) = 1 + 1 + 2 + 1 = 5$$

$$X_1(1) = \sum_{n=0}^{3} x_1(n)e^{-j\pi n/2} = 1 - j - 2 + j = -1$$

$$X_1(2) = \sum_{n=0}^{3} x_1(n)e^{-j\pi n} = 1 - 1 + 2 - 1 = 1$$

$$X_1(3) = \sum_{n=0}^{3} x_1(n)e^{-j3\pi n/2} = 1 + j - 2 - j = -1$$

$$X_1(k) = (5, -1, 1, -1)$$

$$X_2(k) = \sum_{n=0}^{N-1} x_2(n)e^{-j2\pi nk/N} \quad k = 0, 1, \ldots N-1$$

$$X_2(0) = \sum_{n=0}^{3} x_2(n) = 1 + 2 + 3 + 4 = 10$$

$$X_2(1) = \sum_{n=0}^{3} x_2(n)e^{-j\pi n/2} = 1 + 2(-j) + 3(-1) + 4(j) = -2 + j2$$

$$X_2(2) = \sum_{n=0}^{3} x_2(n)e^{-j\pi n} = 1 + 2(-1) + 3(1) + 4(-1) = -2$$

$$X_2(3) = \sum_{n=0}^{3} x_1(n)e^{-j3\pi n/2} = 1 + 2(j) + 3(-1) + 4(-j) = -2 - j2$$

$$X_2(k) = \{10, -2 + j2, -2, -2, -j2\}$$

$$X_3(k) = X_1(k)X_2(k) = \{50, 2 - j2, -2, 2 + j2\}$$

$$x_3(n) = \frac{1}{N}\sum_{k=0}^{N-1} X_3(k)e^{j2\pi nk/N} \quad n = 0, 1, \ldots N - 1$$

$$x_3(0) = \frac{1}{4}\sum_{k=0}^{3} X_3(k) = \frac{1}{4}(50 + 2 - j2 - 2 + 2 + j2) = 13$$

$$x_3(1) = \frac{1}{4}\left[\sum_{k=0}^{4} X_3(k)e^{j\pi k/2}\right]$$

$$= \frac{1}{4}[50 + (2 - j2)j + (-2)(-1) + (2 + j2)(-j)] = 14$$

$$x_3(2) = \frac{1}{4}\left[\sum_{k=0}^{4} X_3(k)e^{j\pi k}\right]$$

$$= \frac{1}{4}[50 + (2 - j2)(-j) + (-2)(-1) + (2 + j2)(j)] = 12$$

x3(n) = {13,14,11,12}