UNIT IV

QUANTIZATION

Quantization

The process of transforming Sampled amplitude values of a message signal into a discrete amplitude value is referred to as Quantization. The quantization Process has a two-fold effect:

1. The peak-to-peak range of the input sample values is subdivided into a finite set of decision levels or decision thresholds that are aligned with the risers of the staircase, and

2. The output is assigned a discrete value selected from a finite set of representation levels that are aligned with the treads of the staircase.

A Quantizer is memory less and the Quantizer output is determined only by the value of a corresponding input sample, independently of earlier analog samples applied to the input.

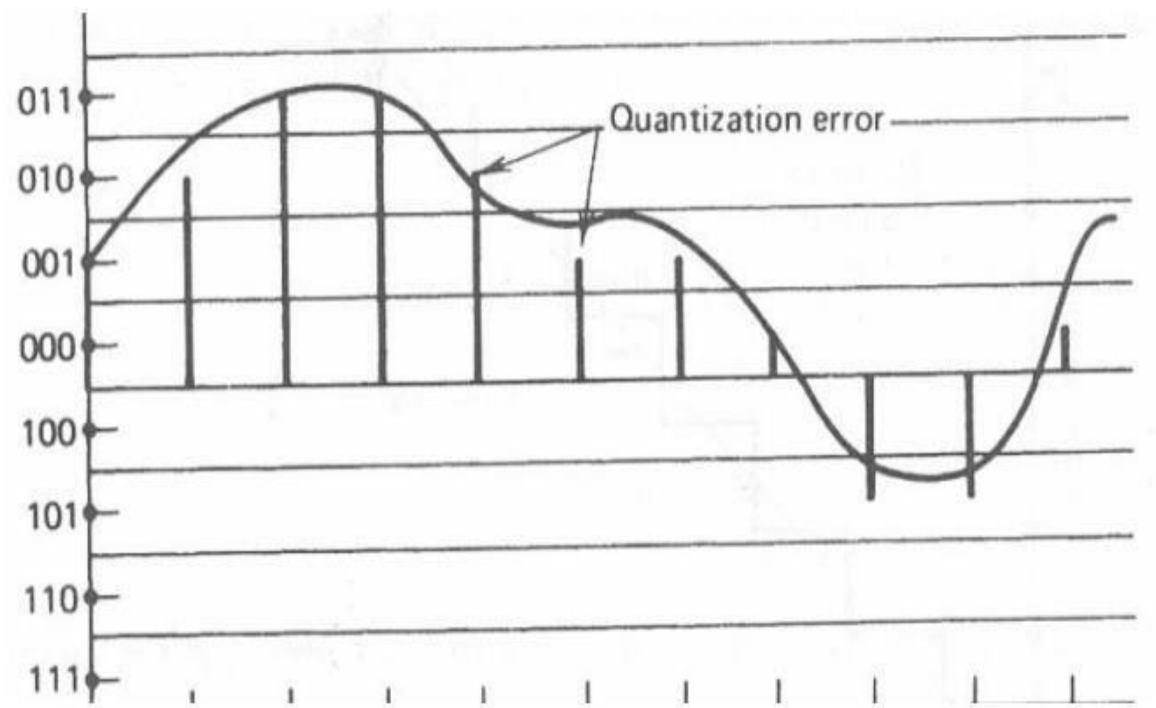

Fig 5.2.1 Quantization Process

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Types of Quantizers

1. Uniform Quantizer
2. Non- Uniform Quantizer

Uniform Quantizer: In Uniform type, the quantization levels are uniformly spaced, whereas in non-uniform type the spacing between the levels will be unequal and mostly the relation is logarithmic.

Types of Uniform Quantizers: (based on I/P - O/P Characteristics)

1. Mid-Rise type Quantizer
2. Mid-Tread type Quantizer

In the stair case like graph, the origin lies the middle of the tread portion in Mid –Tread type where as the origin lies in the middle of the rise portion in the Mid-Rise type.

Mid – tread type:Quantization levels – odd number.

Mid – Rise type: Quantization levels – even number.

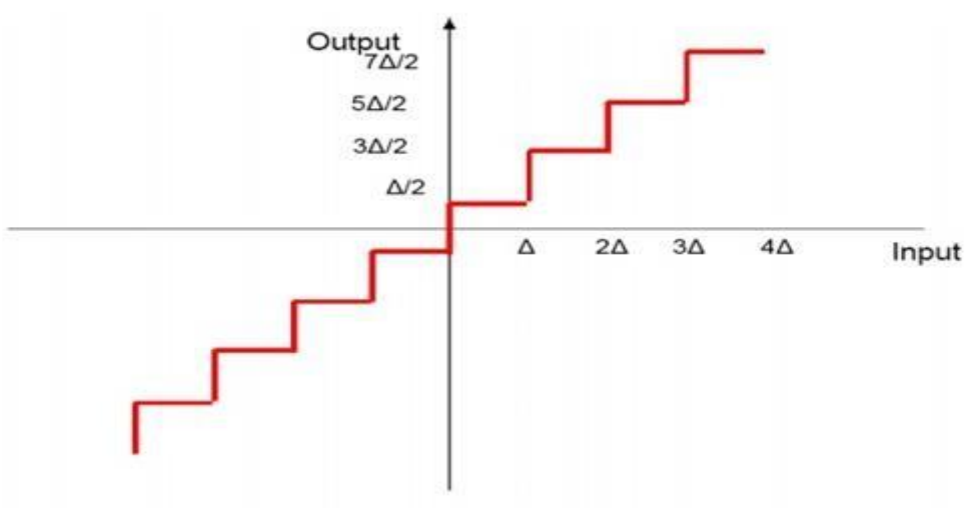

Figure 5.2.2 Input Output Characteristics of Mid Rise Quantizer

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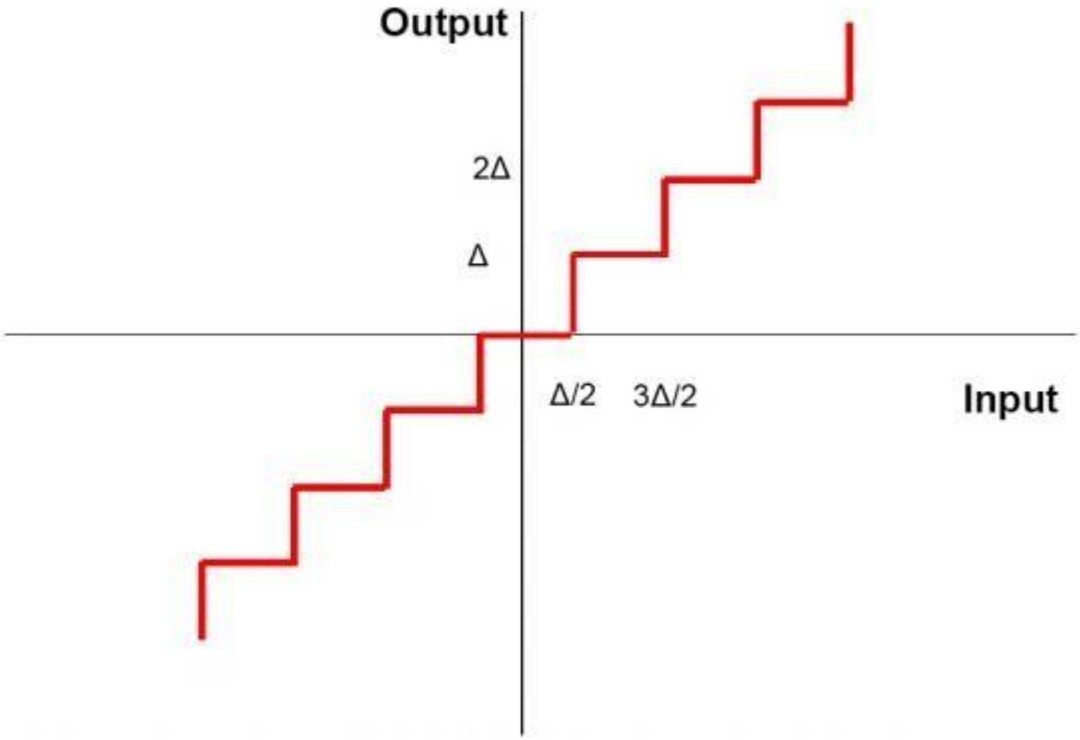

Figure 5.2.3 Input Output Characteristics of Mid Thread Quantizer

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Quantization Noise and Signal-to-Noise

"The Quantization process introduces an error defined as the difference between the input signal, x(t) and the output signal, yt). This error is called the Quantization Noise."

$$q(t) = x(t) – y(t)$$

Quantization noise is produced in the transmitter end of a PCM system by rounding off sample values of an analog base-band signal to the nearest permissible representation levels of the quantizer. As such quantization noise differs from channel noise in that it is signal dependent. Let "Δ‟ be the step size of a quantizer and L be the total number of quantization levels. Quantization levels are 0, ± ., ± 2 ., ±3 ....The Quantization error, Q is a random variable and will have its sample values bounded by [-(Δ/2) < q < (Δ/2)]. If is small, the quantization error can be assumed to a uniformly distributed random variable. Consider a memory less quantizer that is both uniform and symmetric.

L = Number of quantization levels

X = Quantizer input

Y = Quantizer output

The output y is given by

$$Y=Q(x)$$

which is a staircase function that befits the type of mid tread or mid riser quantizer of interest.

Companding of Speech signal

Compander = Compressor + Expander

In Non - Uniform Quantizer the step size varies. The use of a non – uniform quantizer is equivalent to passing the baseband signal through a compressor and then applying the compressed signal to a uniform quantizer. The resultant signal is then transmitted.

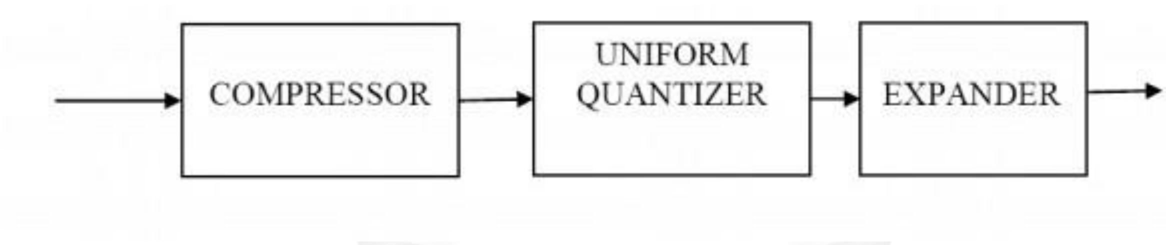

Figure 5.2.3 Model of Non Uniform Quantizer

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At the receiver, a device with a characteristic complementary to the compressor called Expander is used to restore the signal samples to their correct relative level. The Compressor and expander take together constitute a Compander.

- The quantization of the analog form of the signal to discrete form takes place in Quantizer. The sampled analog signal is still analog because though discrete in time the signal amplitude can take any value as it may wish

- The Quantizer forces the signal to takes same discrete values form the continuous amplitude values.
- From the sampled signal $m_s(t)$ new quantized signal $m_q(t)$ is created .
- Whereas it can take any value ,but $m_q(t)$ can take only L discrete values.

$$Vpp = L * q$$

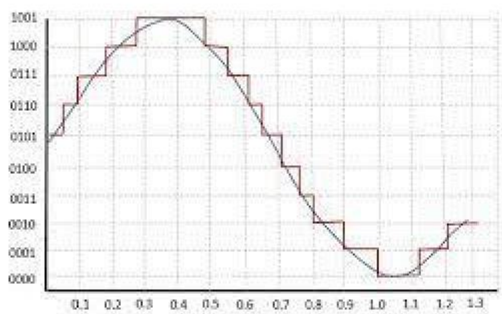

Figure 5.2.4 a Analog signal Quantization

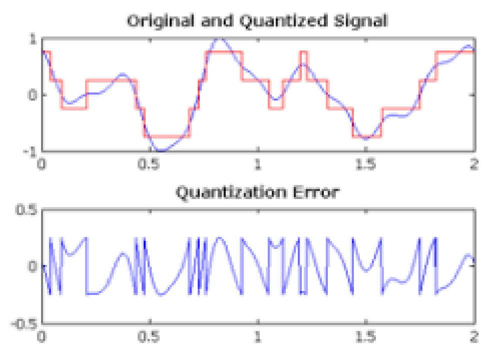

Figure 5.2.4 b Quantization Error

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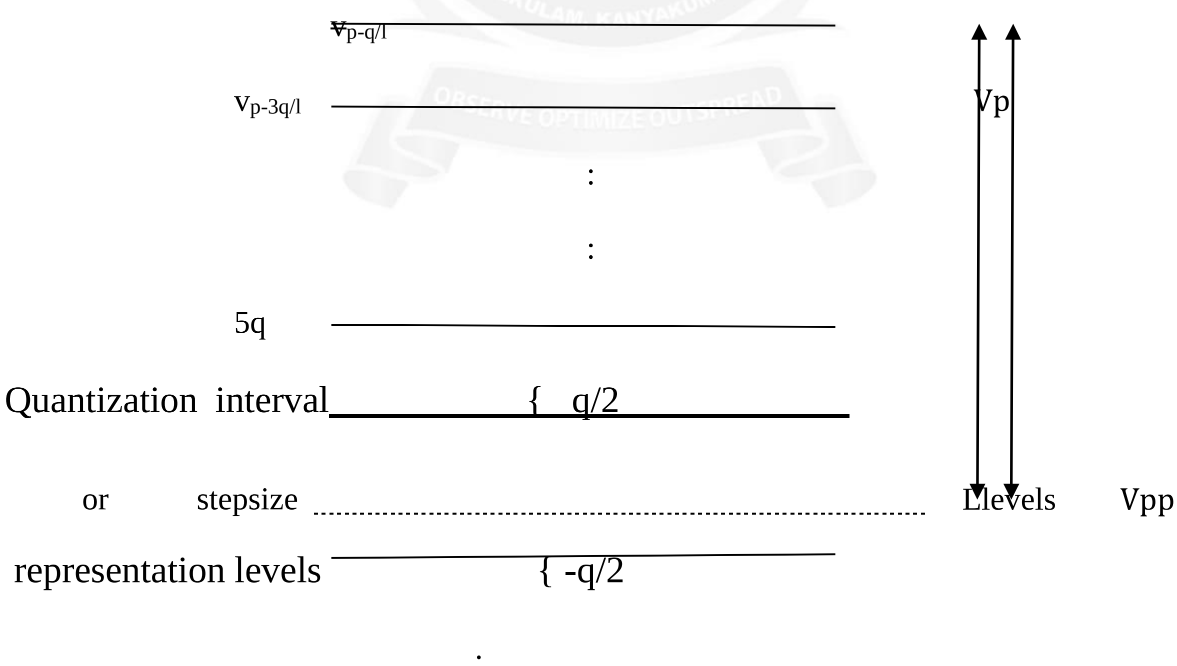

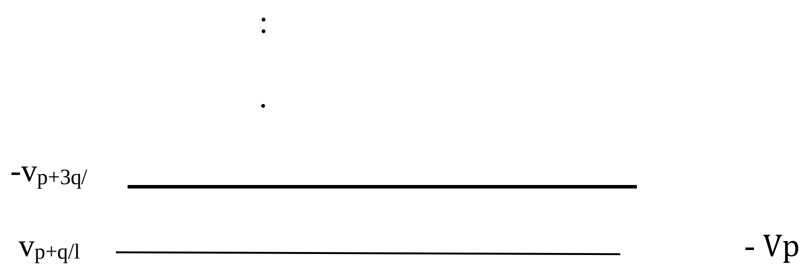

Figure 5.2.5 Signal Quantization with representation levels

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Figure shows the L level linear Quantizer for analog sampled signal $m_s(t)$ for an analog sampled signal $m_s(t)$, Vpp -→ peak to peak voltage range

Vpp=Vp–(-Vp)

Vpp=2Vp volts

The quantized levels levels may assume positive and negative values. The step size between quantization levels are called quantized interval denoted q volts . If the quantization levels are uniformly distributed over the full range ten the Quantizer is called a uniform Quantizer each sample value of the analog waveform is approximated to a Quantizer level.

This approximation process gives rise to an error quantization noise, given by

$e_q(t)=m(t)-m_q(t)$

maximum error in quantization =1/2 q ie)half of the quantile interval ie) ±q/2 volts, let f(m) be the pdf of the Quantizer input signal m(t) in the range ±vp the mean1/square error

$$\overline{eq^2}=\int_{m1-q/2}^{m1+q2} f(m)(m-m1)^2+\int_{m2-q/2}^{m2+q2} f(m)(m-m2)^2+........\int_{mL-q/2}^{mL+q2} f(m)(m_L)^2$$

For large L the quantile interval q becomes quite small .so that assume f(m) to be constant in an interval .without committing any significant error, this constant value may be taken to be the value of f(m) at the quantized values mi in the I the interval

Hence the noise expression becomes,

$$\overline{eq^2}=(f^{I}+f^{II}+.....f^{L})\int_{-q/2}^{q/2} x^2\,dx$$

$$\overline{eq^2} = (f^I + f^{II} + \ldots\ldots f^L)q^3/12$$

$$\overline{eq^2} = (f^I + f^{II} + \ldots\ldots f^L)q^2/12$$

Now are the respective probabilities that when m(t) at the I st , IInd ………… L th quantile interval 1/3 For large L the RMS inside the bracket approximately gives the area under the PDF, curve ,So

$$(f_q^I + f_q^{II} + \ldots\ldots f_q^L) = 1$$

So the value the quantization noise power becomes

$$\overline{eq^2} = q^2/12$$

So it is clear that quantization noise increases with increase in the size of quantile interval.
The uniform quantization in quantile interval is given by

$$Q = Vpp/L - 1$$

In most practical Quantizer L is indeed large. so e increase no of levels, so the quantization noise decreases. The peak signal power of the unquantised signal is

$$Vp^2 = (\frac{Vpp}{2})^2)$$

$$\text{Noise power} = q^2/12$$

Noise power=signal power/noise power

$$SNR = 3L^2$$

Non uniform quantization

The step size is not fixed .it values according to the input amplitude

Case 1

The step size is small at low input signal levels. Hence the quantization error is also small the quantization noise power ratio is also improved at low signal levels.

Case 2

The step size is higher at high input signal levels .here the signal to noise power ratio remains almost same throughout the range of quantizer.

Necessity of non-uniform quantization

In uniform quantization the step size remains same throughout the range of Quantizer

For low signal amplitude the maximum quantization error is quite high ,but for high signal amplitude the maximum quantization error is small. This problem arises because of uniform quantization .,

Speech and music signals are characterized by large crest factor .ie) for such signal the ratio of peak to RMS value is very high,

Crest factor =Peak value/RMS value

Quantization noise is directly related to step size

P-→normalised signal power

if we normalize the signal x(t) then xmax-=1

crest factor $=1/\sqrt{p}$

The round of process in the quantization introduces an error. The difference between input signal m(t) and output signal mq(t) is called as quantization error.

$$eq(t) = m(t) - mq(t)$$

Consider an input m of continuous amplitude in the range of $Vpp = Vp - (-Vp) = 2Vp\ volts.$

The uniform Quantizer of the mid riser type

The total amplitude range=$Mmax - (-Mmax) = 2\,Vp$

The step size $\Delta = 2Vp/L$

If Δ is small the number of representation levels 'L' is sufficiently large

The probability during function of the quantization error Q as

$$fQ(t) = \{1/2 \quad \frac{\Delta}{2} < q < \frac{\Delta}{2} \quad \}$$
$$0 \quad oterwise$$

The variance $\sigma_Q{}^2 = E[Q^2]$=$1/\Delta \int_{-\Delta/2}^{\Delta/2} q^2 dq$

$=1/\Delta[q^3{}_{/3-\Delta/2}^{\Delta/2}$

$=\Delta^2/12$

Let R denote number of bits per sample usd as the construction of binary code.

$$L = 2^R$$

The peak signal power

$$v_p^2 = L^2 D^2/4$$

Companding

The crest factor can be given by

$$Kcr = peak\ value/RMS\ value$$

Speech signal has high crest value. This implies that the speech signal are having amplitude values near zero. So if uniform Quantizer are used for speech most of the time quantizes will be more Than the signal slope This is known as granular noise One solution is tapering quantizing levels.ie) step size can be made smaller signals and larger for larger signals But this type of Quantizer with varying step is practically difficult to implement.

A most practical approach is to predictor the signal by a logarithmic compression and then put into an uniform Quantizer. This compressed and quantized signal is transmitted through the channel and then can be undistorted at the receiver by the logarithmic compression algorithm This process is known as companding A device which is used for companding is known as compander In a compander the true function has a larger slope for small signals and smaller slope for larger signals. Thus a given signal change at small magnitudes will carry uniform Quantizer through more steps. After compression the distorted signal is used as a input of a linear [uniform Quantizer] At the receiver an inverse of compression called expansion is applied so that over all transmission is not distorted This processing pair(compression and expansion)is called Companding. It is similar to pre emphasis and de emphasis in FM. This in amplitude domain and in frequency domain.

Advantages of Non- Uniform Quantization

1. Higher average signal to quantization noise power ratio than the uniform Quantizer when the signal pdf is non uniform which is the case in many practical situation.
2. RMS value of the Quantizer noise power of a non – uniform Quantizer is substantially proportional to the sampled value and hence the effect of the Quantizer noise is reduced.