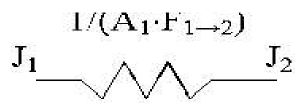
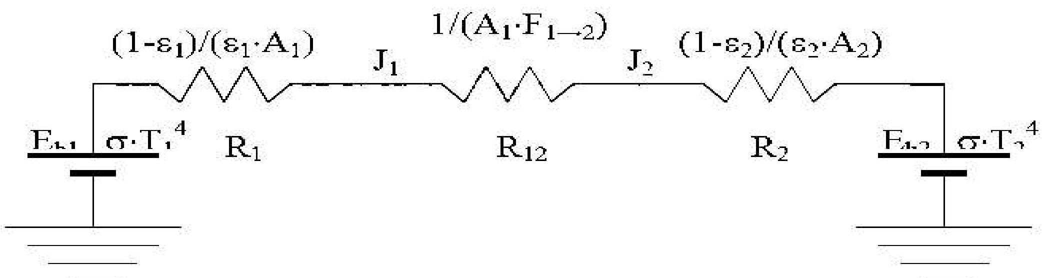
4.24 Solution of Analogous Electrical Circuits.

• Large Enclosures

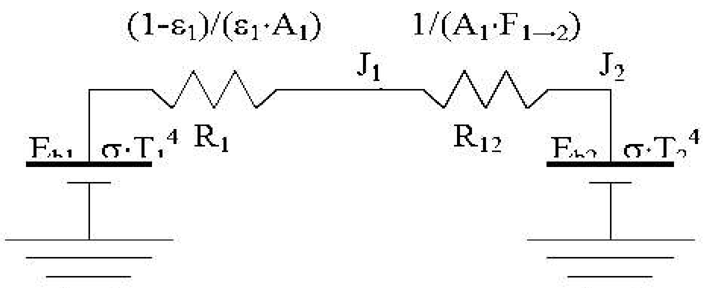
Consider the case of an object, 1, placed inside a large enclosure, 2. The system will consist of two objects, so we proceed to construct a circuit with two radiosity nodes



Now we ground both Radiosity nodes through a surface resistance.



Since A_2 is large, $R_2 = 0$. The view factor, $F_{1\to2} = 1$



Sum the series resistances:

$$R_{Series} = (1-\varepsilon_1)/(\varepsilon_1 \cdot A_1) + 1/A_1 = 1/(\varepsilon_1 \cdot A_1)$$

Ohm's law:

$$i = \Delta V/R$$

or by analogy:

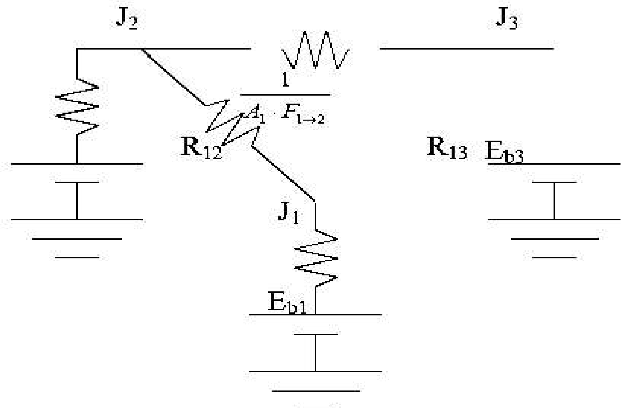
$$q = \Delta E_b/R_{Series} = \varepsilon_1 \cdot A_1 \cdot \sigma \cdot (T_1^4 - T_2^4)$$

You may recall this result from Thermo I, where it was introduced to solve this type of radiation problem.

- Networks with Multiple Potentials

Systems with 3 or more grounded potentials will require a slightly different solution, but one which students have previously encountered in the Circuits course.

The procedure will be to apply Kirchoff's law to each of the Radiosity junctions.



$$\sum_{i=1}^{3} q_i = 0$$

In this example there are three junctions, so we will obtain three equations. This will allow us to solve for three unknowns.

Radiation problems will generally be presented on one of two ways:

1. The surface net heat flow is given and the surface temperature is to be found.

2. The surface temperature is given and the net heat flow is to be found.

Returning for a moment to the coal grate furnace, let us assume that we know (a) the total heat being produced by the coal bed, (b) the temperatures of the water walls and (c) the temperature of the super heater sections.

Apply Kirchoff's law about node 1, for the coal bed:

$$q_1 + q_{2\to1} + q_{3\to1} = q_1 + \frac{J_2 - J_1}{R_{12}} + \frac{J_3 - J_1}{R_{13}} = 0$$

Similarly, for node 2:

$$q_2 + q_{1\to2} + q_{3\to2} = \frac{E_{b2} - J_2}{R_2} + \frac{J_1 - J_2}{R_{12}} + \frac{J_3 - J_2}{R_{23}} = 0$$

(Note how node 1, with a specified heat input, is handled differently than node 2, with a specified temperature.

And for node 3:

$$q_3 + q_{1\to 3} + q_{2\to 3} = \frac{E_{b3} - J_3}{R_3} + \frac{J_1 - J_3}{R_{13}} + \frac{J_2 - J_3}{R_{23}} = 0$$

The three equations must be solved simultaneously. Since they are each linear in J, matrix methods may be used:

$$\begin{bmatrix} -\frac{1}{R_{12}} - \frac{1}{R_{13}} & \frac{1}{R_{12}} & \frac{1}{R_{13}} \\ \frac{1}{R_{12}} & -\frac{1}{R_2} - \frac{1}{R_{12}} - \frac{1}{R_{13}} & \frac{1}{R_{23}} \\ \frac{1}{R_{13}} & \frac{1}{R_{23}} & -\frac{1}{R_3} - \frac{1}{R_{13}} - \frac{1}{R_{23}} \end{bmatrix} \cdot \begin{bmatrix} J_1 \\ J_2 \\ J_3 \end{bmatrix} = \begin{bmatrix} -q_1 \\ -\frac{E_{b2}}{R_2} \\ -\frac{E_{b3}}{R_3} \end{bmatrix}$$

The matrix may be solved for the individual Radiosity. Once these are known, we return to the electrical analogy to find the temperature of surface 1, and the heat flows to surfaces 2 and 3.

Surface 1: Find the coal bed temperature, given the heat flow:

$$q_1 = \frac{E_{b1} - J_1}{R_1} = \frac{\sigma \cdot T_1^4 - J_1}{R_1} \Rightarrow T_1 = \left[\frac{q_1 \cdot R_1 + J_1}{\sigma}\right]^{0.25}$$

Surface 2: Find the water wall heat input, given the water wall temperature:

$$q_2 = \frac{E_{b2} - J_2}{R_2} = \frac{\sigma \cdot T_2^4 - J_2}{R_2}$$

Surface 3: (Similar to surface 2) Find the water wall heat input, given the water wall temperature:

$$q_3 = \frac{E_{b3} - J_3}{R_3} = \frac{\sigma \cdot T_3^4 - J_3}{R_3}$$

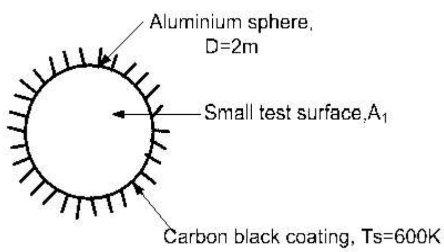
Module 9: Worked out problems

1. A spherical aluminum shell of inside diameter D=2m is evacuated and is used as a radiation test chamber. If the inner surface is coated with carbon black and maintained at 600K, what is the irradiation on a small test surface placed in the chamber? If the inner surface were not coated and maintained at 600K, what would the irradiation test?

Known: Evacuated, aluminum shell of inside diameter D=2m, serving as a radiation test chamber.

Find: Irradiation on a small test object when the inner surface is lined with carbon black and maintained at 600K.what effect will surface coating have?

Schematic:



Assumptions: (1) Sphere walls are isothermal, (2) Test surface area is small compared to the enclosure surface.

Analysis: It follows from the discussion that this isothermal sphere is an enclosure behaving as a black body. For such a condition, the irradiation on a small surface within the enclosure is equal to the black body emissive power at the temperature of the enclosure. That is

$$G_1 = E_b(T_s) = \sigma T_s^4$$

$$G_1 = 5.67 \times 10^{-8} W/m^2.K(600K)^4 = 7348 W/m^2$$

The irradiation is independent of the nature of the enclosure surface coating properties.

Comments: (1) The irradiation depends only upon the enclosure surface temperature and is independent of the enclosure surface properties.

(2) Note that the test surface area must be small compared to the enclosure surface area. This allows for inter-reflections to occur such that the radiation field, within the enclosure will be uniform (diffuse) or isotropic.

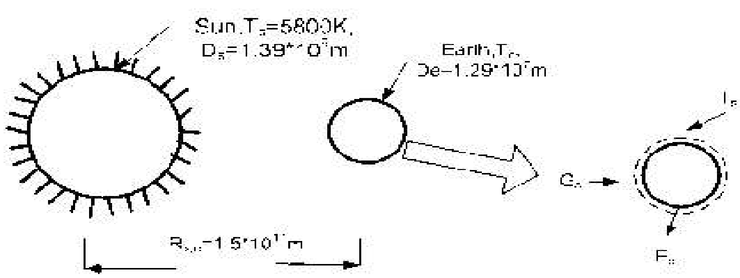
(3) The irradiation level would be the same if the enclosure were not evacuated since; in general, air would be a non-participating medium.

2 Assuming the earth's surface is black, estimate its temperature if the sun has an equivalently blackbody temperature of 5800K.The diameters of the sun and earth are 1.39*109 and 1.29*107m, respectively, and the distance between the sun and earth is 1.5*1011m.

Known: sun has an equivalently blackbody temperature of 5800K. Diameters of the sun and earth as well as separation distances are prescribed.

Find: Temperature of the earth assuming the earth is black.

Schematic:



Assumptions: (1) Sun and earth emit black bodies, (2) No attenuation of solar irradiation enroute to earth, and (3) Earth atmosphere has no effect on earth energy balance.

Analysis: performing an energy balance on the earth

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$A_{e,p} . G_S = A_{e,s} . E_b(T_e)$$

$$(\pi D_e^2 / 4) G_S = \pi D_e^2 \sigma T_e^4$$

$$T_e = \left(G_S / 4\sigma\right)^{1/4}$$

Where $A_{s,p}$ and $A_{e,s}$ are the projected area and total surface area of the earth, respectively. To determine the irradiation GS at the earth's surface, perform an energy bounded by the spherical surface shown in sketch

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$\pi D_S^2 . \sigma T_S^4 = 4\pi [R_{S,e} - D_e / 2]^2 G_S$$

$$\pi (1.39 \times 10^9 m)^2 \times 5.67 \times 10^{-8} W / m^2 . K (5800 K)^4 =$$

$$4\pi [1.5 \times 10^{11} - 1.29 \times 10^7 / 2]^2 m^2 \times G_S$$

$$G_S = 1377.5 W / m^2$$

Substituting numerical values, find

$$T_e = (1377.5 W / m^2 / 4 \times 5.67 \times 10^{-8} W / m^2 . K^4)^{1/4} = 279 K$$

Comments:

(1) The average earth's temperature is greater than 279 K since the effect of the atmosphere is to reduce the heat loss by radiation.

(2) Note carefully the different areas used in the earth energy balance. Emission occurs from the total spherical area, while solar irradiation is absorbed by the projected spherical area.

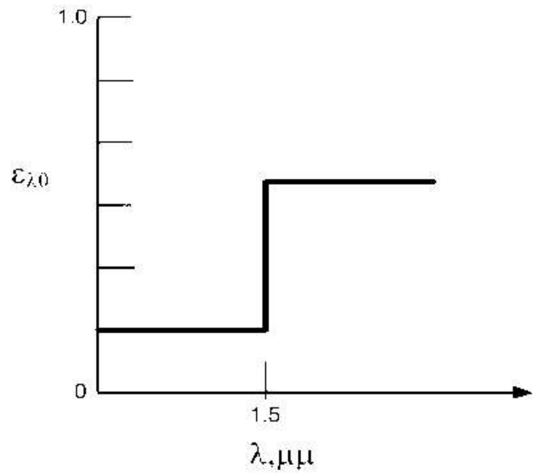
3 The spectral, directional emissivity of a diffuse material at 2000K has the following distribution.

Determine the total, hemispherical emissivity at 2000K.Determine the emissive power over the spherical range 0.8 t0 2.5 μm and for the directions 0≤θ≤30°.

Known: Spectral, directional emissivity of a diffuse material at 2000K.

Find: (1) The total, hemispherical emissivity, (b) emissive power over the spherical range 0.8 t0 2.5 μm and for the directions 0≤θ≤30°.

Schematic:



Assumptions: (1) Surface is diffuse emitter.

Analysis: (a) Since the surface is diffuse,ελ,θ is independent of direction; from Eq. $\varepsilon_{\lambda,\theta} = \varepsilon_\lambda$

$$\varepsilon(T) = \int_0^\infty \varepsilon_\lambda(\lambda) E_{\lambda,b}(\lambda,T) d\lambda / E_b(T)$$

$$\varepsilon(T) = \int_0^{1.5} \varepsilon_1) E_{\lambda,b}(\lambda,2000) d\lambda / E_b) + \int_0^{1.5} \varepsilon_2 E_{\lambda,b}(\lambda,2000) d\lambda / E_b$$

Written now in terms of F $_{(0\to\lambda)}$, with F $_{(0\to1.5)}$ =0.2732 at λT=1.5*2000=3000μm.K, find

$$\varepsilon(2000K) = \varepsilon_1 F_{(0\to1.5)} + \varepsilon_2 \left[1 - F_{(0\to1.5)}\right] = 0.2\times0.2732 + 0.8[1 - 0.2732] = 0.636$$

(b) For the prescribed spectral and geometric limits,

$$\Delta E = \int_{0.8}^{2.5} \int_0^{2\pi} \int_0^{\pi/6} \varepsilon_{\lambda,\theta} I_{\lambda,b}(\lambda,T) \cos\theta \sin\theta \, d\theta \, d\phi \, d\lambda$$

where $I_{\lambda,e}(\lambda,\theta,\phi) = \varepsilon_{\lambda,\theta} I_{\lambda,b}(\lambda,T)$. Since the surface is diffuse, $\varepsilon_{\lambda,\theta} = \varepsilon_{\lambda}$, and nothing $I_{\lambda,b}$ is independent of direction and equal to $E_{\lambda,b}/\pi$, we can write

$$\Delta E = \left\{ \int_0^{2\pi} \int_0^{\pi/6} \cos\theta \sin\theta \, d\theta \, d\phi \right\} \frac{E_b(T)}{\pi} \frac{\int_{0.8}^{1.5} \varepsilon_1 E_{\lambda,b}(\lambda,T) d\lambda}{E_b(T)} + \frac{\int_{1.5}^{2.5} \varepsilon_2 E_{\lambda,b}(\lambda,T) d\lambda}{E_b(T)}$$

Or in terms $F_{(0\to\lambda)}$ values,

$$\Delta E = \left\{ \phi \Big|_0^{2\pi} \times \frac{\sin^2\theta}{2} \Big|_0^{\pi/6} \right\} \frac{\sigma T^4}{\pi} \left\{ \varepsilon_1 [F_{(0\to1.5)} - F_{(0\to0.8)}] \right\} + \varepsilon_2 [F_{(0\to2.5)} - F_{(0\to1.5)}] \}$$

From table $\lambda T = 0.8 \times 2000 = 1600 \mu m.K$ $\quad F_{(0\to0.8)} = 0.0197$

$\lambda T = 2.5 \times 2000 = 5000 \mu m.K$ $\quad F_{(0\to2.5)} = 0.6337$

$$\Delta E = 2\pi \times \frac{\sin^2 \pi/6}{2} \frac{5.67\times10^{-8} 2000^4}{\pi} \frac{W}{m^2} \{0.2[0.2732 - 0.0197] + [0.80.6337 - 0.2732]\}$$

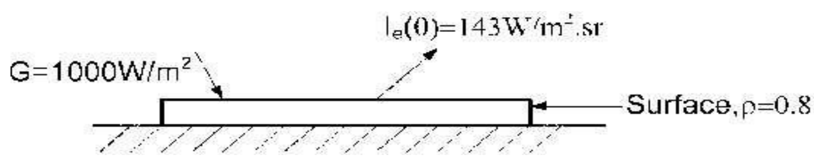
$$\Delta E = 0.25 \times (5.67\times10^{-8} \times 2000^4 W/m^2 \times 0.339 = 76.89 W/m^2$$

4. A diffusely emitting surface is exposed to a radiant source causing the irradiation on the surface to be $1000W/m^2$.The intensity for emission is $143W/m^2$.sr and the reflectivity of the surface is 0.8.Determine the emissive power ,E(W/m^2),and radiosity ,J(W/m^2),for the surface. What is the net heat flux to the surface by the radiation mode?

Known: A diffusely emitting surface with an intensity due to emission of Is=$143W/m^2$.sr and a reflectance ρ=0.8 is subjected to irradiation=$1000W/m^2$.

Find: (a) emissive power of the surface, E (W/m^2), (b) radiosity, J (W/m^2), for the surface, (c) net heat flux to the surface.

Schematic:



Assumptions: (1) surface emits in a diffuse manner.

Analysis: (a) For a diffusely emitting surface, $I_s(\theta) = I_e$ is a constant independent of direction. The emissive power is

$$E = \pi I_e = \pi sr \times 143 W/m^2.sr = 449 W/m^2$$

Note that π has units of steradians (sr).

(b) The radiosity is defined as the radiant flux leaving the surface by emission and reflection,

$$J = E + \rho G = 449 W/m^2 + 0.8 \times 1000 W/m^2 = 1249 W/m^2$$

(c) The net radiative heat flux to the surface is determined from a radiation balance on the surface.

$$q''_{net} = q''_{rad,in} - q''_{rad,out}$$

$$q''_{net} = G - J = 1000 W/m^2 - 1249 W/m^2 = -249 W/m^2$$

Comments: No matter how the surface is irradiated, the intensity of the reflected flux will be independent of direction, if the surface reflects diffusely.

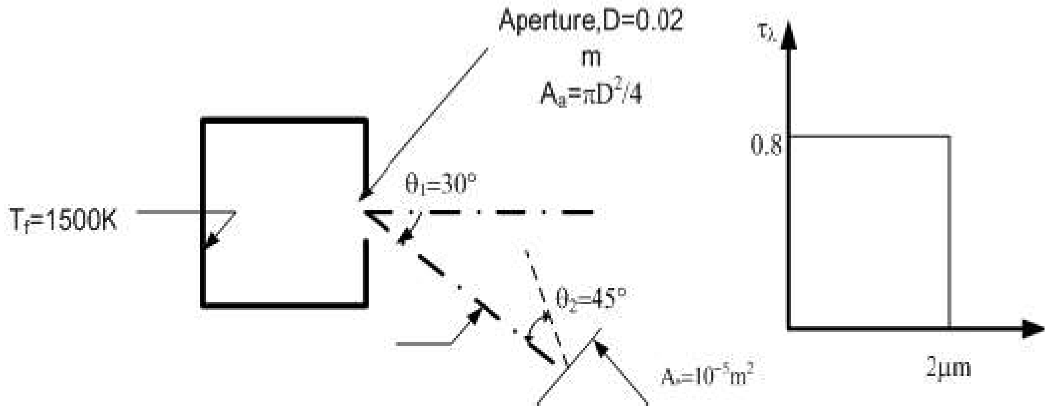
5. Radiation leaves the furnace of inside surface temperature 1500K through an aperture 20mm in diameter. A portion of the radiation is intercepted by a detector that is 1m from the aperture, as a surface area $10^{-5}m^2$, and is oriented as shown.

If the aperture is open, what is the rate at which radiation leaving the furnace is intercepted by the detector? If the aperture is covered with a diffuse, semitransparent material of spectral transmissivity $\tau\lambda=0.8$ for $\lambda \leq 2\mu m$ and $\tau\lambda=0$ for $\lambda > 2\mu m$, what is the rate at which radiation leaving the furnace is intercepted by the detector?

Known: Furnace wall temperature and aperture diameter. Distance of detector from aperture and orientation of detector relative to aperture.

Find: Rate at which radiation leaving the furnace is intercepted by the detector, (b) effect of aperture window of prescribed spectral transmissivity on the radiation interception rate.

Schematic:



Assumptions:

(1) Radiation emerging from aperture has characteristics of emission from a black body, (2) Cover material is diffuse, (3) Aperture and detector surface may be approximated as infinitesimally small.

Analysis: (a) the heat rate leaving the furnace aperture and intercepted by the detector is

$q = I_e A_s \cos\theta\, w_{a-a}$ Heat and Mass Transfer

$$I_e = \frac{E_b(T_f)}{\pi} = \frac{\sigma T_f^4}{\pi} = \frac{5.67\times10^{-8}(1500)^4}{\pi} = 9.14\times10^4 W/m^2.sr$$

$$w'_{s-a} = \frac{A''}{r^2} = \frac{A_s.\cos\theta^2}{r^2} = \frac{10^{-5}m^2\cos45°}{(1m)^2} = 0.70710^{-5}.sr$$

Hence

$$q = 9.14\times10^4 W/m^2.sr[\pi(0.02)m^2/4]\cos30°\times0.707\times10^{-5}sr = 1.76\times10^{-4}W$$

(b) With the window, the heat rate is

$$q = \tau(I_e A_a \cos\theta_1 w_{a-a})$$

where τ is the transmissivity of the window to radiation emitted by the furnace wall.

$$\tau = \frac{\int_0^\infty \tau_\lambda G_\lambda d_\lambda}{\int_0^\infty G_\lambda d_\lambda} = \frac{\int_0^\infty \tau_\lambda E_{\lambda,b}(T_f) d\lambda}{\int_0^\infty E_{\lambda,b} d\lambda} = 0.8\int_0^2 (E_{\lambda,b} / E_b) d\lambda = 0.8F_{(0\to 2\mu m)}$$

with $\lambda T = 2\mu m \times 1500K = 3000\mu m.K$, from table $F(0 \to 2\mu m) = 0.273$.

hence with $0.273 \times 0.8 = 0.218$, find

$$q = 0.218 \times 1.76 \times 10^{-4}\,W = 0.384 \times 10^{-4}\,W$$

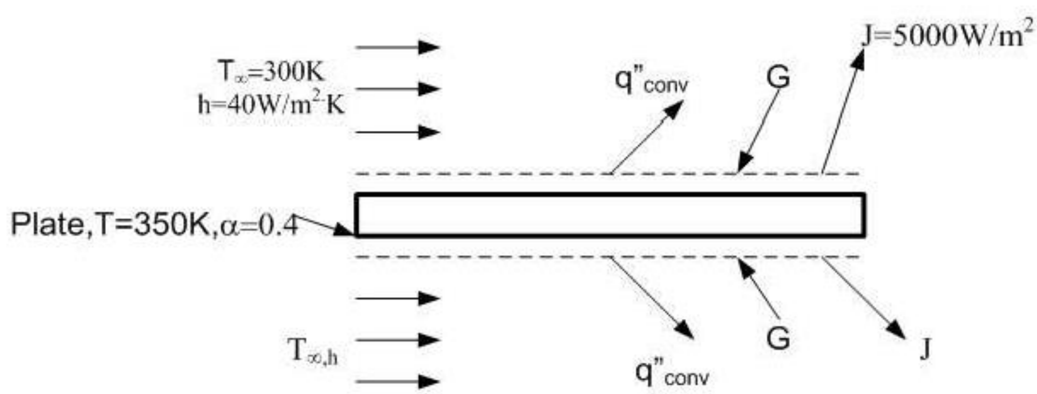
6.A horizontal semitransparent plate is uniformly irradiated from above and below, while air at T=300K flows over the top and bottom surfaces. providing a uniform convection heat transfer coefficient of h=40W/m2.K.the total, hemispherical absorptivity of the plate to the irradiation is 0.40.Under steady-state conditions measurements made with radiation detector above the top surface indicate a radiosity(which includes transmission, as well as reflection and emission) of J=5000W/m2,while the plate is at uniform temperature of T=350K

Determine the irradiation G and the total hemispherical emissivity of the plate. Is the plate gray for the prescribed conditions?

Known: Temperature, absorptivity, transmissivity, radiosity and convection conditions for a semi-transparent plate.

Find: Plate irradiation and total hemispherical emissivity.

Schematic:



Assumptions: From an energy balance on the plate

E_{in}- E_{out}

2G=2q"$_{conv}$+2J

Solving for the irradiation and substituting numerical values,

G=40W/m^2.K (350-300) K+5000W/m^2=7000W/m^2

From the definition of J

$$J = E + \rho G + \tau G = E + (1 - \alpha)G$$

Solving for the emissivity and substituting numerical values,

$$\varepsilon = \frac{J - (1 - \alpha)G}{\sigma T^4} = \frac{(5000\,W / m^2) - 0.6(7000\,W / m^2)}{5.67 \times 10^{-8}\,W / m^2 .K^4 (350\,K)^4} = 0.94$$

Hence

$$\alpha \neq \varepsilon$$

And the surface is not gray for the prescribed conditions.

Comments: The emissivity may also be determined by expressing the plate energy balance as

$$2\alpha G = 2q''_{conv}\,2E$$

hence

$$\varepsilon \sigma T^4 = \alpha G - h(T - T_\infty)$$

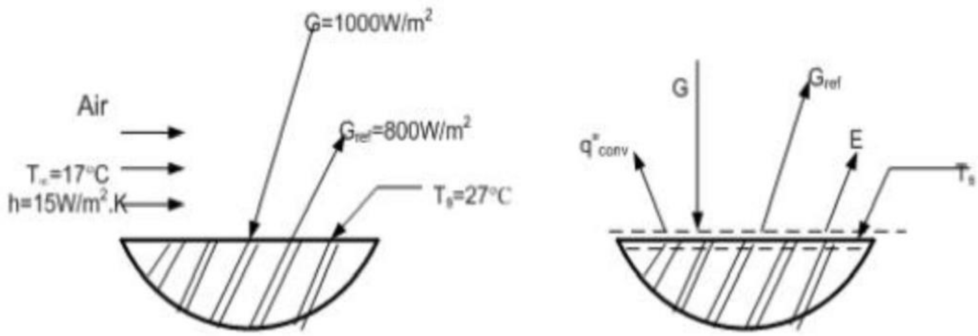
$$\varepsilon = \frac{0.4(7000W / m^2) - 40W / m^2 .K(50K)}{5.67 \times 10^{-8} W / m^2 .K4(350K)^4} = 0.94$$

7 An opaque, gray surface at 27°C is exposed to irradiation of 1000W/m2, and 800W/m2 is reflected. Air at 17°C flows over the surface and the heat transfer convection coefficient is 15W/m2.K.Determine the net heat flux from the surface.

Known: Opaque, gray surface at 27°C with prescribed irradiation, reflected flux and convection process.

Find: Net heat flux from the surface.

Schematic:



Assumptions:

1) Surface is opaque and gray,

2) Surface is diffuse,

3) Effects of surroundings are included in specified irradiation.

Analysis: From an energy balance on the surface, the net heat flux from the surface is

$$q''_{net} = \dot{E}''_{out} - E''_{in}$$

$$q''_{net} = q''_{conv} + E + G_{ref} - G = h(T_s - T_\infty) + \varepsilon\sigma T_s^4 + G_{ref} - G$$

$$\varepsilon = \alpha = 1 - \rho = 1 - (G_{ref} / G) = 1 - (800/1000) = 1 - 0.8 = 0.2$$

where $\rho = G_{ref}/G$**. the net heat flux from the surface**

$$q''_{net} = 15W/m^2.K(27-17)K + 0.2 \times 5.67 \times 10^{-8} W/m^2.K^4(27+273)^4 K^4 + 800W/m^2 - 1000W/m^2$$

$$q''_{net} = (150 + 91.9 + 800 - 1000)W/m^2 = 42W/m^2$$

Comments: (1) For this situation, the radiosity is

$$J = G_{ref} + E = (800 + 91.9)W/m^2 = 892W/m^2$$

The energy balance can be written involving the radiosity (radiation leaving the surface) and the irradiation (radiation to the surface).

$$q''_{net,out} = J - G + q''_{conv} = (892 - 1000 + 150) W/m^2 = 42 W/m^2$$

Note the need to assume the surface is diffuse, gray and opaque in order that Eq (2) is applicable.

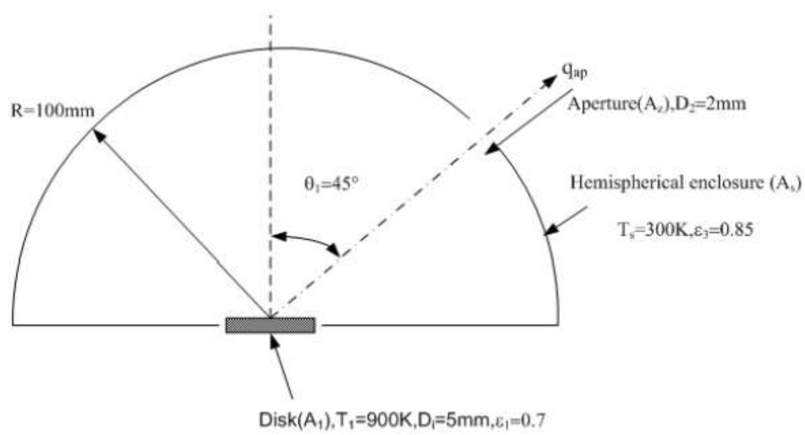
8. A small disk 5 mm in diameter is positioned at the center of an isothermal, hemispherical enclosure. The disk is diffuse and gray with an emissivity of 0.7 and is maintained at 900 K. The hemispherical enclosure, maintained at 300 K, has a radius of 100 mm and an emissivity of 0.85.

Calculate the radiant power leaving an aperture of diameter 2 mm located on the enclosure as shown.

Known: Small disk positioned at center of an isothermal, hemispherical enclosure with a small aperture.

Find: radiant power [μW] leaving the aperture.

Schematic:



Assumptions: (1)Disk is diffuse-gray,(2) Enclosure is isothermal and has area much larger than disk,(3) Aperture area is very small compared to enclosure area, (4) Areas of disk and aperture are small compared to radius squared of the enclosure.

Analysis: the radiant power leaving the aperture is due to radiation leaving the disk and to irradiation on the aperture from the enclosure. That is

$$q_{ap} = q_{1\to 2} + G_2 . A_2$$

The radiation leaving the disk can be written in terms of the radiosity of the disk. For the diffuse disk

$$q_{1\to 2} = \frac{1}{\pi} J_1 . A_1 \cos\theta_1 . w_{2-1}$$

and with $\varepsilon = \alpha$ for the gray behavior, the radiosity is

$$\mathrm{J}_1 = \varepsilon_1 E_b(T_1) + \rho G_1 = \varepsilon_1 \sigma T_1^4 + (1-\varepsilon_1)\sigma T_3^4$$

Where the irradiatin G_1 is the emissive power of the black enclosure, $\mathrm{E}_b(\mathrm{T}_3)$;

$\mathrm{G}_1 = \mathrm{G}_2 = \mathrm{E}_b(\mathrm{T}_3)$. The solid angle $\omega_{2\text{-}1}$ follows

$$\omega_{2-1} = A_2 / R^2$$

Combining equations. (2), (3) and (4) into eq.(1) with G2=σT^4_3, the radiant power is

$$q_{ap} = \frac{1}{\pi}\sigma[\varepsilon_1 T_1^4 + (1-\varepsilon_1)T_3^4]A_1 \cos\theta_1 . \frac{A_2}{R^2} + A_2 \sigma T_3^4$$

$$q_{ap} = \frac{1}{\pi} 5.67\times 10^{-8} W/m^2.K^4 [0.7(900K)^4 + (1-0.7)(300K)^4 \frac{\pi}{4}(0.005m)^2 \cos 45^\circ \times$$

$$\frac{\pi/4(0.002m)^2}{(0.100m)^2} + \frac{\pi}{4}(0.002m)25.67\times 10^{-8} W/m^2.K^4 (300K)^4$$

$$q_{ap} = (36.2 + 0.19 + 1443)\mu W = 1479\mu W$$