STRUCTURE OF FIR FILTERS

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Structures of FIR Filters:

Explain with neat sketches the Structure of FIR filters. [Nov/Dec-2012]

The realization of FIR filter is given by

- Transversal structure.
- Linear phase realization
- Polyphase realization.

Transversal structure:

It contains two forms of realization such as,

- Direct form realization
- Cascade form realization.

Direct form realization:

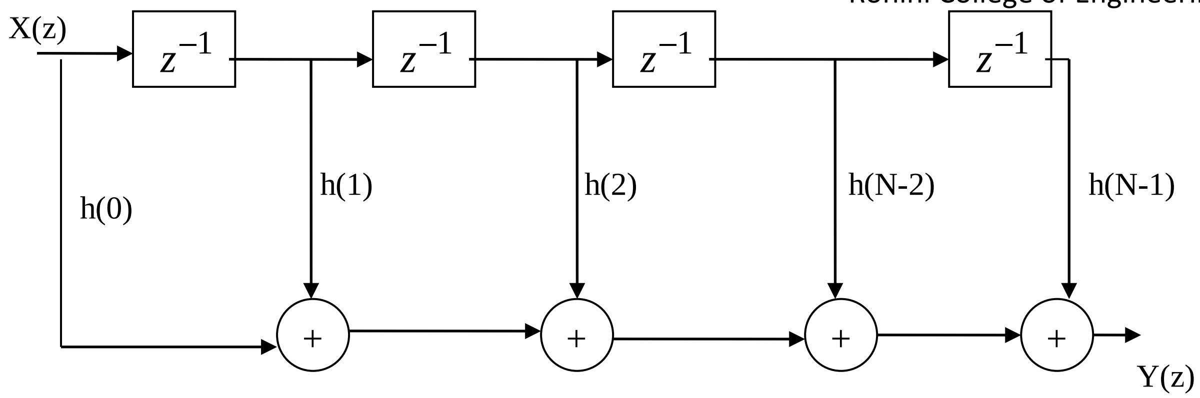
The system function of an FIR filter can be written as

$$H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-n}$$

$$= h(0) + h(1)\, z^{-1} + h(2)\, z^{-2} + ... + h(N-1)\, z^{-(N-1)} \qquad \text{eq(1)}$$

$$Y(z) = h(0)X(z) + h(1)z^{-1}X(z) + h(2)z^{-2}X(z) + ... + h(N-1)\, z^{-(N-1)}\, X(z) \qquad \text{eq(2)}$$

This structure is known as direct form realization. It requires N multipliers, N-1 adders, and N-1 delay elements.



Cascade Realization:

Problem 1: Determine the direct form Realization of the following system function. (Nov/Dec-14)

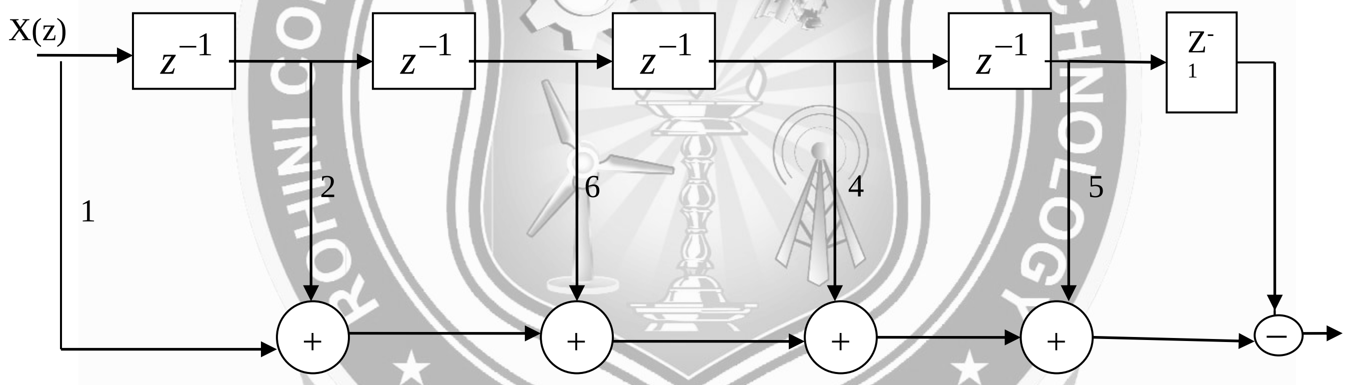
$H(z) = 1 + 2z^{-1} + 6z^{-2} + 4z^{-3} + 5z^{-4} + 8z^{-5}$

Solution:

Given: The system function is $H(z) = 1 + 2z^{-1} + 6z^{-2} + 4z^{-3} + 5z^{-4} + 8z^{-5}$

$$H(z) = \frac{Y(z)}{X(z)} = 1 + 2z^{-1} + 6z^{-2} + 4z^{-3} + 5z^{-4} + 8z^{-5}$$

$$Y(z) = X(z) + 2z^{-1}X(z) + 6z^{-2}X(z) + 4z^{-3}X(z) + 5z^{-4}X(z) + 8z^{-5}X(z)$$



Problem 2: Obtain the cascade realization of system function $H(z) = (1 + 2z^{-1} - z^{-2})(1 + z^{-1} - z^{-2})$ (May/June-12) (Nov/Dec-10)

Solution:

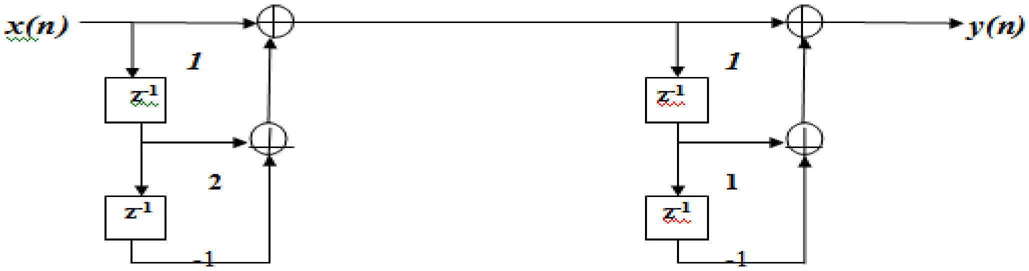
$$H(z) = H_1(z)\,H_2(z)$$

Where $H_1(z) = 1 + 2z^{-1} - z^{-2}$ and $H_2(z) = 1 + z^{-1} - z^{-2}$

$$H_1(z) = \frac{Y_1(z)}{X_1(z)} \Rightarrow Y_1(z) = X_1(z) + 2z^{-1}X_1(z) - z^{-2}X(z) \qquad \text{eq(1)}$$

$$H_2(z) = \frac{Y_2(z)}{X_2(z)} \Rightarrow Y_2(z) = X_2(z) + z^{-1}X_2(z) - z^{-2}X(z) \qquad \text{eq(2)}$$

The equation (1) and equation (2) can be realized in direct form and can be cascaded as shown in figure.



H.W 1 : Obtain the direct form realization for the following system function.

1. $H(z) = 1 + 2z^{-1} - 3z^{-2} - 4z^{-3} + 5z^{-4}$

2. $H(z) = \left(1 - \frac{1}{4}z^{-1} + \frac{3}{8}z^{-2}\right)\left(1 - \frac{1}{8}z^{-1} - \frac{1}{2}z^{-2}\right)$

H.W 2: Obtain the cascade form realization for the following system function.

1. $H(z) = 1 + \frac{5}{2}z^{-1} + 2z^{-2} + 2z^{-3}$

2. $H(z) = (1 + 2z^{-1})\left(1 + \frac{1}{2}z^{-1} + z^{-2}\right)$

Obtain the linear phase realization of the system function. [Nov/Dec-10]

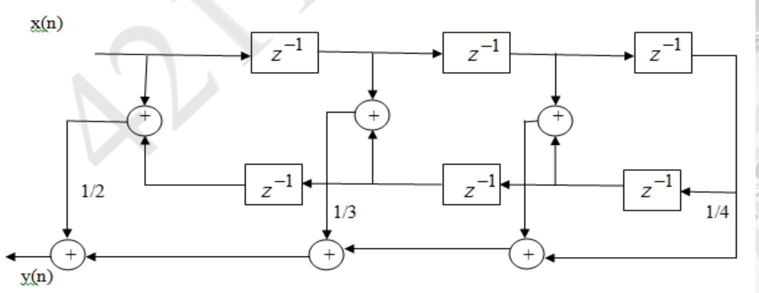
$$H(z) = \frac{1}{2} + \frac{1}{3}z^{-1} + z^{-2} + \frac{1}{4}z^{-3} + z^{-4} + \frac{1}{3}z^{-5} + \frac{1}{2}z^{-6}$$

Solution:

By inspection we find system function H(z) is that of a linear phase FIR filter and,

h(n)=h(N-1-n)

Therefore, we can realize the system function as shown in Figure.



Lattice Structure:

The lattice structure formulas are,

$$\alpha_m(0) = 1$$
$$\alpha_m(m) = k_m$$
$$\alpha_m(k) = \alpha_{m-1}(k) + \alpha_m(m)\alpha_{m-1}(m-1)$$

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Consider an FIR lattice filter with co-efficients $K_1 = \frac{1}{2}; K_2 = \frac{1}{3}; K_3 = \frac{1}{4}$. Determine the FIR fil the direct form structure. [Nov/Dec-2013] [Nov/Dec-2015]

Solution:

Given: The FIR lattice filter with co-efficients are $K_1 = \frac{1}{2}; K_2 = \frac{1}{3}; K_3 = \frac{1}{4}$

$$\alpha_3(0) = 1; \quad \alpha_3(3) = K_3 = \frac{1}{4}$$

$$\alpha_2(2) = K_2 = \frac{1}{3}; \alpha_1(1) = K_1 = \frac{1}{2}$$

We know,

$$\alpha_m(k) = \alpha_{m-1}(k) + \alpha_m(m)\alpha_{m-1}(m-1)$$

For m=2 and K=1

$$\alpha_2(1) = \alpha_1(1) + \alpha_2(2)\alpha_1(1)$$
$$= \frac{1}{2} + \frac{1}{3}.\frac{1}{2} = \frac{2}{3}$$

For m=3 and K=1

$$\alpha_3(1) = \alpha_2(1) + \alpha_3(3)\alpha_2(2)$$

$$= \frac{2}{3} + \frac{1}{4}.\frac{1}{3} = \frac{3}{4}$$

For m=3 and K=2

$$\alpha_3(2) = \alpha_2(2) + \alpha_3(3)\alpha_2(1)$$

$$= \frac{1}{3} + \frac{1}{4}.\frac{2}{3} = \frac{1}{2}$$

$\therefore$ *The lattice filter coeficients are* $\alpha_3(0) = 1; \alpha_3(1) = \frac{3}{4}; \alpha_3(2) = \frac{1}{2}; \alpha_3(3) = \frac{1}{4}$

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