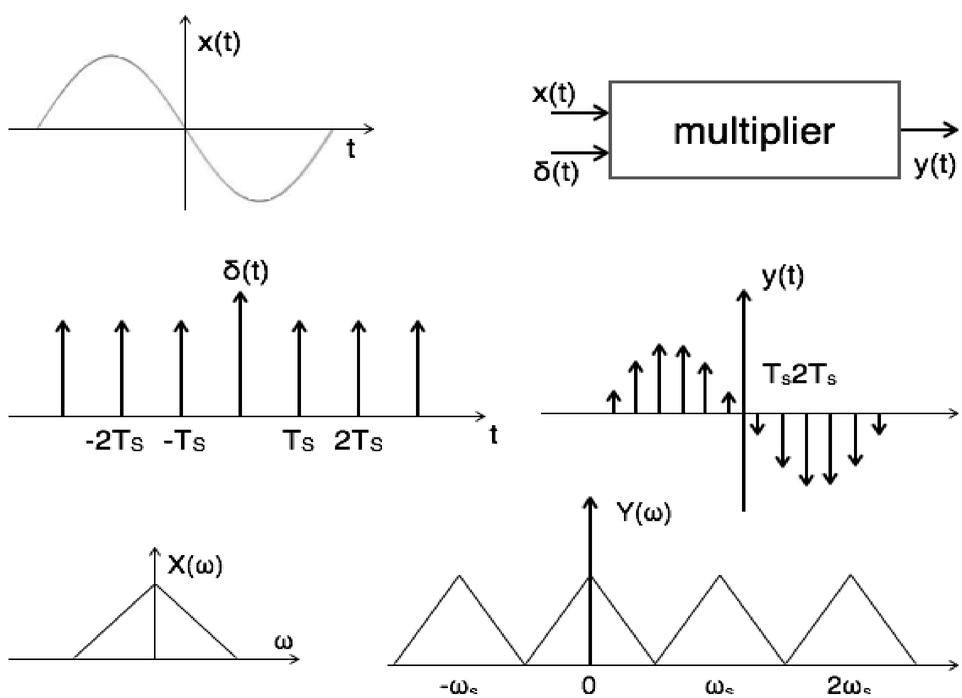
SAMPLING

A continuous time signal can be represented in its samples and can be recovered back when sampling frequency f_s is greater than or equal to the twice the highest frequency component of message signal. i. e.

$$f_s \geq 2f_m$$

Proof: Consider a continuous time signal x(t). The spectrum of x(t) is a band limited to f_m Hz i.e. the spectrum of x(t) is zero for $|\omega|>\omega_m$.

Sampling of input signal x(t) can be obtained by multiplying x(t) with an impulse train δ(t) of period T_s. The output of multiplier is a discrete signal called sampled signal which is represented with y(t) in the following diagrams:



Here, you can observe that the sampled signal takes the period of impulse. The process of sampling can be explained by the following mathematical expression:

Sampled signal $y(t)=x(t).\delta(t)$......(1)

The trigonometric Fourier series representation of δ(t) is given by

$$\delta(t) = a_0 + \Sigma_{n=1}^{\infty}(a_n \cos n\omega_s t + b_n \sin n\omega_s t) \;\ldots\ldots(2)$$

Where $a_0 = \frac{1}{T_s}\int_{\frac{-T}{2}}^{\frac{T}{2}} \delta(t)dt = \frac{1}{T_s}\delta(0) = \frac{1}{T_s}$

$$a_n = \frac{2}{T_s}\int_{\frac{-T}{2}}^{\frac{T}{2}} \delta(t)\cos n\omega_s\, dt = \frac{2}{T_2}\delta(0)\cos n\omega_s 0 = \frac{2}{T}$$

$$b_n = \frac{2}{T_s}\int_{\frac{-T}{2}}^{\frac{T}{2}} \delta(t)\sin n\omega_s t\, dt = \frac{2}{T_s}\delta(0)\sin n\omega_s 0 = 0$$

SUBSTITUTE THE ABOVE VALUES IN 2

$$\therefore\ \delta(t) = \frac{1}{T_s} + \Sigma_{n=1}^{\infty}(\frac{2}{T_s}\cos n\omega_s t + 0)$$

Substitute δ(t) in equation 1.

$$\rightarrow y(t) = x(t).\delta(t)$$

$$= x(t)[\frac{1}{T_s} + \Sigma_{n=1}^{\infty}(\frac{2}{T_s}\cos n\omega_s t)]$$

$$= \frac{1}{T_s}[x(t) + 2\Sigma_{n=1}^{\infty}(\cos n\omega_s t)x(t)]$$

$$y(t) = \frac{1}{T_s}[x(t) + 2\cos\omega_s t.\,x(t) + 2\cos 2\omega_s t.\,x(t) + 2\cos 3\omega_s t\ .\,x(t)\ldots\ldots]$$

Taking FOURIER TRANSFORM ON BOTH SIDES

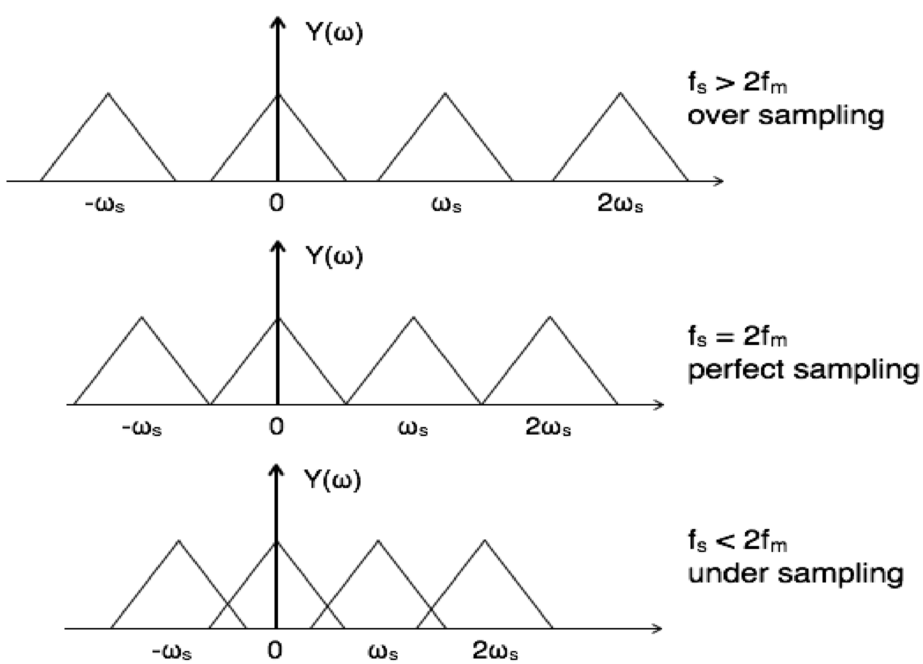
$$Y(\omega) = \frac{1}{T_s}[X(\omega) + X(\omega - \omega_s) + X(\omega + \omega_s) + X(\omega - 2\omega_s)$$

$$+ X(\omega + 2\omega_s) + \dots]$$

$$\therefore\ Y(\omega) = \frac{1}{T_s}\Sigma_{n=-\infty}^{\infty} X(\omega - n\omega_s) \qquad where\ n = 0, \pm 1, \pm 2, \dots$$

To reconstruct x(t), you must recover input signal spectrum X(ω) from sampled signal spectrum Y(ω), which is possible when there is no overlapping between the cycles of Y(ω).

Possibility of sampled frequency spectrum with different conditions is given by the following diagrams:



Aliasing Effect

The overlapped region in case of under sampling represents aliasing effect, which can be removed by

- considering $f_s > 2f_m$
- By using anti-aliasing filters

Nyquist Rate

It is the minimum sampling rate at which signal can be converted into samples and can be recovered back without distortion.

Nyquist rate $f_N = 2f_m$ hz

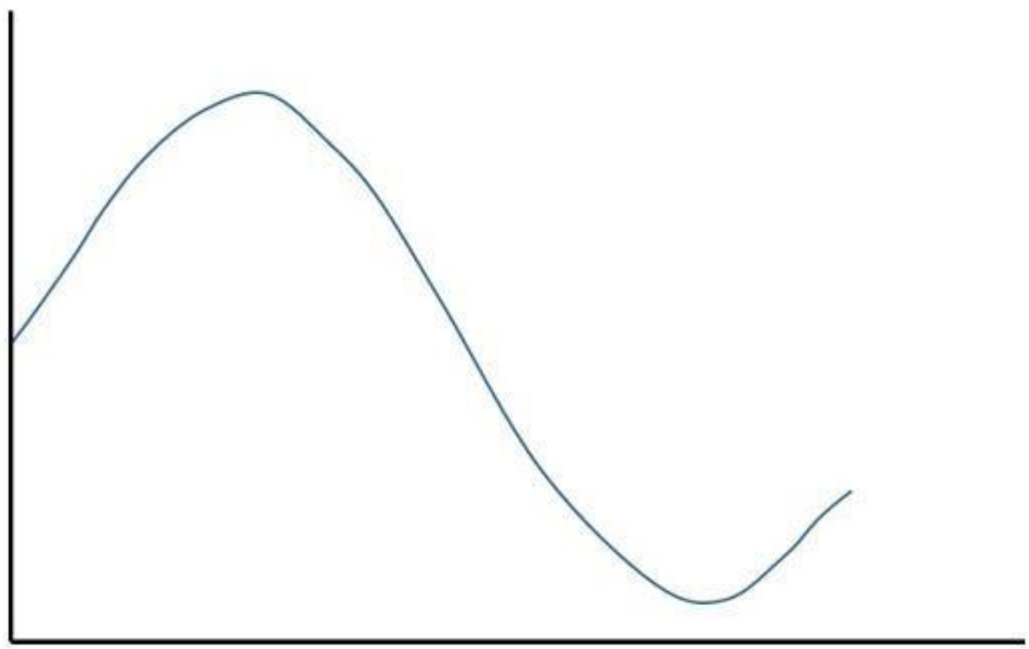
Nyquist interval = $\frac{1}{fN} = \frac{1}{2fm}$seconds

QUANTIZATION

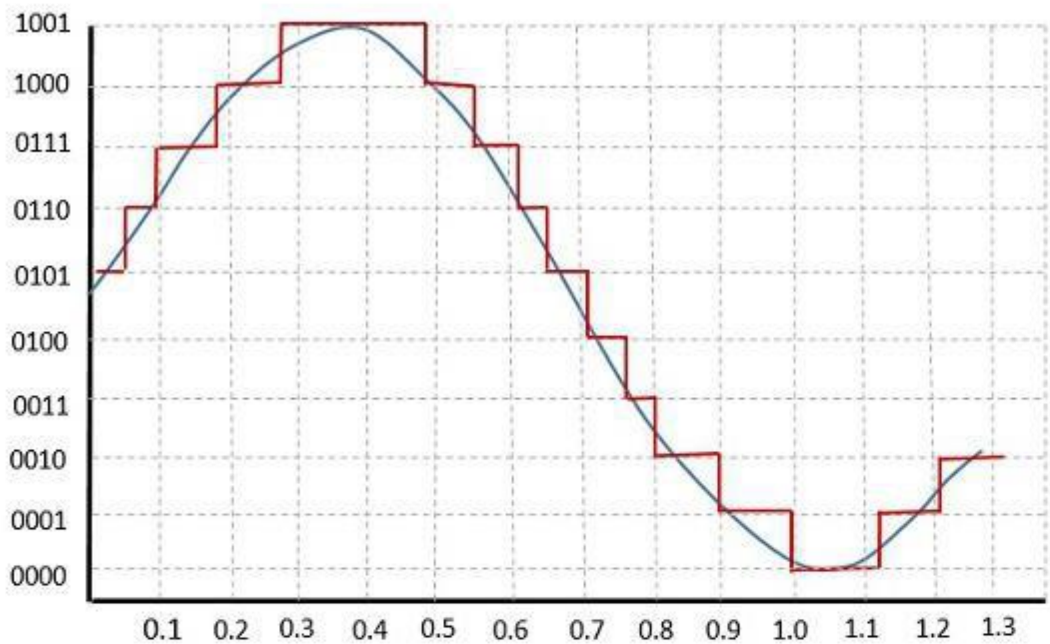
The digitization of analog signals involves the rounding off of the values which are approximately equal to the analog values. The method of sampling chooses a few points on the analog signal and then these points are joined to round off the value to a near stabilized value. Such a process is called as Quantization.

Quantizing an Analog Signal

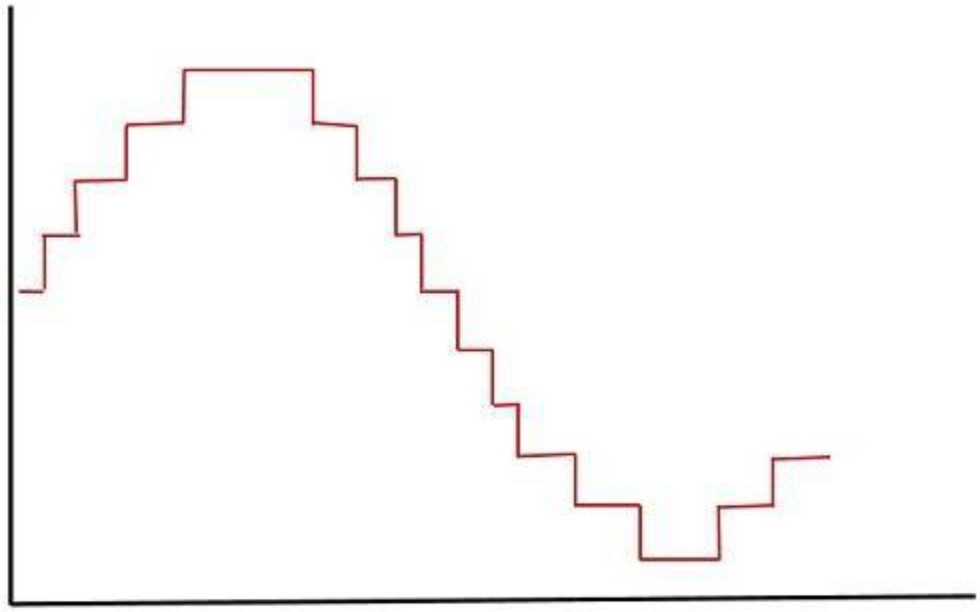
The analog-to-digital converters perform this type of function to create a series of digital values out of the given analog signal. The following figure represents an analog signal. This signal to get converted into digital, has to undergo sampling and quantizing.



The quantizing of an analog signal is done by discretizing the signal with a number of quantization levels. **Quantization** is representing the sampled values of the amplitude by a finite set of levels, which means converting a continuous-amplitude sample into a discrete-time signal.The following figure shows how an analog signal gets quantized. The blue line represents analog signal while the brown one represents the quantized signal.



Both sampling and quantization result in the loss of information. The quality of a Quantizer output depends upon the number of quantization levels used. The discrete amplitudes of the quantized output are called as **representation levels** or **reconstruction levels**. The spacing between the two adjacent representation levels is called a **quantum** or **step-size**.The following figure shows the resultant quantized signal which is the digital form for the given analog signal.



This is also called as **Stair-case** waveform, in accordance with its shape.

Types of Quantization

There are two types of Quantization - Uniform Quantization and Non-uniform Quantization.

The type of quantization in which the quantization levels are uniformly spaced is termed as a **Uniform Quantization**. The type of quantization in which the quantization levels are unequal and mostly the relation between them is logarithmic, is termed as a **Non-uniform Quantization**.

There are two types of uniform quantization. They are Mid-Rise type and Mid-Tread type. The following figures represent the two types of uniform quantization.

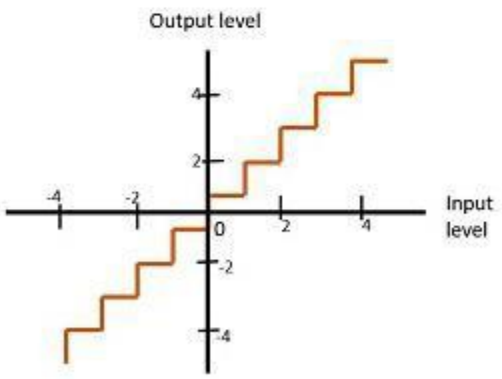


Fig 1 : Mid-Rise type Uniform Quantization

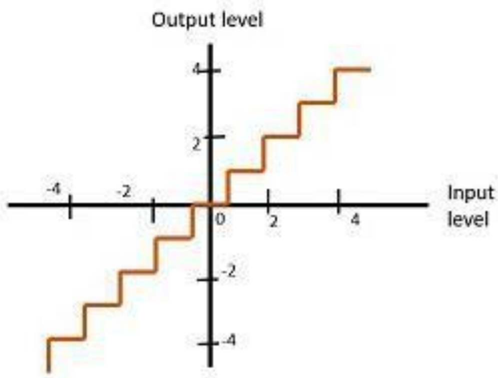


Fig 2 : Mid-Tread type Uniform Quantization

Figure 1 shows the mid-rise type and figure 2 shows the mid-tread type of uniform quantization.

- The **Mid-Rise** type is so called because the origin lies in the middle of a raising part of the stair-case like graph. The quantization levels in this type are even in number.

- The **Mid-tread** type is so called because the origin lies in the middle of a tread of the stair-case like graph. The quantization levels in this type are odd in number.

- Both the mid-rise and mid-tread type of uniform quantizers are symmetric about the origin.