2.7 Expression for Critical Thickness of Insulation for a Cylindrical Pipe

Let us consider a pipe, outer radius r_1 as shown in Fig. 2.18. An insulation is added such that the outermost radius is r a variable and the insulation thickness is $(r - r_1)$. We assume that the thermal conductivity, k, for the insulating material is very small in comparison with the thermal conductivity of the pipe material and as such the temperature T_1, at the inside surface of the insulation is constant. It is further assumed that both h and k are constant. The rate of heat flow, per unit length of pipe, through the insulation is then,

$\dot{Q}/L = 2\pi\left(T_1 - T_\infty\right)/\left(\ln\left(r/r_1\right)/k + 1/hr\right)$, where T_∞ is the ambient temperature.

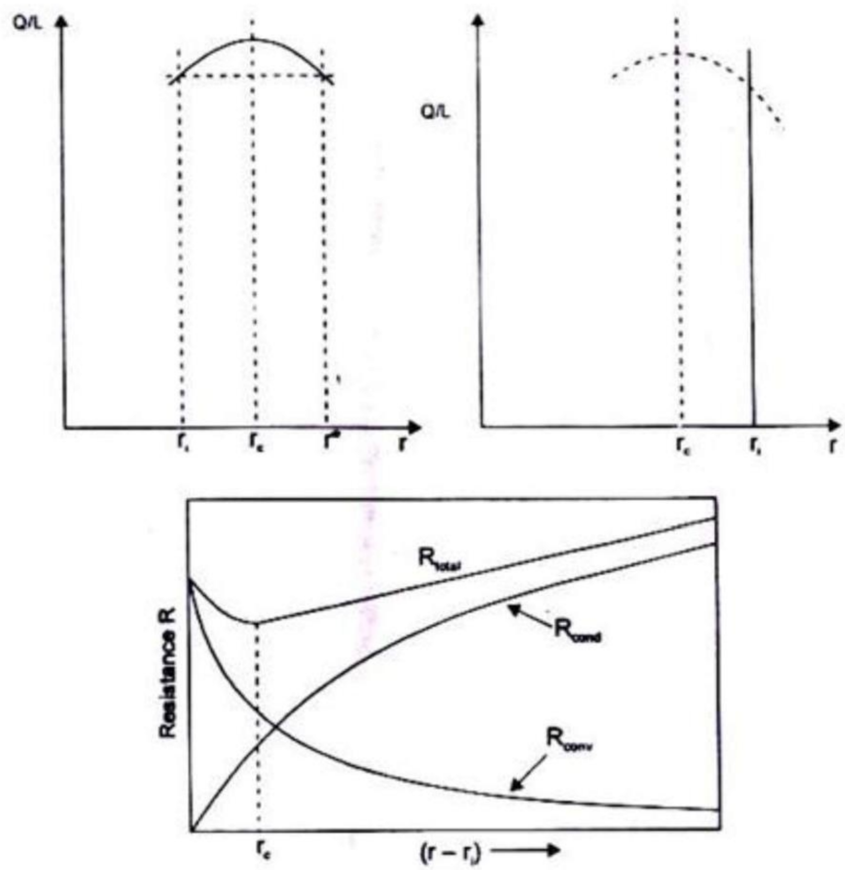

Fig 2.8 Critical thickness for pipe insulation

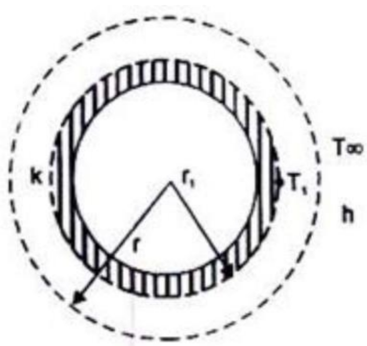

Fig 2.9 critical thickness of insulation for a pipe

An optimum value of the heat loss is found by setting $\frac{d(\dot{Q}/L)}{dr} = 0$.

$$\text{or,}\quad \frac{d(\dot{Q}/L)}{dr} = 0 = -\frac{2\pi(T_1 - T_\infty)(1/kr - 1/hr^2)}{(\ln(r/r_1)/k + 1/hr^2)}$$

or, $(1/kr) - (1/hr^2) = 0$ and $r = r_c = k/h$ (2.21)

where r_c denote the 'critical radius' and depends only on thermal quantities k and h.

If we evaluate the second derivative of (Q/L) at $r = r_c$, we get

$$\left.\frac{d^2(Q/L)}{dr^2}\right|_{r=r_c} = -2\pi(T_1 - T_\infty)\left[\frac{\frac{k}{hr} + \ln\left(\frac{r}{r_1}\right)\left(\frac{2k}{hr} - 1\right) - 2\left(1 - \frac{k}{hr}\right)^2}{\frac{1}{kr}\left(\frac{k}{h} + r\ln\left(\frac{r}{r_1}\right)\right)}\right]_{r=r_c}$$

$$= -\left[2\pi(T_1 - T_\infty)h^2/k\right]/\left[1 + \ln r_c/r_1\right]^2$$

Which is always a negative quantity. Thus, the optimum radius, $r_c = k/h$ will always give a maximum heal loss and not a minimum.

2.8. An Expression for the Critical Thickness of Insulation for a Spherical Shell

Let us consider a spherical shell having an outer radius r_1 and the temperature at that surface T_1 . Insulation is added such that the outermost radius of the shell is r, a variable. The thermal conductivity of the insulating material, k, and the convective heat transfer coefficient at the outer surface, h, and the ambient temperature T_∞ is constant. The rate of heat transfer through the insulation on the spherical shell is given by

$$\dot{Q} = \frac{(T_1 - T_\infty)}{(r - r_1)/4\pi k r r_1 + 1/h 4\pi r^2}$$

$$\frac{d\dot{Q}}{dr} = 0 = \frac{4\pi(T_1 - T_\infty)(1/kr^2 - 2/hr^3)}{\left[(r - r_1)/k r r_1 + 1/hr^2\right]^2}$$

which gives, $1/Kr^2 - 2/hr^3 = 0$;

or $\quad r = r_c = 2\ k/h \qquad (2.22)$

2.9 Heat and Mass Transfer

Example 2.5 Hot gases at 175°C flow through a metal pipe (outer diameter 8 cm). The convective heat transfer coefficient at the outside surface of the insulation (k = 0.18 W /mK) IS 2.6 W m1K and the ambient temperature IS 25°C. Calculate the insulation thickness such that the heat loss is less than the uninsulated case.

Solution: (a) Pipe without Insulation

Neglecting the thermal resistance of the pipe wall and due to the inside convective heat transfer coefficient, the temperature of the pipe surface would be 175°C.

$\dot{Q}/L = h \times 2\pi r\ (T_1 - T_\infty) = 2.6 \times 2 \times 3.14 \times .04\{175\text{-}25) = 98$ W/m (b) Pipe Insulated. Outermost Radius, r*

$$\dot{Q}/L = 98 = (T_1 - T_\infty) / \left(\frac{1n(r^*/4)}{2\pi \times 0.18} + \frac{100}{2.6 \times 2\pi \times r^*}\right)$$

or $\frac{150}{98}$ = 08841n (r*/4)+6.12/r*; which gives r* = 13.5 cm.

Therefore, the insulation thickness must be more than 9.5 cm.

(Since the critical thickness of insulation is $r_c = k/h = 0.18/2.6 = 6.92$ cm, and is greater than the radius of the bare pipe, the required insulation thickness must give a radius greater than the critical radius.)

If the outer radius of the pipe was more than the critical radius, any addition of insulating material will reduce the rate of heat transfer. Let us assume that the outer radius of the pipe is 7 cm ($r > r_c$)

$\dot{Q}/L$, without insulation = hA (ΔT) = 2.6 × 2 ×3.142 × 0.07× (175-25)

= 171.55 W/m

By adding 4 cm thick insulation, outermost radius = 7.0 + 4.0 = 11.0 cm.

and $\dot{Q}/L = (175 - 25) / \left[\frac{\ln(11/7)}{2\pi \times 0.18} + \frac{1}{2.6\pi \times 2 \times 0.11}\right] = 133.58 W/m.$

Reduction in heat loss = $\frac{171.55 - 133.58}{171.55} = 0.22$ or 22%.

Example 2.6 An electric conductor 1.5 mm in diameter at a surface temperature of 80°C is being cooled in air at 25°C. The convective heat transfer coefficient from the conductor surface is $16 W/m^2K$. Calculate the surface temperature of the conductor when it is covered with a layer of rubber insulation (2 mm thick, k = 0.15 W /mK) assuming that the conductor carries the same current and the convective heat transfer coefficient is also the same. Also calculate the increase in the current carrying capacity of the conductor when the surface temperature of the conductor remains at 80°C.

Solution: When there is no insulation,

$\dot{Q}/L = hA(\Delta T) = 16 \times 2 \times 3.142 \times 0.75 \times 10^{-3} = 4.147$ W/m

When the insulation is provided, the outermost radius = 0.75 + 2 = 2.75 mm

$\dot{Q}/L = 4.147 = (T_1 - 25) / \left(\frac{\ln 2.75/0.75}{2\pi \times 0.15} + \frac{1000}{16 \times 2\pi \times 2.75}\right)$

or $T_1 = 45.71^{o}C$

i.e., the temperature at the outer surface of the wire decreases because the insulation adds a resistance.

The critical radius of insulation, $r_c = k/h = 0.15/16 = 9.375$ mm

i.e., when an insulation of thickness (9.375 - 0.75) = 8.625 mm is added, the heat transfer rate would be the maximum and the conductor can carry more current. The heat transfer rate with outermost radius equal to $r_c = 9.375$ mm

$$\dot{Q}/L = (80 - 25) \Big/ \left(\frac{1n\,9.375/0.75}{2\pi \times 0.15} + \frac{1000}{16 \times 2\pi \times 9.375}\right) = 14.7 \text{ W/m}$$

The rate of heat transfer is proportional to $(\text{current})^2$, the new current I_2 would be:

$I_2/I_1 = (14.7 / 4.147)^{1/2} = 1.883$

or, the current carrying capacity can be increased 1.883 times. But the maximum current capacity of wire would be limited by the permissible temperature at the centre of the wire.

The surface temperature of the conductor when the outermost radius with insulation is equal to the critical radius, is given by

$$\dot{Q}/L = 4.147 = (T\text{-}25/ \left(\frac{1n\,9.375/0.75}{2 \times 3.142 \times 0.15} + \frac{1000}{16 \times 2 \times 3.142 \times 9.375}\right)$$

or $\quad T = 40.83°C.$