CLOSURE PROPERTIES OF CFL

Context Free Languages (CFLs) are accepted by pushdown automata. Context free languages can be generated by context free grammars, which have productions (substitution rules) of the form : A -> ρ (where A ∈ N and ρ ∈ (T ∪ N)* and N is a non-terminal and T is a terminal)

Properties of Context Free Languages

Union : If L1 and L2 are two context free languages, their union L1 ∪ L2 will also be context free. For example,

L1 = { $a^nb^nc^m$ | m >= 0 and n >= 0 } and L2 = { $a^nb^mc^m$ | n >= 0 and m >= 0 }

L3 = L1 ∪ L2 = { $a^nb^nc^m$ ∪ $a^nb^mc^m$ | n >= 0, m >= 0 } is also context free.

L1 says number of a's should be equal to number of b's and L2 says number of b's should be equal to number of c's. Their union says either of two conditions to be true. So it is also context free language.

So CFL are closed under Union.

Concatenation : If L1 and If L2 are two context free languages, their concatenation L1.L2 will also be context free. For example,

L1 = { a^nb^n | n >= 0 } and L2 = { c^md^m | m >= 0 }

L3 = L1.L2 = { $a^nb^nc^md^m$ | m >= 0 and n >= 0} is also context free.

L1 says number of a's should be equal to number of b's and L2 says number of c's should be equal to number of d's. Their concatenation says first number of a's should be equal to number of b's, then number of c's should be equal to number of d's. So, we can create a PDA which will first push for a's, pop for b's, push for c's then pop for d's. So it can be accepted by pushdown automata, hence context free.

So CFL are closed under Concatenation.

Kleene Closure : If L1 is context free, its Kleene closure L1* will also be context free. For example,

L1 = { a^nb^n | n >= 0 }

L1* = { a^nb^n | n >= 0 }* is also context free.

So CFL are closed under Kleen Closure.

Intersection and complementation : If L1 and If L2 are two context free languages, their intersection L1 ∩ L2 need not be context free. For example,

L1 = { $a^nb^nc^m$ | n >= 0 and m >= 0 } and L2 = ($a^mb^nc^n$ | n >= 0 and m >= 0 }

L3 = L1 ∩ L2 = { $a^nb^nc^n$ | n >= 0 } need not be context free.

L1 says number of a's should be equal to number of b's and L2 says number of b's should be equal to number of c's. Their intersection says both conditions need to be true, but push down automata can compare only two. So it cannot be accepted by pushdown automata, hence not context free.

Similarly, complementation of context free language L1 which is ∑* – L1, need not be context free.